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The Einstein-Podolsky-Rosen function for infinitely separated particles in a classical context

Abstract

This paper shows that the Einstein-Podolsky-Rosen function, for two quantum particles of equal mass moving in 3D space, moving away from each other until they approach the quantum-classical boundary, verifies a classical equation.

Keywords: Einstein-Podolsky-Rosen function, general schrodinger equation, quantum classical boundary

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J D Bulnes,[1] M A I Travassos,[1] J. López-Bonilla[2]

[1]Department of Exact Sciences and Technology, Federal University of Amapá, Brazil
[2]ESIME-Zacatenco, National Polytechnic Institute, Mexico

Correspondence: J D Bulnes, Department of Exact Sciences and Technology, Federal University of Amapá (UNIFAP), Rod. J. Kubitschek, 68903-419, Macap'a, AP, Brazil

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Introduction

Ninety years ago, in the article,[1] co-authored by A. Einstein, B. Podolsky, and N. Rosen, a theoretical framework was defined for discussing the (incomplete, according to EPR) nature of the quantum mechanical model,[i] this discussion was later supplemented by Einstein himself,[5] inaugurating and generating a broad literature.[ii] It is known that Prof. Einstein offered several arguments to support his view that quantum mechanics was no more than a provisional theory, regardless of whether it produced correct results. Essentially, the intrinsically probabilistic nature of quantum mechanics' predictions was not compatible with Prof. Einstein's vision of a correct physical description of nature.

In article,[1] there is an essential element within the general considerations and developments presented by its authors: a function for a physical system consisting of two quantum particles, *I* and *II*, which had previously interacted but subsequently stopped interacting. This function, which will be the focus of this article, is given by expression (7) of,[1] which we rewrite below:

$$\Psi(x_1,x_2)=\sum_n \psi_n(x_2)u_n(x_1) \qquad (1)$$

Where $u_1(x_1), u_2(x_1), \ldots$ represents the set of eigenfunctions corresponding to an *arbitrary observable A*, defined for subsystem *I*, with x_1 representing the variable used to describe, precisely, the functions of this first subsystem. The set of functions $\psi_1(x_2)$, $\psi_2(x_2), \ldots$ corresponds to subsystem *II*, with x_2 referring to the variable used to describe the second subsystem.

Note that, by writing the total state (1) as an expansion in the base elements of an observable of system *I*, as stated in,[1] one is assuming that the Hilbert space for the complete system is the same as for system *I*. In,[1] it is asserted that its function (7), or function (1) above, represents the state of the combined system for any instant in time; in particular, for an instant after the interval in which the particles interacted.

Some recent results

From the point of view of identifying the manifestations of a physical world that presents several sectors, one of which is described classically, as well as another, distinct from the first, which is described quantumly, there is the problem, not yet completely resolved, of describing the "boundary" between these sectors. Consider the transition from the quantum to the classical context; then, regardless of whether the function that define the state of a quantum system is of the separable or inseparable type, the "collapse" of its quantum description would be inevitable.[iii] For two quantum particles moving in opposite directions, in a vacuum, and not only separate from each other, but separate until they reach the classical quantum boundary; that is, until they reach a separation of macroscopic dimensions, such a collapse would occur. Thus, functions corresponding to this specific situation could not verify the Schroedinger equation, but a classical equation, which one?

On the other hand, it has been shown recently[11] that for an analytic function (for two quantum particles moving in vacuum, in opposite directions, in a 1-dimensional space), which results from an extension of the usual Schroedingeransatz,[11] as follows:

$$\Psi_1(x_1,x_2,t)=C(x_1-x_2)\,exp\{(i/\hbar)\,S(x_1,x_2,t)\}$$

S being the action function for two particles with the same mass parameter, *m*, and *C* a decreasing exponential function, the function collapses ψ_1 as a result of which the function *C* vanishes at the quantum-classical boundary; that is, when $(x_1-x_2)\to\infty$. It was shown in[11] that the function *C* verifies a quantum equation and that the function ψ_1 verifies, "surprisingly", a classical equation:

$$\frac{\partial^2\Psi_1}{\partial x_1^2}+\frac{\partial^2\Psi_1}{\partial x_2^2}=0$$

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[i] Among the most recent works addressing aspects of quantum mechanics are the articles.[2-4]

[ii] Some recent examples of aspects of this famous work are the articles.[6-9]

[iii] Let us consider the following simplified example. In the case of giant atoms, or Rydberg atoms,[10] it is possible to make a classical description from the quantum context for the state of the outermost electron of a giant atom, which may be a few millimeters in classical diameter, as the proper superposition of many electronic states, with principal quantum numbers n verifying $n > 100$, producing a state approximately similar in shape to a Dirac delta, thus corresponding to the *localization* of the electron, as expected in a classical description of a particle.

In,[11] when evaluating the mathematical fact that ψ_1 is a quantum function that verifies a classical equation, it was understood that this can be explained by assuming that in the context of approaching the quantum-classical boundary the time variable would be irrelevant if the collapse of the quantum description would happen regardless of how quickly the quantum-classical boundary is reached, which seems reasonable. In this case, the time derivative term in the general Schrödinger equation could be ignored, resulting in the classical equation above. This correspondence is not surprising, since the same Schrödinger equation was originally derived[iv] from a classical equation, the Hamilton-Jacobi equation. Thus, we see that there is a suitable context, the approximation to the quantum-classical boundary, in which the Schrödinger equation "becomes" a classical equation. Returning to the function of Einstein, Podolsky, and Rosen, function (1), we want to determine which equation it verifies in the context of the approximation to the quantum-classical boundary. To do this, we will show that ψ, in (1) an equation like (3) could not be verified, with $g(\vec{x}_1,\vec{x}_2)\neq 0$. Let's see this.

Mathematical development

Let's consider the three-dimensional case, that is, in which $\vec{x}_1$ represents a set of three coordinate variables *x, y, z* for subsystem *I*. Let's assume, for simplicity, that the mass parameters of particles (subsystems) *I* and *II* are equal. The general Schrdinger equation for two free particles, with the same mass parameter *m*, is written as:

$$\frac{-\hbar^2}{2m}\nabla_1^2\Psi+\frac{-\hbar^2}{2m}\nabla_2^2\Psi=i\hbar\frac{\partial}{\partial t}\Psi \qquad (2)$$

Function (1), which does not contain the time variable t, correctly represents, as EPR claims, the state of the combined system at any instant $t>T$, with T being the instant at which the particles ceased to interact. The function ψ, since it does not present time dependence, would be expected to verify equation (2) without the time derivative term $(\partial\psi/\partial t=0)$. However, to verify or rule out the possibility that it verifies a different equation, let us consider equation[v]

$$\frac{-\hbar^2}{2m}\nabla_1^2\Psi+\frac{-\hbar^2}{2m}\nabla_2^2\Psi=g(\vec{x}_1,\vec{x}_2) \qquad (3)$$

which also does not present time dependence. The value of the function g in (3) will be defined by general requirements consistent with quantum mechanics and by logical consistency arguments. After substituting (1) into (3) we have,

$$-\frac{\hbar^2}{2m}\sum_n\psi_n(\vec{x}_2)\nabla_1^2u_n(\vec{x}_1)-\frac{\hbar^2}{2m}\sum_n u_n(\vec{x}_1)\nabla_2^2\,\psi_n(\vec{x}_2)=g(\vec{x}_1,\vec{x}_2). \qquad (4)$$

Equation (4) can be integrated with respect to the variables $\vec{x}_1$. Doing so, we obtain the expression

$$-\frac{\hbar^2}{2m}\sum_n\psi_n(\vec{x}_2)\int d^3\vec{x}_1\nabla_1^2u_n(\vec{x}_1)-\frac{\hbar^2}{2m}\sum_n\nabla_2^2\,\psi_n(\vec{x}_2)\int d^3\vec{x}_1u_n(\vec{x}_1)=\int d^3\vec{x}_1\;g(\vec{x}_1,\vec{x}_2).$$

(5)

Next, let us consider Green's identity,

$$\int d^3x\;\varphi\nabla^2\vartheta=\oint d\vec{s}.\,\varphi\nabla\vartheta-\int d^3x\;\nabla\varphi.\nabla\vartheta \qquad (6)$$

in which the integrals appearing at the extremes of expression (6) must be evaluated over a region of space (which occupies a certain volume) and the integral at the center must be evaluated over the surface that bounds the considered volume. For convenience, it is considered that φ is a constant function, the simplest being $\varphi(\vec{x})=1$, $\forall\vec{x}\in D_\varphi$ and $\vartheta=u_n$; in this case, we can write,

$$\int d^3\vec{x}_1\;\nabla_1^2u_n=\int d\vec{s}\cdot\nabla_1u_n \qquad (7)$$

but the integral on the right vanishes if we consider a spherical surface of arbitrarily large radius due to the mathematical fact that the functions u_n must be of the type square integrable, as required by quantum mechanics for all states accessible to the considered system (subsystem *I*, in this case). Thus, we arrive at the following result:

$$-\frac{\hbar^2}{2m}\sum_n\nabla_2^2\psi_n(\vec{x}_2)\int d^3\vec{x}_1\;u_n(\vec{x}_1)=\int d^3\vec{x}_1\;g(\vec{x}_1,\vec{x}_2)=G(\vec{x}_2). \qquad (8)$$

It is clearly understood that the function g must be fixed in equation (3) and that in expression (8) the function g should not change depending on which observable A is considered to introduce the corresponding eigenfunctions.[vi] Thus, to express in (8) this independence of the observable, it must be imposed that:

$$G(\vec{x}_2)=C_1 \qquad (9)$$

Where C_1 is a numerical constant? The second part of this development is analogous to the previous one. We perform a multiple integration of expression (4) with respect to the variables $\vec{x}_2$. The expression equivalent to (5), for this new case, is the following,

$$-\frac{\hbar^2}{2m}\sum_n\nabla_1^2u_n(\vec{x}_1)\int d^3\vec{x}_2\psi_n(\vec{x}_2)-\frac{\hbar^2}{2m}\sum_n u_n(\vec{x}_1)\int d^3\vec{x}_2\nabla_2^2\psi_n(\vec{x}_2)=\int d^3\vec{x}_2\;g(\vec{x}_1,\vec{x}_2)$$

(10)

The particular expression arising from Green's identity that we will use in this case is the following,

$$\int d^3\vec{x}_2\;\nabla_2^2\phi_n=\int d\vec{s}\cdot\nabla_2\phi_n \qquad (11)$$

Next, we are faced with the situation of requiring that the functions ϕ_n be all square-integrable; moreover, considering an arbitrarily large surface for the integral on the right-hand side of (11), we have that these integrals are zero; so, we write,

$$-\frac{\hbar^2}{2m}\sum_n\nabla_1^2u_n(\vec{x}_1)\int d^3\vec{x}_2\;\psi_n(\vec{x}_2)=\int d^3\vec{x}_2\;g(\vec{x}_1,\vec{x}_2)=H(\vec{x}_1) \qquad (12)$$

Here, by the same argument considered in the paragraph below expression (8), we necessarily arrive at the result: $H(\vec{x}_1)=C_2$ (13)

where C_2 is a constant, the same for any set of Eigen functions considered for system *I*.

[iv] It may be interesting to mention that only recently has it been shown that the transition from the Hamilton-Jacobi equation to the Schroedinger equation, and vice versa, which in the literature is performed approximately, can in fact be performed exactly, as shown in.[12]

[v] Expression (3) is merely formal; it is not a differential equation per se, since in this expression ψ is known and g is undefined. A partially similar situation occurs with the Euler-Lagrange differential equation of classical mechanics: knowing the Lagrangian of a particle and inserting it into the Euler-Lagrange equation, we obtain the physical trajectory. Here we see the great importance of the methods of mathematical physics, through which works such as will find application in more elaborate and formal problems related to the Schrödinger equation.[13-15]

[vi] The function ψ is the same regardless of the basis $\{u_n\}$ expressing it, but the integrals of the elements of any of these bases would not produce the same result in (8).

Discussion

From the above expressions it follows that the only possibility for both expressions (9) and (13) to be satisfied simultaneously is that the value $g(\vec{x}_1,\vec{x}_2)$ does not depend on either $\vec{x}_1$ or $\vec{x}_2$; in other words, that $g(\vec{x}_1,\vec{x}_2)$ is a constant. Furthermore, since in quantum mechanics the functions that describe quantum states are of the square-integrable type, thus corresponding to a context in which these functions vanish at "infinity"[vii] and based on the general hypothesis that quantum functions, as well as their derivatives, are continuous, we conclude that g can only assume a zero value, in other words $g(\vec{x}_1,\vec{x}_2)=0$, for all points in the domain of this function. Thus, from (3), we arrive at equation:

$$\frac{-\hbar^2}{2m}\nabla_1^2\Psi+\frac{-\hbar^2}{2m}\nabla_2^2\Psi=0\Rightarrow\nabla_1^2\Psi+\nabla_2^2\Psi=0 \quad (14)$$

Equation (14) corresponds to the generalization, for the 3-dimensional case of the result found in, in the 1-dimensional case, shown here in Section "Some recent results".

Equation (14) for the total function of two quantum particles with the same mass, moving away from each other until they reach the quantum-classical boundary, can be viewed (operationally) as arising from the general Schrödinger equation in the situation where the term with the time derivative has been suppressed, based on the argument that the time variable turns out to be irrelevant, as was assumed, here too, in the context of approaching the quantum-classical boundary.

Conclusion

It has been shown that the Einstein, Podolsky, Rosen function, for two quantum particles of equal mass, moving away from each other until they approach the quantum-classical boundary, which was characterized in the passages for expressions (8) and (12), verifies the classical equation given in (14), provisionally called (for lack of a better understanding) the classical Schrödinger equation.

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Conflicts of interest

The Authors declares that there are no conflict of interest.

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[vii] The definition of "infinity" depends on the specific physical context being considered.

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