Review Article

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Quantum-vacuum and dark-entities in the Universe: experimental test of the vacuum inhomogeneity

Abstract

A Test-experiment based on the Spontaneous Emission (SE) from optically excited Cr atoms in a ruby crystal is proposed. The system, to be carried into the skyes by a mission spacecraft, provides a unique and most direct measurement apparatus of the density ρ_{vac} of the inhomogeneous QED vacuum-field in the Universe. As such it may contribute to enlighten several puzzling cosmological processes as, for instance, the ones related to the very nature of QED vacuum, of dark-energy and of dark-matter. A measured large inhomogeneity of ρ_{vac} is expected to resolve the enigma of the cosmological constant and several aspects of the Hubble constant tension.

Keywords: Spontaneous Emission, QED, Dark energy, Dark matter, Hubble tension

Volume 9 Issue 2 - 2025

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Received: April 1, 2025 | **Published:** April 10, 2025

Introduction

According to several Authors[1,2] the vacuum energy remains the simplest model for dark energy and dark matter which could drive the accelerated expansion of the Universe. In particular, the concept of *homogeneous vacuum* has been adopted and carefully studied for a long time to account for several unsolved astrophysical problems as the small value of the measured *cosmological constant* and for the so-called *Hubble constant tension* where the size of the vacuum energy evaluated by quantum theory differs by more than 130 order of magnitude from the ρ_{vac} vacuum energy estimated with the cosmological observations.[3,4] In particular the homogeneous vacuum approach was adopted since the old debate between Albert Einstein and George Lemaitre in the years 1927-1933 involving in later years Willem De Sitter and Hermann Weyl.[3] However, it is well known that the Universe's itself is far from being homogeneous and then one can suppose that at least a partial resolution of the mentioned universe's enigmas can be found by a careful theoretical and experimental consideration of the more complex problem of *inhomogeneous* vacuum, the one that is necessarily interacting in General Relativity (GR).[2]

In general it is possible to give a consistent description of vacuum dynamics and in particular of the relativistic equations of motion for inhomogeneous perturbations given a covariant prescription for the vacuum energy and then construct gauge-invariant vacuum perturbations.[2] However, all dynamical theories on this subject face the so far unsolved problem of knowing the real size of the inhomogeneities of vacuum field with respect to his homogeneous counterpart, at least within the relevant context of Quantum Electrodynamics (QED). The present paper is intended to contribute to the concrete resolution of the problem by presenting the detailed project of a reliable experiment, based on a well-known fundamental process of atomic physics: the Spontaneous Emission (SE) of photons, a process that, according to QED, is directly "stimulated" by the Electro-Magnetic (EM) vacuum field.[5] In Chapter 2 the theory of the homogeneous QED vacuum field on the basis of the concepts of "modes" and of the corresponding quantum "harmonic-oscillators" in the Minkowski space is outlined ending with the standard calculation of the "cosmological constant" Λ. In Chapter 3 several comments are devoted to the promotion of the above concepts in the curved space of GR. A simple theory is also given of the long range gravitational forces acting on the vacuum field close to a massive astronomical body.[6] In Chapter 4 a detailed account of the Test (SE) experiment involving Chromium atoms in a ruby crystal is given. There the conceptual foundations lying at the basis of the proposed test-experiment and then his actual reliability in spacetime applications are discussed and demonstrated on the basis of an extended series of published laboratory experiments on atomic SE. Concluding remarks about the *cosmological-constant* problem and the possible interesting SE processes in the cepheid stars and in black-holes are the subjects of Chapter 5.

Homogeneous QED vacuum field

The structure of the QED homogeneous vacuum in a flat Minkowski spacetime is identified by the geometrical concept of normal "mode", identified by the vector **k** and by each one of the two orthogonal polarizations of the transverse (EM) field. In a quantum mechanical context each mode **k** corresponds to a "*quantum harmonic oscillator*" characterized by an oscillator *frequency* ω_k, with a dynamics imposed by the properties of the quantum Bose operators, i.e. the photon "creation" ($\hat{a}_k^\dagger$) and "annihilation" ($\hat{a}_k$) operators, acting in the Fock space.[5] The eigenvalue of the operator ($\hat{a}_k^\dagger\hat{a}_k$) in that space is the number of photons ($\check{n}_k$) in the k-mode. The energy of each oscillator is: $\mathcal{E}_k=[\check{n}_k\hbar\omega_k+ ½\hbar\omega_k]$. Assuming the absence of photons in all modes, $\check{n}_k=0$, the energy of the field is the "*zero-point vacuum-energy*": $\mathcal{E}_{vac} = [\Sigma_k ½\hbar\omega_k]$ where the sum is over all k-modes and the two field polarizations. Because of the massive *vacuum-energy* all k-modes are *gravitationally active* even in absence of photons: this is a counterintuitive non-classical property of the vacuum field. In the context of the *homogeneous vacuum*, the total number of such modes in the Universe may be calculated on the basis of standard QED. In a frequency interval dω and in a volume V, the number of modes is:

$$dn_k = [\omega^2 d\omega/(\pi^2c^3)]\times V \equiv d\eta_k \times V \quad (1)$$

where η_k is the *homogeneous* density of states of photon k-modes. By integration respect to frequencies between: ω=0 and: $\tilde{\omega}=(2\pi c/Ł)$, the total number of harmonic-oscillators in the Universe with volume V is obtained: $n_T = \Sigma_k\eta_k\times V = [\tilde{\omega}^3/(3\ \pi^2c^3)]\times V$. In the context of cosmology the limit length Ł is generally identified with the *Planck length*: $L_p \equiv (\hbar G/c^3)^{½}=1.161\times10^{-33}$ cm, being $m_p \equiv \hbar(cL_p)^{-1}$ the *Planck mass* and: $G \equiv \hbar c(m_p)^{-2}= 6.673\times10^{-8}$ cm^{3}g^{-1}sec^{-2} the *gravitational constant*. The corresponding overall "*zero-point vacuum-energy*" density ρ_{vac} in the k-mode *homogeneity* condition is obtained by a ω-integration of the function: ($\frac{1}{2}\hbar\omega dn_k$). The condition: $Ł = L_p$ leads to the value of the total vacuum energy in the Universe: $\mathcal{E}_{vac} = 8.6\times10^{125}$ (eV/cm^3) × V(cm^3). The "weight" of 1 cm^3 of vacuum, the size of a little stone, is: $m_{vac} \equiv (\rho_{vac}/c^2) = 1{,}52 \times 10^{90}$ Kg. The enormous value of the "zero-point vacuum-energy density" ρ_{vac} evaluated by QED is to be compared with the recently *measured cosmological constant*: $\Lambda \equiv (8\pi G/c^4)\times\rho_{vac}$ proportional to the very small value: $\rho_{vac} \approx 10^{-3}$(eV/cm^3). These numbers express the well-known disastrous failure of the "*naturalness*" of the modern Quantum Field Theory (QFT), i.e. the: "Λ-puzzle".[7] In the context of GR it may also be defined a vacuum energy density having an energy-momentum tensor:$(T^\mu{}_\nu)_{vac} = [pg^\mu{}_\nu - (\rho+p)\, u^\mu u_\nu]_{vac}$ proportional to the metric and with $(\rho+p)_{vac} = 0$, i.e. implying a *negative vacuum-pressure*: $p_{vac} = -\rho_{vac}$.[2,8] In any case the simple addition of the cosmological term ($g_{\mu\nu}\Lambda$) within the Einstein ΛCDM equation appears to imply a simple and useful conception of the zero-point vacuum field in the Universe interpreted as a "homogeneous" fluid. However, the Universe appears to be a quite complex inhomogeneous entity made at least of thousand billions of stars, galaxies, black-holes etc. We believe that many cosmological puzzles can perhaps be understood and resolved by consideration of the basic *inhomogeneity* of the vacuum field: an endeavour that has already been tackled, albeit only theoretically, by the recent scientific literature.[1-4] In the present paper only the QED vacuum is considered as the dominant vacuum. However the "overall vacuum" filling the Universe is also contributed by all fields of the Standard Model, e.g. the Higgs field,[9,10] the scalar Weyl field etc.[8]

In writing the present paper a fundamental problem, both semantic and linguistic, was raised by the spacetime curvature due to GR gravitation. In the Minkowski flat space there is a natural set of k-modes, the ones introduced in this Chapter, that are closely associated with the rectangular coordinate system (x, y, z, t).[5] These natural coordinates are associated with the Poincaré group, the action of which leaves the Minkowski line element unchanged. In curved spacetime the Poincaré group is no longer a symmetry group, in general there will be no Killing vectors to define positive frequency modes and a vacuum state is not uniquely defined. For instance an accelerated detector in flat space would detect Unruh radiation while an inertial detector sees vacuum. This is indeed an argument of large relevance in the very context of the present proposal.[3,6,11] Within the GR context the construction of a quantum vacuum state or of a complete set of modes becomes a very complex theoretical problem that appears not solvable in general.[6] With this proviso, in the present paper we shall continue adopting the usual k-mode language as an inevitable approximation, referred to as "*Minkowski approximation*", by keeping in mind the real physical aspect of our present problem, i.e. the measurement of the distribution in spacetime of the vacuum energy density.

Inhomogeneous QED vacuum field. The "dark" entities

In the Universe the gravitation process pervasively affects every cosmological structure carrying energy/mass including the zero-point vacuum energy contributed by all the quantum fields. In the present paper we propose the conjecture according to which large amounts of "quantum harmonic-oscillators", i.e. k-modes carrying energy/mass, can move in space being gravitational attracted by the massive structure of the stars, galaxies and galaxy clusters. Indeed, according to GR any ray of light follows a *null-geodesic* and the "ray-bending process" i.e. the deflection of light-rays in presence of a mass, is *independent* of the light intensity i.e. of the corresponding photon number.[3,11] This implies a general gravitational-geometrical process which also acts in the zero-photon condition, i.e. in the *vacuum*. Indeed, according to quantum mechanics it is possible approximately to say that the "light-ray", i.e. a coherent superposition of k-modes, does not disappear when no photons are present: it is invisible but can be somewhat "deflected" because of the intrinsic gravitational activity of all k-modes. This implies that the density of the vacuum quantum harmonic-oscillators locally increases in the vicinity of the large masses in the Universe and diminishes elsewhere: there the vacuum should become highly *inhomogeneous*.

Within the assumed "*Minkowski approximation"* we shall represent formally this local k-mode accumulation/rarefaction process by multiplying the k-mode *homogeneous* density η_k defined by Eq. (1) by a local "*homogeneity parameter*", a real number: $\xi \equiv \xi(X) \geq 0$, where X represents the coordinates of a point P in spacetime: $\tilde{\eta}_k(X) \equiv \eta_k \times \xi(X)$. Furthermore, $\zeta(X) \equiv \log_e \xi(X)$ is zero for the *homogeneous* distribution and positive/ negative for mode accumulation/rarefaction, i.e. for the *inhomogeneous* distribution.

If the simple statements just expressed, implying gravity-induced delocalization in the spacetime of all vacuum fields is truthful, the enigma implied by the real nature of the "*dark matter"* and "*dark energy"* appears to be resolved. The large, massive, invisible and transparent "halos" of unseen non-barionic mass surrounding the galaxies – e.g. detected by gravitational lensing effects – may at least partially consist of high density "zero-point vacuum energy" which, by the very definition of "vacuum", is devoid of quantum particles.[12,13] Since every quantum detector, of any kind – including our own eyes – is only excited via the annihilation of the quantum particles of the field under measurement, these "halos" of dense dark mass/energy surrounding the galaxies are necessarily "transparent" to any act of measurement on that field. In other words, the *dark energy* and the *dark matter* are dark simply because all the quantum detectors available for measurement are necessarily blind. However, since the "dark" fields may be gravitationally active, it is possible that they could be investigated by the methods of advanced Gravitational Wave (GW) spectroscopy. Furthermore, it is highly reasonable to think that the invisible massive, inhomogeneous halos of "zero-point vacuum energy" surrounding the galaxies actively contribute to their internal dynamical stability.[8] The above propositions implying a somewhat unusual, counterintuitive process of QED vacuum-energy attraction by massive, extended astronomical bodies and the consequent vacuum field inhomogeneity, may be understood by the following simple model calculation.Assume, within a flat Minkowski spacetime, a massive, plane, infinitely extended material slab with mass-density (ρ) and thickness (z). A small (1 cm^3) volume of *zero-point vacuum-energy* with mass $m_{vac} \equiv [(\rho_{vac}/c^2)\times 1]$ is placed at the spacetime point (P) at a distance (đ) from the slab. Assume: đ >> z. A geometrical point O on the slab, the closest point to P, is the origin of a linear cartesian coordinate (r) drawn on the slab surface: in other words, the segment OP with length đ is orthogonal to the plane of the slab. Calculate the gravitational force acting on the mass m_{vac} by r-integration from r = 0 to r = ∞ of the expression: $[2\pi G\times m_{vac}\times z\times\rho\times đ\times(r^2 + đ^2)^{-3/2}\times r dr]$. The result is an attractive Newton force-vector orthogonal to the slab, acting on m_{vac} with modulus: $|F| = 2\pi G[z\times\rho\times m_{vac}]$. Note that $|F|$ is proportional to the slab thickness and to the slab mass density, as expected. But, more interesting, being independent of the distance đ, **F** is a *long-range* force capable of acting on distant spatial portions of the vacuum field.When translated into the cosmological context this calculation shows a plausible origin of the "halos" of vacuum-field circling the galaxies and then of the expected spatial inhomogeneities of the vacuum field.

Citation: Martini FD. Quantum-vacuum and dark-entities in the Universe: experimental test of the vacuum inhomogeneity . *Phys Astron Int J.* 2025;9(2):64–67. DOI: 10.15406/paij.2025.09.00365

In summary, because of the intrinsic structural inhomogeneity of the Universe and of the dynamical gravitational effects of the Universe's structure on all quantum fields, all statements above imply a general *inhomogeneity* of all the "zero-point vacuum fields". Nevertheless, as previously stressed, these ones are invisible to the human observer and to his quantum detectors, and may remain a source of mysteries and enigmas. In order to resolve them and to understand the corresponding scientific implications, the direct knowledge of the dynamics of the inhomogeneous vacuum field in the spacetime should be an important and necessary scientific endeavour of modern cosmology. The Test-experiment to be described in the next Chapter could represent a unique, concrete and radical resolution of the problem. The experiment consists of the detection of the Spontaneous Emission (SE) of photons emitted by atoms interacting with the set of EM k-modes, i.e. with the related quantum-oscillators, $\tilde{\eta}_k(X)$ in different positions P in spacetime. The probability of finding an atom in a ground state (*g*) at time t given that it was in the excited state (*e*) at t=0 is given by the standard quantum theory of the interaction (Eq.2):[14]

$$\left|c(g,t;e,o)\right|^2 = \alpha\left[\frac{2\pi c}{3e^2}(\tilde{\omega})^2\right]|\mu|^2\int_0^\infty d\,\tilde{\eta}_k \times \frac{4\sin^2\left(\frac{1}{2}(\omega-\tilde{\omega})^2 t\right)}{\omega(\omega-\tilde{\omega})^2} \quad (2)$$

being: $\alpha=[e^2/(4\pi\varepsilon_o\hbar c)]\approx(1/137)$ the *fine-structure* constant, e the electron charge, $\tilde{\omega}\equiv\omega_{eg}$ the atom's resonant SE frequency, $\boldsymbol{\mu}_{eg}\equiv\left|X_{eg}\right|^2$ the atomic dipole matrix element between the ground and excited states involved in SE and: $\tilde{\eta}_k\equiv\eta_k\times\xi(X)$. The integral in Eq. 2 shows that the probability of the SE process is indeed proportional to the density $\tilde{\eta}_k$ of the EM field's k-modes "*filtered"* by the narrow atomic resonance at frequency $\tilde{\omega}$ expressed by the highly peaked resonant function appearing in the integral. Some further straightforward calculations lead to the final SE expression probability per second at the point P(X) over the solid angle $\Omega = 4\pi$ (Eq.3):[14]

$$P_{SE}(X) = d/dt\left|c(g,t;e,o;X)\right|^2 = \omega_{eg}{}^3\left|\mu_{eg}\right|^2/(3\pi\hbar\varepsilon_0 c^3)\times\xi(X) = (4\alpha\tilde{\omega}^3\left|X_{eg}\right|^2/3c^2)\times\xi(X) \quad (3)$$

Equations (2), (3) shows that the spontaneous emission rate is indeed proportional to the *fine-structure constant* and to the square of the dipole matrix element multiplied with the *filtered* density of states of photon modes. The filtering process due to the atomic resonance does not spoil the information on the density of EM k-modes supplied by the proposed test-experiment. The Equations (1),(2),(3) with $\xi(X) = 1$, i.e. $\zeta(X)=0$, implying the *homogeneity* condition are the results of the standard quantum EM field-atom interaction theory expressed by all standard books on QFT.[5,9,14] The "inhomogeneous" condition ($\xi(X) \neq 1$, i.e.: $\zeta(X) \neq 0$) applies to most of the content of present paper.

Experiment: Atomic spontaneous emission

The test-experiment must be carried in the skyes by a spacecraft. The data, consisting merely of the size of electronic pulses emitted by a photomultiplier, should be radio-transmitted back to the earth. In the experiment (Figure 1) a small number N of Cr Atoms in a ruby crystal (0,05% doping by chromium atoms within a synthetic sapphire matrix of Al_2O_3) from atomic level E_1 to E_3 are excited by focusing on a tiny ruby disk a pumping light flash, with small duration:dt<5×10[8] sec = 50ns, of incoherent light (or coherent light, e.g. by the 2[nd] harmonics of a mode-locked or Q-switched Nd-Yag laser) emitted by a device (SP) in a sizable frequency range centered on the wavelength: $\lambda_p\approx$ 5500 Å. Within the pumping process a glass optical filter (IF_1) is used to transmit the pump radiation at ~ λ_p wavelength and to block efficiently the SE radiation with at: λ_{SE} = 6943 Å. The atomic excitation is rapidly (≈1 ns) transferred within the crystal to the metastable level E_2. The exponential Spontaneous Emission (SE) decay of each ruby atom from E_2 to the ground-state E_1 follows, with a wavelength $\lambda_{SE} =(2\pi c/\tilde{\omega})$=6943 Å and a very long SE lifetime (on the Earth): $\tau_{SE} \approx 3\times10^{-3}$ sec = 3ms. The experiment is carried out at a low level of pump excitation (microjoules) in order to avoid any self-stimulated-emission, i.e. a ruby-laser operation. Since, according to QED the SE emission probability (P_{SE}) is proportional to the EM vacuum k-mode density $\tilde{\eta}_k$, the decay SE time is: $\tau_{SE}= C\times(\xi)^{-1}$ and the peak size of the electronic pulse (I_{SE}) realized at the output of the photomultiplier (PM) detecting the λ_{SE} radiation is proportional to $N\times\xi$. In front of the PM the λ_{SE} radiation is efficiently transmitted, and the pump with $\lambda_p\approx$ 5500 Å very efficiently rejected by a narrowband optical interference filter (IF_2). Each electronic pulse realized at the PM output is easily time-integrated to get a reference pulse with peak size (I_N) proportional to N. Then, the ratio: $I_{vac}(X)= (I_{SE}/I_N)$ is proportional *only* to the value of $\tilde{\eta}_k(X)$ measured at the *spacetime location* (X) where each SE experiment is carried out: precisely, X is the point of origin of SE within the ruby crystal. A nice feature of the experiment is that the output $I_{vac}(X)$ is independent of the pump intensity, of the number N of excited atoms and, at least in principle, of the signal attenuation implied by the data transmission. The in-flight experiment is repeated, under remote control, at successive spacetime locations X's. The result is a *map* of the distribution of $\tilde{\eta}_k(X)$ in the Universe and then of the local value of $\rho_{vac}(X)$ since this quantity, arising from the ω-integration of the function ($\frac{1}{2}\hbar\omega d\tilde{\eta}_k$), is also proportional to $\xi(X)$.

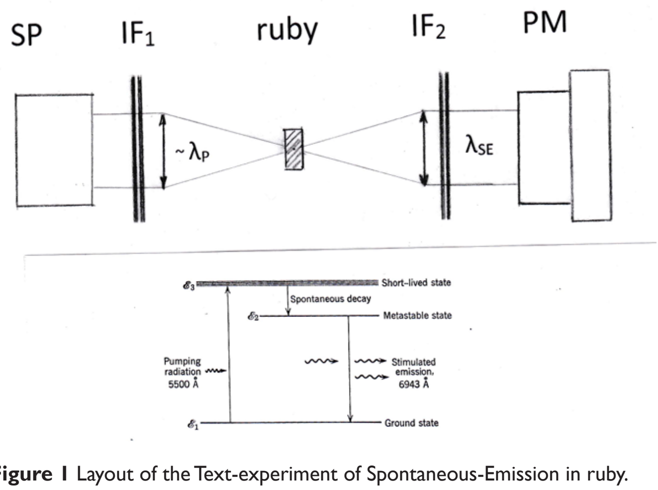

Figure 1 Layout of the Text-experiment of Spontaneous-Emission in ruby.

The consistency of the actual experimental SE results with the QED theory in "critical" boundary conditions, and therefore the actual *reliability* of the present proposal, have been thoroughly tested by an extended theoretical and experimental search carried out over many years at the Quantum Optics and Quantum Computation Laboratory at the Università "La Sapienza" in Roma.[15-20] In all these experiments an optical "microcavity", sometimes dubbed as "Casimir-microcavity" (because of the QED Casimir-effect)[5] was instrumental in the process of realizing the local controlled change of the EM vacuum-field density $\tilde{\eta}_k$ in the presence of atoms EM excited in local controlled space locations. This controlled double-superposition of space-

DOI: 10.15406/paij.2025.09.00365

localized $\tilde{\eta}_k$ EM-density with space-localized EM-excited-atoms was carried out within the Casimir microcavity geometry consisting of two extended, parallel plane optical mirrors placed at a mutual distance (d) of the order of the molecular SE wave-length, e.g.: $d < ½ \lambda_{SE}$, $d = ½ \lambda_{SE}$, $d = \lambda_{SE}$, $d >> \lambda_{SE}$ etc. etc. At the end many interesting results were experimentally attained, e.g. the SE lifetime as a linear function of the inverse of the vacuum EM density: $\tau_{SE}(\xi^{-1})$,[15] the (first) zero-threshold Laser action,[16] the Raman SE probability as a linear function of ξ etc.[19,20] To summarize, the positive results of all the above experiments are now at the basis of our confidence about the soundness, usefulness and timeliness of the present proposal.

Cosmological constant. Conclusion

Let us suppose that by the proposed experiment a very large inhomogeneity of $\tilde{\eta}_k$ is found. This could happen if the SE measurement spacetime point X_1 is close e.g. to a neutron star or to a blackhole, and another point X_2 is close e.g. to the Earth. Then: $\rho_{vac}(X_1) >> \rho_{vac}(X_2)$ and: $\Lambda(X_1) >> \Lambda(X_2)$. In that case the very conceptual and dynamical significance of the *cosmological-constant* appears to be lost together with the global, universal character of the Einstein ΛCDM theory.[3] Then possibly a "Λ-average" property should be introduced in the theory. In general, if a large inhomogeneity of $\Lambda(X)$ shall be found several enigmas related to the "Λ-puzzle" and to the *Hubble constant tension* could possibly be understood and resolved, at least in principle.[4] Within an assembly of atoms or molecules excited by a stationary and constant external source an increase of SE atomic emission probability, P_{SE} and then a reduction of the SE lifetime τ_{SE} proportional to $(P_{SE})^{-1}$, results in a larger average fluorescence light emission because of the ensuing faster dynamical recycling of the overall atomic excitation. On the other hand, according to QED the probability P_{SE} is proportional to ρ_{vac} and ρ_{vac} is increased by any local increase of the gravitational field because of an ensuing gravity-induced delocalization of the inhomogeneous QED vacuum. The conjecture expressed in the present paper may then directly relate the luminosity of any cosmological object to the intensity of the local gravitational field. This may perhaps partially explain the periodic change of light emission from the Cepheid stars corresponding to the periodic change of size of the stars: a smaller size corresponding to a larger local gravitational field. This process could also explain the very brilliant circular fluorescent halos appearing, in a context of extreme gravity, around the black-holes reported by the two beautiful experimental pictures recently obtained by the Event Horizon Telescope (EHT).

Because of the simple and fundamental QED process lying at the conceptual basis of the apparatus reported in the paper, we believe that the present proposal is a reliable and efficient contribution to cosmology and to the physics of the quantum fields. We acknowledge enlightening discussions with: Ersilia Vaudo, Hans-Thomas Elze and Carlo Rovelli.

Conflict of interest

None.

Funding

No funding was necessary for the implementation of the paper. The Author was not supported by any Grant.

Data availability

All data are contained within the paper.

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