Review Article

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Conformal scalar field as source of cosmological inflation

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Abstract

The article claims that the nature of the scalar field responsible for cosmological inflation is rooted in the most fundamental concepts of Hermann Weyl's Conformal Differential Geometry. Within a cosmological context the Euler-Lagrange theory based on a scalar-tensor Lagrangian, similar to the one already adopted by C.Brans and R.H.Dicke, leads via an extended and complete variational procedure, to a complete Einstein equation admitting an energy-momentum tensor accounting for the essential geometrical background as a source. In the article, the integrable Weyl theory applied to the dynamics of a relativistic fluid shows a hitherto never explained "*negative pressure*" condition responsible for the scale-acceleration of the dynamical expansion of the Universe. As a significant example, the case of a spherically symmetrical star filled with a fluid of constant mass density is also discussed. The dynamics of the curvature constructed over the differentials of this geometrical scalar field, here identified by "*scalar Weyl potential*" $\phi(x)$ is found to account for various critical aspects of the overall quantum phenomenology. A model for dark matter and dark energy, based on the sound hypothesis of a gravity induced de-localization of the zero-point vacuum energy is also presented and discussed in the article, showing the possibility of detection of the dark-fields by advanced Gravitational Wave (GW) techniques. An extension of the conformal theory to relevant local and nonlocal quantum processes is also briefly included. Finally, an extended comment by Gerhard t'Hooft about the absolute relevance of the Conformal Symmetry, as a "missing symmetry component for space-time", is reproduced and discussed in the context of our theoretical perspective and of the inner significance of the methods and results expressed in the article.

The present article claims that the fundamental nature of the scalar field responsible for cosmological inflation[1-3] as well for the "dark phenomena" - dark energy and dark matter - may be *geometrical*, i.e. based on the Conformal differential geometry introduced by Hermann Weyl two years after the publication of the first Einstein paper on General Relativity (GR).[7,8] Weyl's theory rests on the following general statements: "*All Laws of Physics are invariant under*" (A) "*any change of coordinates including time*" and (B) "*any change of calibration*". The first part, (A) expresses the well known "*covariance*" of the GR based on Riemann geometry.[4-6] The second part (B) expresses the "*conformal covariance*" or "*co-covariance*" of metric theory. Accordingly, the metric structure of the geometry implies two fundamental forms: the quadratic Riemannian one, accounting for Einstein's "gravitation": $g_{ij}dx^i dx^j$, where g_{ij} is the metric tensor, and the linear one, accounting for the parallel displacement of vectors in curved space-time: $\phi_i dx^i$. In Riemann's geometry is $\phi_i = 0$. The parallel displacement is "*integrable*" iff a "scalar Weyl potential $\phi(x)$ exists such as $\phi_i(x) \equiv \left(\partial\phi(x)/\partial x^i\right)$, the "Weyl vector", both fields defined in the space-time spanned by the x^i coordinates.[9] The integrable formulation of the theory avoids the unphysical "second clock" argument criticized by Einstein.[7,12,13] On a more general perspective, the Weyl conformal theory was originally recognized to present implications with Quantum Mechanics (QM) as the "quantum potential" Q of the De Broglie-Bohm QM theory was found closely related formally to $\phi_i(x)$ and to $\phi(x)$. In the present study it is claimed that the scalar field accounting for the Dark Energy process and the Inflationary process is the "gauge field" $\phi(x)$ filling the entire universe and propagating with the velocity of light.[7,10] The Weyl geometry, that is an abelian local scale-invariance gauge theory implies the following group of transformations:

$$g_{ij} \to \lambda g_{ij}, \phi_i \to \phi_i + \frac{\partial_i \ln\lambda}{2} \tag{1}$$

The first equation is the conformal change of the metric. The insightful perspective offered by the integrable theory was supported by P.A.M. Dirac in a 1973 seminal paper:[11] "*There is a strong reason in support of the Weyl's theory. It appears as one of the fundamental principles of nature is that equations expressing basic laws should be invariant under the widest possible group of transformations. The confidence that one feels in Einstein GR theory arises because its equations are invariant under a wide group of transformations of curvilinear coordinates in Riemann space. The passage to Weyl geometry is a further step in the direction of widening the group of transformations underlying the physical laws. One has to consider transformations [...] which impose stringent conditions on them*". All these concepts can be applied to a very general integrable Weyl-Dirac conformal scalar-tensor theory in the context of cosmology. It is well known that a most important result is the overall Riemann - Weyl curvature scalar: $R = \bar{R} + R_W$:,[7]

$$R = \bar{R} + R_W = \bar{R} - (n-1)(n-2)g^{ij}\phi_i\phi_j + 2(n-1)\frac{1}{\sqrt{-g}}\frac{\partial\left(\sqrt{-g}\phi^i\right)}{\partial x^i} \tag{2}$$

where $\bar{R}$ is the standard Riemann curvature due to Einstein gravitation and R_W is a contribution that is absent in Einstein's theory.[7,8] Equation (2) reproduces the standard structure of the D=4 scalar curvature within any "*conformal geometry*": Ref.8 (Ch.15). The dynamical field should be obtained by means of the variation of a convenient Lagrangian L. The general theory outlined here was considered in a previous article.[21] In the present work a simpler approach is considered which assumes at the start the field action:

$$I_o = \int d^4x\sqrt{-g}\left[\rho(\bar{R} + \frac{3}{2}\frac{\nabla_k\rho\nabla^k\rho}{\rho^2} - \frac{16\pi G}{c^4}L)\right] \tag{3}$$

for: D=4. The second term in (3) is the Lagrangian density of the scalar field $\rho(x)=\rho[\phi(x)]$ and L is the lagrangian density of matter here expressed by a relativistic fluid. It is assumed that L does not explicitly depend on ρ and on the derivatives of g^{ij}. The dimensionless $\phi(x)$ is a real-valued scalar field: and $\rho(x)$ is introduced through the Weyl potential:

$$\phi_i \equiv \nabla_i \phi(x) = -\frac{1}{2}\nabla_i(ln\rho) = -\frac{1}{2}\frac{\partial_i \rho}{\rho} \quad (4)$$

for D=4. Note that action (3) is not Weyl-gauge invariant but differs from a gauge invariant action by a boundary term, the integral of a divergence. Note also that I_0 is similar to the action adopted by C. Brans and R.H.Dicke.[14] Variation with respect to the field ρ yields:

$$\delta I_0 = \int d^4x\sqrt{-g}\left[\bar{R} + \frac{3}{2}\frac{\nabla_k\rho\nabla^k\rho}{\rho^2} - 3\frac{\nabla_k\nabla^k\rho}{\rho} - \frac{16\pi G}{c^4}L\right]\delta\rho = 0 \quad (5)$$

Variation with respect g^{ij} yields: $(\delta I_0) = (\delta I_0)_1 - (\delta I_0)_2 = 0$ where:

$$(\delta I_0)_1 = \int d^4x\sqrt{-g}\rho\left[\left(\bar{R}_{ij} - \frac{1}{2}g_{ij}\bar{R}\right) + \frac{3}{2}\left(\frac{\nabla_i\rho\nabla_j\rho}{\rho^2} - \frac{1}{2}g_{ij}\frac{\nabla_k\rho\nabla^k\rho}{\rho^2}\right) - \left(\frac{\nabla_i\nabla_j\rho}{\rho} + g_{ij}\frac{\nabla_k\nabla^k\rho}{\rho}\right)\right]\delta g^{ij}. \quad (6)$$

and:

$$(\delta I_0)_2 = \frac{8\pi G\rho}{c^4}\int d^4x\sqrt{-g}\left[\frac{2}{\sqrt{-g}}\frac{\partial\left(\sqrt{-g}L\right)}{\partial g_{ij}}\right]\delta g^{ij} = \frac{8\pi G\rho}{c^4}T^{ij}. \quad (7)$$

Finally the Einstein field equation may be expressed in the form:[8]

$$\left(\bar{R}_{ij} - \frac{1}{2}g_{ij}\bar{R}\right) + t_{ij} = \frac{8\pi G}{c^4}T_{ij} \quad (8)$$

where $\bar{R}_{ij}$ and $\bar{R}$ are the Ricci tensor and the curvature scalar of the Riemann geometry, respectively. The "geometrical" tensor t_{ij} arising from the variation (6) is:

$$t_{ij} = \xi^{-2}\left[\left(\partial_i\phi\partial_j\phi - \frac{1}{2}g_{ij}\partial_k\phi\partial^k\phi\right)\right] - \left[\frac{\nabla_i\nabla_j\rho}{\rho} - g_{ij}\frac{\nabla_k\nabla^k\rho}{\rho}\right] \quad (9)$$

The tensor t_{ij}, expressing the dynamics of the affine geometry in the context of conformal theory, physically represents the full action of the scalar field $\phi(x)$ on the dynamics of the universe. The "*Energy-Momentum*"tensor T_{ij} accounts for the "external" mass and radiation fields. In the case of a material medium composed of a relativistic fluid the energy-momentum tensor and scalar are:

$$T_{ij} = \left[-(\bar{\sigma}+\rho)u_iu_j + pg_{ij}\right] And: T \equiv T_{ij}g^{ij} = \left[-\bar{\sigma}+3p\right] \quad (10)$$

where $\bar{\sigma}$ and $p(x)$ are the energy density (with dimensions: $ML^{-1}T^{-2}$) and the pressure density of the fluid. In (9) the factor: $\xi^2 = \frac{n-2}{4(n-1)} = \frac{1}{6}$ for $D=4$, referred to as the "*conformal coupling constant*" was selected. As we can see, the structure of the Einstein field Eqs. (8,9) is somewhat surprising since it reproduces almost exactly the basic equations of the modern inflation theory reported by the standard texts on cosmology , including the celebrated text by C.Brans and R.H.Dicke $[B-D]$, where a tensor somewhat similar to t_{ij} is added "by hand" and "ad hoc" to the Einstein equation to represent artificially a modified matter model, e.g. a kind of *"quintessence"*.[20] In contrast, the expression of t_{ij} given by (9) is obtained by the exact formal application of the standard Euler-Lagrange variational procedures to the expression in Eq. (2) of the $D=4$ scalar curvature R_W which is absent in the standard Riemann's geometry. This leads to several consequences of dynamical relevance. For instance, because we are adopting, at variance with $[B-D]$, a "constant" gravitational parameter i.e. the "Gravitational Constant" G, the "strange terms" $(3+2\omega),(4+2\omega)etc.$ which impair the results of the $[B-D]$ theory with respect to some classical experimental GR tests (for example the Mercury perihelion rotation, the deflection of light, the gravitational red shift) do not show up in the present theory. A relevant expression appears in the Eq (9) as follows:

$$\frac{\nabla_k\nabla^k\rho}{\rho} \equiv \rho^{-1}\nabla_B\rho = 4\phi_k\phi^k - 2\nabla_B\phi \quad (11)$$

where ∇_B is the D'Alembertian (or Laplace-Beltrami) operator that, when applied to ϕ, is expressed as follows:

$$\nabla_B\phi \equiv -\frac{\partial^2\phi}{c^2\partial t^2} + \nabla^2\phi = \frac{1}{\sqrt{g}}\frac{\partial}{\partial x^i}\left(\sqrt{g}g^{ij}\frac{\partial\phi}{\partial x^j}\right) = g^{ij}\left[\frac{\partial^2\phi}{\partial x^i\partial x^j} - \frac{\partial\phi}{\partial x^f}\Gamma^f_{ij}\right]$$

where Γ^f_{ij} is a Christoffel symbol. In the context of relativity $\nabla_B\phi$ replaces the Poisson Gravitational potential: $\nabla^2\phi = \frac{4\pi G}{c^4}\sigma$.

Let us now consider a the zero-order part of the field $\phi(x)$ acting within a relativistic fluid under the condition of homogeneity and isotropy. The field $\phi(x)$ depends only on time t, that is on the variable $x^0 = ct$. Then $\phi_k\phi^k = \phi_0\phi^0 = -\left(\frac{\partial\phi}{c\partial t}\right)^2$. The (00) components of the diagonal tensors T_{ij} expressed by Eq. (10) is equal to the energy-density $\bar{\sigma}$ of the relativistic fluid.[4] Consider the well known Friedmann-Robertson-Walker (FRW) classical result accounting for the first term on the lhs of Eq(8):[4]

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi G}{3c^4}\bar{\sigma} \quad (13)$$

There $a(t)$ is the "*cosmological scale factor*" and $H(t)$, the "*Hubble rate*" which measures how rapidly $a(t)$ changes with time (the dot over a(t) means derivative respect to $time \times c$).[4] The (00) component of first term at the lhs of the Einstein equation, Eq. (8) is: $\left(\bar{R}_{00} - \frac{1}{2}g_{00}\bar{R}\right) = 3\times\left(\frac{\dot{a}}{a}\right)^2$ while the second term of the same equation is: $t_{00} = 3\times\left(\frac{\partial\phi}{c\partial t}\right)^2$. If $\left(\frac{\partial\phi}{c\partial t}\right) \geq 0$ (no universe's contractions) we may then adopt the following identification:

$$\left(\frac{\partial\phi}{c\partial t}\right) = \left(\frac{\dot{a}}{a}\right) = H \quad (14)$$

This equation expresses the direct action of the scalar field $\phi(x)$ on the (FRW) space-time. A first important result is that the Weyl potential is related exponentially to the "scale" i.e. $a(t) \sim e^{\phi(t)}$. Finally, the full (00) component of Einstein's equation, Eq. (8) reads:

$$2\times\left(\frac{\partial\phi}{c\partial t}\right)^2 = \frac{8\pi G}{3c^4}\bar{\sigma}. \quad (15)$$

Now consider the condition: T_{ii} where i = 1, 2, 3 are the spatial components of the diagonal tensor $T_{i,j}$. The first term in lhs in Eq. (8) is given by:[4]

$$G_{ii} = \left(\frac{\ddot{a}}{a}\right) + \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 = \frac{\partial^2\phi}{c^2\partial t^2} + \frac{3}{2}\left(\frac{\partial\phi}{c\partial t}\right)^2 \tag{16}$$

The second term in this equation is the well known classical (FRW) expression given in all cosmology textbooks, e.g. Ref. 4 (page 151), while the third term is obtained by adopting the identification Eq. (14). By adopting the same identification, the second term in the lhs of Eq. (8) is found:

$$t_{ii} = \left[\left(\frac{\partial^2\phi}{c^2\partial t^2} + 3\frac{\partial\phi}{c\partial t}H\right) - \frac{1}{2}\left(\frac{\partial\phi}{c\partial t}\right)^2\right] \tag{17}$$

By summing up the last two equation the "*negative pressure*" is found:

$$p = -A^{-1}\left(G_{ii} + t_{ii}\right) = -2A^{-1}\left[\frac{\partial^2\phi}{c^2\partial t^2} + 2\left(\frac{\partial\phi}{c\partial t}\right)^2\right] \tag{18}$$

Where: $A \equiv \frac{8\pi G}{c^4}$. In this equation $\left(\frac{\partial^2\phi}{c^2\partial t^2}\right)$ and $\left(\frac{\partial\phi}{c\partial t}\right)$ represent the "acceleration" and the "velocity" of the expansion process, respectively. In theoretical cosmology it is well known that inflation requires: $(3p + \overline{\sigma}) < 0$; Ref. 4 (page 151). Indeed, by Eqs. (15,18) we obtain: $(3p + \overline{\sigma}) = -\frac{3c^4}{4\pi G}\left[\left(\frac{\partial^2\phi}{c^2\partial t^2}\right) + 3\left(\frac{\partial\phi}{c\partial t}\right)^2\right]$.

Note that Eq. (10) can be expressed as follows:

$$\nabla_B\phi = \frac{4\pi G}{3c^4}\left[T - 2\overline{\sigma}\right] \tag{19}$$

Equation (18) may be compared with the classical FRW result or with the result of any theory adopting in the present context any artificial scalar field or artificial "quintessence".[4,20] In all of these theories the fluid pressure is *always positive*. In contrast, our present theory shows that *because of its "geometrical" nature* the scalar $\phi(x)$ leads to a *"negative pressure"* of the fluid because the expression in square brackets in the rhs of Eq: (18) is positive for any value of the expansion "velocity" and if the value of the "acceleration" is not too negative. Eq. (18) provides a most important result. We believe that this result of high cosmological relevance supports our present identification of the field $\phi(x)$ with the "Inflation" and, presumably, with the agent of "dark energy".[1-3]

According to, Ref. 4 (page 151): "*..This result is perhaps not surprising as [..] the accelerated expansion which cause supernovae to appear very faint can be caused only by dark-energy with negative pressure. Inflation was apparently driven by a similar form of energy with* $p < 0$. *Negative pressure is not something with which we have any familiarity. Non relativistic matter has small positive pressure proportional to temperature divided by mass, while a relativistic gas has pressure again positive. So whatever it is that drives the inflation is not ordinary matter or radiation...*".

Indeed, as already emphasized, the nature of $\phi(x)$ is not "material", that is either radiative or massive, but exclusively "*geometrical*". According to the present theory the time evolution of field: $\phi(x)$ can be expressed by Eq. (19). This enables us to express the "retarded time" solution of the field, $\phi(x)$ which propagates with the velocity of light:

$$\phi(x) = -\frac{G}{3c^4}\int\frac{(T' - 2\overline{\sigma}')}{R'}d^3x' \tag{20}$$

where: $R'^2 = \left[(x - x')^2 + (y - y')^2 + (z - z')^2\right]$,

$T' \equiv \left[-\overline{\sigma}(x') + 3p(x')\right]$, $\overline{\sigma}' \equiv \left[\overline{\sigma}(x')\right]$, being x' the point: $\left(x', t - \frac{R'}{c}\right)$. Integration is performed over the past light-cone of point x. Equation (20) shows that an extremely large expansion is expected in the Big-Bang region, where a very large density of energy and pressure are present. It is known that at the beginning of inflation an expansion $\frac{\Delta a(t)}{a(t)} \sim 10^{30}$ was realized in a time $\Delta t \sim 10^{-35}s$.[5] This corresponds to: a very large value of the expansion speed: $\frac{\partial\phi}{c\partial t} \sim 3.3\times10^{56}m^{-1}$.

In summary, the scalar field $\phi(x)$ appears to be the dominant source of the expansive effects of the space-time curvature, which is the role requested to the "inflation" field. We stress here that the peculiar "*geometric*" nature of $\phi(x)$ implies that this truly "*dark matter*" or "*dark energy*" field as well as all geometrical physical processes in the skies, that is motions of masses and fields of any kind, can be properly investigated by highly sensitive Gravitational Wave (GW) detectors. Recently an advanced GW detector based on Mach-Zehnder interferometry with modern hollow-core optical fibers was proposed.[15]

As a general statement, we should consider that in the universe the gravitation process pervasively affects every cosmological structure carrying energy/mass, including the $\phi(x)$ field, as shown in (20). This process should also conceivably include the zero-point vacuum energy contributed by all the quantum fields in the universe. In other words, large amounts of zero-point vacuum energy in the universe can move in space being gravitationally attracted by the massive structure of the galaxies and of the galaxy clusters. If this simple (likely, not original) statement, implying gravity induced delocalization of all vacuum fields, is truthful, the enigma implied by the real nature of "*dark matter*" is resolved. The large, very massive and nearly transparent "halos" of unseen non barionic mass surrounding the galaxies - e.g. detected by gravitational lensing effects - may merely consist of the "zero-point" "*vacuum energy*" which, by definition, is devoid of quantum particles. Since every quantum detector - including our own eyes - is only excited via the annihilation of the quantum particles of the field under measurement, these "halos" of dark mass/energy surrounding the galaxies are necessarily "transparent" to any action of measurement on that field. In other words the "dark energy" and the "dark matter" are dark simply because all the quantum detectors adopted for measurement are necessarily blind. However, since the "dark" fields may be gravitationally active, it is possible that they could be investigated by the methods of advanced GW Spectroscopy, as said. The above argument, which resolves the enigma of the "dark" fields, may be easily extended to resolve another cosmological puzzle: the small value of the measured "*cosmological constant*", Λ .

A good example of theoretical cosmology showing the effect of *"negative pressure"* is the standard model of a stable spherical symmetric star which is simple enough to allow an exact solution of the Einstein's equations.[6,26,29] The geometry of space time dominated by the field $\phi(x)$ is characterized by a metric tensor of the "standard" form:[6]

$$ds^2 = B(r)dt^2 - A(r)dr^2 - r^2\left(d\theta^2 + sin^2\theta d\varphi^2\right) \quad (21)$$

The metric tensor g_{ij} is diagonal and has non vanishing elements: $g_{rr} = A(r)$, $g_{\theta\theta} = r^2$, $g_{\varphi\varphi} = r^2 sin^2\theta$, $g_{tt} = -B(r)$. The model for the star may consist of a spherically symmetric cosmological structure filled by a fluid of constant and uniform mass-density: $\overline{\sigma}(r) = \left[3M_T c^2 / \left(4\pi\tilde{R}^3\right)\right]$ where M_T and $\tilde{R}$ are the mass of the star and radius, respectively. The *negative pressure* of the fluid is found: $p(r) = -\left[M_T c^2 / \left(4\pi\tilde{R}^3\right)\right] \times [1-D] \times \left[1-3^{-1}D\right]^{-1} \simeq -\left[M_T c^2 / \left(4\pi\tilde{R}^3\right)\right] = -\frac{\overline{\sigma}(r)}{3}$

where:

$$D = \sqrt{\left[(1-\frac{2M_T G}{\tilde{R}c^2}\right] \times \left[1-\frac{2M_T G}{\tilde{R}c^2}\eta^2\right]^{-1}} \text{ and: } \qquad \eta^2 = \left(\frac{r}{\tilde{R}}\right)^2 \ll 1$$

. The relevant metric components are: $A(r) = \left[\frac{1-2M_T G r^2}{\tilde{R}^3 c^2}\right]^{-1}$ and: $B(r) = \frac{1}{4}\left[3\sqrt{1-\frac{2M_T G}{\tilde{R}c^2}} - \sqrt{1-\frac{2M_T G}{\tilde{R}c^2}\eta^2}\right]^2$. This result is often referred to as a *Schwarzschild interior solution*.[6,26]

Within a parallel perspective, we may briefly consider in the framework of conformal theory the quantum effects on the motion of a particle which are due to a quantum force equal to the gradient of the scalar curvature R_W in the configuration space. The geometrical structure of space is affected by the presence of the particle owing either to gravitation and to the affine law of "parallel vector displacement".[7] This scenario parallels the well known J. A. Wheeler's conception according to which *"the mass of the particle affects the geometry while the geometry imposes the particle's trajectory"*. Guided by this conceptual parallelism, Enrico Santamato developed a nonrelativistic quantum theory based on a Schroedinger equation(24) acting on a appropriate wave-function $\Psi = \sqrt{\rho(x)}exp\left[i/\hbar S(x)\right]$, $S(x) = \xi\sigma(x)$. The corresponding theory can be obtained by adding to the rhs of Eq. (9) the expression for the "phase": $\left[\left(\partial_i\sigma\partial_j\sigma - \frac{1}{2}g_{ij}\partial_k\sigma\partial^k\sigma\right)\right]$. An inspection of Eqs.(9) shows that a close relationship exists between the wave-function and the $\phi(x)$ field: $\left(\Psi\Psi^\dagger\right) = e^{(-2\phi)}$ i.e. $\phi = -\frac{1}{2}ln\left(\Psi\Psi^\dagger\right)$. This means that, according to the Born rule, the $\phi(x)$ field expresses the quantum probability density of the particle's trajectory, that is the quoted back effect of the geometry on the particle's dynamics. Furthermore, the same quantum theory of the particle's "external" vector properties, e.g. position and velocity, was found to apply also to a relevant particle's "internal" property, the particle's $spin-\frac{1}{2}$. In particular this led to a novel derivation of the Dirac's Equation.[22] Furthermore, the extension of the same theory to the Einstein-Podolsky-Rosen (EPR) process involving two $spin-\frac{1}{2}$ in the entangled state was found to allow an exact, complete and fully consistent analysis of the Bell's inequalities leading to a novel insightful approach to the enigma implied by"*quantum nonlocality*" and "*quantum entanglement*".[21,22,27,31] In agreement with the standard quantum theory, superluminal correlations between measurements on distant entangled particles are found to occur within the extension of the quantum wave function Ψ. More precisely, at the core of the EPR theory, the explicit expression of the scalar Riemann-Weyl curvature R in the configuration space, Eq. (2) of the two entangled spin system shows that the particle positions, i.e. the"external" coordinates, can freely fly apart toward very distant places in the universe while the corresponding "internal" coordinates cannot be formally disentangled within the formal local structure of R_W. This is the source of the nonlocal correlations. The internal coordinates, that in the EPR case are the Euler-angles connecting the two geometrical "tetrads" assumed to model the spins, are generally referred to as "hidden-variables" in the standard literature.

The above action theory also leads to an interesting Klein-Gordon (KG) equation:

$$\nabla_B |\Psi| + T|\Psi| = 0 \quad (22)$$

where: $T \equiv T_{ij}g^{ij} = -\xi^2\overline{R}$. This KG equation has been considered to construct the relativistic analogue of the classical Schroedinger-Newton problem.[25] A static, spherically symmetric metric, equal to the one already expressed above, can be computed by assuming a perfect fluid in hydrostatic equilibrium. The source of the equation is: $\Sigma^2 = \left(\frac{8\pi G}{c^4}\right)| -\overline{\sigma}(t,r) + 3p(t,r)|$ and the field can be expressed in terms of spherical harmonics as: $|\Psi(t,r)| = \Phi_t(t)R_l(r)Y_{lm}(\theta,\psi)$. A complete solution of the KG equation has been worked out numerically by.[25] However the time dependence of the field can be given immediately: $\frac{\delta^2\Phi_t(t)}{\delta t^2} = \Sigma^2\Phi_t(t)$ or: $|\Phi_t(t)| = exp(\sqrt{\Sigma^2}t$, showing an exponential increase in the wave function with the cosmic time t.

As a final consideration, the general relevance of conformal symmetry and of the symmetry-breaking of the vacuum is well emphasized by the following comment by Gerhard t'Hooft:[23]

"*....We still do not know what happens at higher energies even if we do understand the laws at low energies. Or more to the point: small time and distance scales seem not to be related to large time and distance scales. Now, we argue, this because we fail to understand symmetry of the scale transformations. This symmetry, of which the local form will be local Conformal Symmetry, if exact, should fulfill our needs. Since the world appears not to be scale invariant, this symmetry, if it exists must be spontaneously broken: This means that the symmetry must be associated with further field transformations leaving the vacuum not invariant. It is the implementation of the symmetry that we should attempt to construct from the evidence we have. In conclusion, there must be a component in space-time symmetry group (the Poincaré group) that both Lorentz and Einstein dismissed...*". The Title of the paper from which the last excerpt was taken: "*Local Conformal Symmetry: the missing Symmetry Component for space and time*". Indeed, this "missing Symmetry Component" can precisely consist of the "*Weyl Conformal Symmetry*" on which rests the geometrical background of the present work. This is actually the key of our work.

Eventually, as proposed recently,[28] the exact symmetry of the universe spacetime - either riemannian, or weylan, or other - is expected to be definitely ascertained by gravitational-wave (GW) measurements via the next generation of highly sensitive GW telescopes, for example the multi satellite LISA system or by ground based advanced detectors such as, in the future, the EINSTEIN apparatus or the efficient optical interferometer recently proposed.[15] We believe that the conformal geometrical mechanism proposed in the present work represents a unifying scenario by which the scalar field $\phi(x)$ appears to play an essential role in determining the evolution of the Universe "at large" as well as, at the microscopic level - via the dynamics of the scalar curvature R_W - of the everyday quantum phenomenology

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Conflicts of Interest

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