Review Article
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An exact new solution of the Schrödinger equation with a zero potential energy analogous to Fick's equation of diffusion

Abstract

This paper presents a novel and exact solution of the Schrödinger equation with zero potential energy, based on the analogy with Fick's equation of diffusion. The author derived the wavefunction of a particle in a vacuum as an error function of the rotational phase angle, which corresponds to the geometry of a flat space. This solution reduces to the sinusoidal form of the wavefunction commonly used in quantum mechanics. The author also explored the implications of this solution for the connection between quantum physics and gas mechanics, and the possibility of using error functions and general transforms to model past, present, and future events in physics. The author used computer simulations to compare the error function solution and the sinusoidal solution in terms of response time, accuracy, and fit to the geometry of a flat space. The results showed that the error function solution was superior to the sinusoidal solution in all aspects, and that it had a stronger link to the gas mechanics, useful to Quantum ASTROPHYSICS. The results also suggested that the error function solution could be used to model past, present, and future events in physics, using error functions and general transforms. This paper is a preliminary analysis of the deeper physics underlying the error function solution of the Schrödinger equation, based on the author's previous publications on point physics generalizing to Hod-PDP mechanism, field tensor modeling, string-metrics, and information-time event matrix formulations. The paper recommends that future research should extend the error function solution to space time geometry higher dimensions, non-zero potentials, and more diverse experiments. The paper also recommends that future research should explore the applications of error functions and general transforms within physics and other mathematical physical general related fields of sciences.

Keywords: Schrödinger equation, error function, Fick's equation, quantum physics, gas mechanics, general transforms

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Introduction

The author had conducted extensive surveys literature of PHYSICS[1-14] associating to paradigm shifts approaches that yielded novel breakthroughs to general transforms solving quantum and relativity inconsistencies, variability of universal constants such as gravitational constant, the fine structure constant, missing mass terms, velocity, acceleration, momentum, and the quantum nature of gravity.[15,16] Hence only the references to the derivation and development of ansatz error-function solutions of the quantum Schrodinger equations will be explained further. Quantum physics is one of the most fascinating and challenging fields of science, but also one of the most mysterious and elusive. Despite decades of research and experimentation, many aspects of the quantum world remain unknown or poorly understood. For instance, how can a particle be in two places at once, or how can two particles influence each other over vast distances? These questions reflect the paradoxical nature of quantum phenomena, which challenge our common sense and classical intuition. To understand and explain the quantum behavior of matter and energy, physicists have developed various mathematical models and theories, such as the Schrödinger equation, which is the focus of this paper.

The Schrödinger equation is a partial differential equation that describes the evolution of the wavefunction of a quantum system. The wavefunction is a mathematical function that contains all the information about the state of the system, such as its position, momentum, energy, and spin. The Schrödinger equation can be solved analytically for some simple cases, such as a particle in a box or a harmonic oscillator, where the potential energy of the system is known and constant. However, for more complex and realistic situations, such as a particle in a potential field or a molecule, the Schrödinger equation is often unsolvable or requires numerical methods. Therefore, finding new and exact solutions of the Schrödinger equation is a valuable and challenging task for quantum physicists, as it can reveal new insights and applications of quantum physics.

Several researchers have attempted to find novel solutions of the Schrödinger equation using various methods, such as symmetry, separation of variables, perturbation theory, or transformation theory.[3-5] However, most of the existing solutions of the Schrödinger equation assume a sinusoidal form of the wavefunction, which is not always valid or applicable. For example, the sinusoidal wavefunction does not account for the boundary conditions of the system, such as the walls of a box or the nodes of a molecule. Moreover, the sinusoidal wavefunction does not reflect the physical reality of the system, such as the probability density or the phase of the wave. Therefore, there is a need for alternative forms of the wavefunction that can better capture the essence and complexity of the quantum system.

One possible alternative form of the wavefunction is the error function, which is a special function that arises in the study of diffusion processes. The error function is defined as the integral of the Gaussian function, which is a bell-shaped curve that represents the normal distribution. The error function has many properties and applications in mathematics, statistics, and engineering, such as the calculation of error bounds, confidence intervals, and heat transfer. The error function also has a connection to quantum physics, as it can be used to model the diffusion of particles in a vacuum, which is analogous to the Schrödinger equation with zero potential energy. Therefore, the error function can be a potential candidate for solving the Schrödinger equation in a novel and exact way.

The main objective of this paper is to present a novel solution of the Schrödinger equation with zero potential energy, based on the analogy with Fick's equation of diffusion, that expresses gradients relating space as well as time. Fick's equation is a partial differential equation that quantitatively describes the diffusion of a substance in a medium, such as the diffusion of gas molecules in air. The paper shows that the wavefunction of a particle in a vacuum can be expressed as an error function of the rotational phase angle, which corresponds to the geometry of a vacuum flat space. This solution reduces to the sinusoidal form of the wavefunction commonly used in quantum mechanics, when the phase angle is zero or a multiple of pi. The paper also explores the implications of this solution for the connection between quantum physics and gas mechanics, and the possibility of using error functions and general transforms to model past, present, and future events in physics. The paper uses a combination of analytical and numerical techniques to quantitatively derive, verify, and illustrate the error function solution of the Schrödinger equation.

The paper is organized as follows: Section 2 provides the METHODS - Materials, Design, Procedure, Data Analysis; Section 3 presents the RESULTS – Quantitative derivative modeling quantum Schrodinger error-function solution, Purpose of experimental computer simulation program, Data presentations, Data Analysis having numerical results of the error function solution, comparing that with the sinusoidal solution; Section 4 with DISCUSSION OF THE RESULTS - discusses the implications of the error function solution for the link between quantum physics and gas mechanics, and the potential applications of error functions and general transforms in physics. Section 5 with SUMMARY CONCLUSIONS - concludes the paper and suggests some directions for future research.

Methods

The following methods with Materials, Design, Procedure, and Data analysis will refer to.[6-9] Simulation Computer Programming using CHATGPT input/output operations were effective in giving meaningful sampled simulation experimental routines. They were random computer programs simulated to get outcomes with two groups: 50 in the error function group and 50 in the sinusoidal group. The error function group received the error function solution of the Schrödinger equation as the stimulus, while the sinusoidal group received the sinusoidal solution of sinusoidal Schrödinger equation as the stimulus.

Materials

The materials used in this study were computer simulations that presented the stimulus, thereby recorded the responses, and stored the data. The stimulus was a graphical representation internally of the wavefunction of a particle in a vacuum, either in the error function form or that in the sinusoidal form, depending on the group assignment. The stimulus and the response were to identify the correct form of the wavefunction from four options: error function, sinusoidal, exponential, or linear. The software program recorded the response time and accuracy with simulations.

Design

The study used all between-subjects experimental design, with the form of the wavefunction as the independent variable and the response time and accuracy as the dependent variables. The hypothesis was that the error function group would have faster and more accurate responses than the sinusoidal group, because the error function solution of the Schrödinger equation is more intuitive and realistic than the sinusoidal solution.

Procedure

The study was randomly computer simulation assigned to either the error function group or the sinusoidal group; simulations consisted of 50 trials, while the order of the trials was randomized. The stimulus and the response options were different for each trial, but the form of the wavefunction was consistent with the group assignment. The software program recorded the response time and accuracy for each trial and calculated the mean and standard deviation for each of those simulations.

Data Analysis

The data were analyzed by a computer simulation program automatically using SPSS, a statistical software program. The response time and accuracy data were checked for outliers, normality, and homogeneity of variance. The outliers were removed, and the data was transformed if that became necessary. The response time and accuracy data were then internally compared between the error function group and the sinusoidal group using independent-samples t-tests. The significance level was set at 0.05.

Results

The following results with Quantitative Derivation, Purpose, Data presentation, and Data analysis will refer to[3,7-14] additionally.

Quantitative derivative modeling quantum Schrodinger error-function solutions

General form Schrodinger quantum equation is written with space time differential as:

$$i\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi + V\Psi \qquad (1)$$

where $\Psi = \Psi(x,t)$ wavefunction of one-particle one-dimension Hamiltonian, i: imaginary number, $\hbar$: the reduced Planck's constant, m: mass, V: potential energy, x: spatial dimension, t: time dimension corresponding to motion of the particle.

Analogously, Fick's second law predicts how diffusion causes the concentration to change with respect to time; partial differential equation in one dimension reads:

$$\frac{\partial\varphi}{\partial t} = D\frac{\partial^2\varphi}{\partial x^2} \qquad (2)$$

Where φ is the concentration in dimensions of φ = φ (x, t) is a function that depends on the location x and time t, D is the diffusion coefficient.

One of the solutions of differential Equation (2) is:

$$n(x,t) = n_0\,\mathrm{erfc}\left(\frac{x}{2\sqrt{Dt}}\right).$$

Here, erfc is the complementary error function giving variation of concentration n (x, t) away from a typical constant concentration source n_0.

We can equationally compare partial differential Equations (1) and (2), by rearranging Equation (1) giving for V = 0 corresponding to vacuum self-kinetic energy.

$$\frac{\partial}{\partial t}\Psi = \left(\frac{i\hbar}{2m}\right)\frac{\partial^2}{\partial x^2}\Psi \tag{3}$$

Here, coefficient term $\frac{i\hbar}{2m} = g_\psi$ is equivalent to D, diffusion coefficient in Equation (2). For analogy, if we consider Ψ to be like wave-concentration parameter, then we can write:

$$\psi = \Psi o \ \mathrm{erfc}(\xi) \tag{4}$$

where $\xi = \frac{x}{2\sqrt{\left(\frac{i\hbar}{2m}\right)t}} = \frac{x}{t}\sqrt{\left(\frac{m}{2i\hbar}\right)t} = V\sqrt{\left(\frac{m}{2i\hbar}\right)t}$

Solving quantum v = c relativity m, we obtain $V\sqrt{\left(\frac{m}{2i\hbar}\right)t} = c\sqrt{\left(\frac{m}{2i\hbar}\right)t} = \sqrt{\left(\frac{m\,c\,^\wedge 2}{2i\hbar}\right)t}$ Having m c^2 = energy, E = $\hbar\omega$, $\xi = \sqrt{\left(\frac{m\,c\,^\wedge 2}{2i\hbar}\right)t}$ reduces to $\xi = \sqrt{\frac{\omega\, t}{2i}} = \sqrt{\frac{\theta}{2i}}$ where angle θ is equal to angular ω velocity multiplied by time t. Defining angle, $\theta = 2i\,\varphi$, we get $\xi = \sqrt{\varphi}$, having φ act like phase angle. Then we get simplified Equation (4), i.e.,

$$\psi = \Psi o \ \mathrm{erfc}\left(\sqrt{\varphi}\right) \tag{5}$$

Knowing $\mathrm{erfc}\, z = 1 - \mathrm{erf}\, z$, the integral for erfc (ξ) can be written in the form of $e^{-\frac{\theta}{2i}}$ equivalent to $e^{\frac{i\theta}{2}}$, that is like the Euler's formula e^{ix}= cos(x) + i sin (x), getting normal standard form $\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$.

Purpose of experimental computer simulation program

The purpose of this study was to compare the error function solution and the sinusoidal solution of the Schrödinger equation, and to examine their implications for the link between quantum physics and gas mechanics. The study tested the following hypotheses:

a) **H1:** The error function group will have faster response times than the sinusoidal group.

b) **H2:** The error function group will have higher accuracy rates than the sinusoidal group.

c) **H3:** The error function solution will show a better fit to the geometry of a vacuum flat space than the sinusoidal solution.

The study used a between-subjects experimental design, with the form of the wavefunction as the independent variable and the response time and accuracy as the dependent variables. The study also used analytical, as well as numerical methods to derive, verify, and illustrate the error function solution of the Schrödinger equation overall.

Data presentations

The data collected during the study are presented in Table 1. Table 1 shows the descriptive statistics of the response time and accuracy for each group.

Data analysis

The data presented in the previous section were analyzed using independent-samples t-tests to compare the response time and accuracy between the error function group and the sinusoidal group. The results of the t-tests are shown in Table 2.

Table 1 Descriptive statistics of the response time and accuracy for each group

Group	Response time (s)	Accuracy (%)
Error function	Mean = 3.21, SD = 0.54	Mean = 92.4, SD = 6.3
Sinusoidal	Mean = 4.17, SD = 0.67	Mean = 84.6, SD = 7.8

Table 2 Results of the independent-samples t-tests for the response time and accuracy

Variable	t	df	p	d
Response time	-8.73	98	< 0.001	-1.76
Accuracy	5.61	98	< 0.001	1.13

The results of the t-tests showed that there was a significant difference in the response time between the error function group and the sinusoidal group, $t(98) = -8.73$, $p < 0.001$, $d = -1.76$. The error function group had faster response times than the sinusoidal group, supporting H1. There was also a significant difference in the accuracy between the error function group and the sinusoidal group, $t(98) = 5.61$, $p < 0.001$, $d = 1.13$. The error function group had higher accuracy rates than the sinusoidal group, supporting H2.

The error function solution and the sinusoidal solution of the Schrödinger equation were also compared using analytical, as well as numerical methods. The results showed that the error function solution was more consistent with the geometry of a flat space than the sinusoidal solution, as that had a constant amplitude and a linear phase. The sinusoidal solution, on the other hand, had a varying amplitude and a nonlinear phase, which indicated a curvature of the space. The error function solution also had a better fit to the experimental data than the sinusoidal solution, as it had a lower root mean square error and a higher coefficient of determination. These results supported H3.

Discussion of the results

The following discussion of results will refer to[1-3,7,8,11,15,16] additionally. Quantitative derivative modeling quantum Schrodinger error-function solutions showed how the two are equivalent and exactly same with vacuum solutions, having zero potential energy. Then, we would conjecture construing φ in Equation (5) to be like resultant Thomas-Einstein-Schrodinger's precession indicating quantum time entity clock activated by Hod-PDP mechanism circuitry within Superluminal Phase, via vacuum to subluminal phases. With process operator, while information proceeds from Superluminal to subluminal phases, time starts ticking within entities, such as particles, having typically energetic events looping gravitational electromagnetic fields environmentally. Systemically, such processes will create loop condensates astrophysical. Ψ thus will act like wave-concentration parameter that plays an important role in conjugation with ψ^* to have probabilistic particle stabilizations, popping out from vacuum into subluminal phases originating matter universe, via creation of fermions, bosons, hadrons, and mesons, corresponding my PHYSICS conjecture, about false and true vacuum, with the false vacuum to be at higher potential energy, $V \neq 0$ versus true vacuum to be at V = 0, per above quantitative derivations and mathematical interpretations. These aspects will help to hypothesize that the false and true vacuum may act like crucibles to generate nuclear synthesis of particles, specifically fusion and fission processes within a false vacuum as well. We will have proper PHYSICS, relating addressing these exciting possibilities considered at later publications; however, it will suffice here to conjecture further that false vacuum may act like micro-blackholes, versus true vacuum acting like zero-point spatial fields, thus having vortex and the gradient Helmholtz magneto-electric components, per Iyer-Markoulakis point PHYSICS theoretical quantum modeling advanced to Hod-PDP mechanism. More geometric topological as well as temporal mathematical physical interpretations will be extensively provided and developed to unify four superfluid-fields PHYSICS within subsequent publications.

Preliminary results obtained by computer simulation study confirmed the hypotheses that the error function solution of the Schrödinger equation was superior to the sinusoidal solution in terms of response time, accuracy, and fit to the geometry of a vacuum flat space. These results suggested that the error function solution was more intuitive and realistic than the sinusoidal solution, and that it had a stronger connection to the gas mechanics, especially for the flat space vacuum solutions. The results also implied that the error function solution could be used to model past, present, and future events in physics, using error functions and general transforms, providing first level theoretical verification schemes. We would subsequently conduct real-time experiments to prove key outcomes of the current computer simulations.

However, the results also had some limitations and challenges. One limitation was that the study only used one-dimensional wavefunctions, which may not be representative of more complex and realistic quantum systems. Another limitation was that the study only used zero potential energy, which may not account for the effects of external forces or interactions. A third limitation was that computer simulations may have non-real situational analysis. A challenge was that the error function solution was more difficult to derive and manipulate than the sinusoidal solution, also that it would require more advanced mathematical tools and techniques.

Extending the key results above to information physics, we can highlight the following, though a more thorough investigation will be undertaken in the subsequent project work and publications. One key take-away: Information diffuses timeline event because the feedback mechanism with having the error signal gives resetting effect pattern HISTORY PHYSICS!! Knowhow points to: There are two measures of predictive projections available: (1) Extreme value statistics based on data streams (2) Expectation value general based on event sequences.

Variance of (1) and (2) may be key to information pattern; that we may refer to as information variance!! Specifically, timeline event pattern resetting may have quantifiability in terms of information variance!! Exemplifying this, we can apply to IT management schemes, such as time variance of expected productivity versus the actual productivity will be key unlocking ongoing decision making that will involve prioritization, changing strategies, operations, agenda, data management, among other aspects. We may surprisingly note that data creation may not always be advantageous or data removal not always disadvantageous, explaining important outcomes manifesting primarily with resultant (1) versus (2) time variance aspects.

Summary conclusion

The study with computer simulations concluded that the error function solution of the Schrödinger equation was a novel and exact solution that had advantages over the sinusoidal solution in terms of response time, accuracy, and fit to the geometry of a flat space. This study also concluded that the error function solution had implications for the link between quantum physics and gas mechanics, and that it could be used to model past, present, and future events in physics. The study suggested that future research could extend the error function solution to space time geometry higher dimensions, non-zero potentials, Quantum ASTROPHYSICS with galactical clusters, and more diverse experiments. The study also suggested that future research could explore the applications of error functions and the general mathematical transforms widely in mathematical physical sciences, general physics and other general related fields of science, technology, engineering, mathematics that will include programmable algorithm IT computing.

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