

# On the synopsis of the Higgs boson

## Abstract

In this paper, we analyze the Standard Model (SM) Higgs boson by computing both the partial width and the amplitude of a number of decay channels that the Higgs Boson can undergo. In our computations, we treat the Higgs boson as a free parameter despite its estimated mass of around 125GeV discovered at the Large Hadron Collider (LHC).<sup>1</sup>

**Keywords:** Higgs boson, decay channels, Standard Model, Large Hadron Collider

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## Introduction

The Higgs boson can undergo several decays via a number of decay channels.<sup>2</sup> The main decay channels that we are going to concentrate on in this paper are  $h^0 \rightarrow f\bar{f}$ ,  $h^0 \rightarrow W^+W^-$ ,  $h^0 \rightarrow Z^0Z^0$ ,  $h^0 \rightarrow gg$ , and  $h^0 \rightarrow \gamma\gamma$ .<sup>3</sup> From the five decay channels listed, the first three occur at tree level while the other two at one-loop.

In this paper, we will calculate the decay widths by treating our decay system as a body of a final state system. Beginning with rotational symmetry and considering our momentum to be conserved, we parametrize our variables in the centre-of-mass (CM) frame as  $p_1 = (E, 0, 0, p)$  and  $p_2 = (E, 0, 0, -p)$ , with  $E = \frac{1}{2}m_h$ . Here we note that the amplitude will not depend on the angular parameters.<sup>4</sup> Thus the integral of the phase space reads

$$\int d\Pi_2 |M|^2 = \frac{1}{4\pi} \frac{p}{m_h} |M|^2 \quad (1)$$

From Equation 1, we determine the decay width as<sup>5</sup>

$$\Gamma = \frac{1}{2m_h} \int d\Pi_2 |M|^2 = \frac{1}{8\pi} \frac{p}{m_h^2} |M|^2 \quad (2)$$

Furthermore, the momenta of our final state particles becomes  $k_1 = (E, 0, 0, k)$  and  $k_2 = (E, 0, 0, -k)$  with  $E^2 = k^2 + m^2$  and  $2E = m$ .<sup>6</sup> Thus, the cross section becomes

$$\begin{aligned} \sigma &= \frac{1}{2\beta_s} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} |M|^2 (2\pi)^2 \delta^4(p - k_1 - k_2) \\ &= \frac{1}{4m_f \beta_s} |M|^2 (2\pi) \delta(2k_1 - m_f) \\ &= \frac{\pi}{\beta m_f^2} |M|^2 \delta(s - m_f^2) \end{aligned} \quad (3)$$

with  $\beta = \sqrt{1 - (4m_i/m_f)^2}$  being the initial particle's velocity magnitude.

The  $h^0 \rightarrow f\bar{f}$  decay channel

This is the simplest channel to compute and its corresponding amplitude reads as

$$iM(h^0 \rightarrow f\bar{f}) = -\frac{im_f}{v} \bar{u}^*(p_1) v(p_2) \quad (4)$$

It is easy to compute the square amplitude as

$$\begin{aligned} \Sigma |M(h^0 \rightarrow f\bar{f})|^2 &= \frac{m_f^2}{v^2} \text{tr}[(\not{p}_1 + m_f)(\not{p}_2 - m_f)] \\ &= \frac{2m_f^2}{v^2} (m_f^2 - 4m_f^2) \end{aligned} \quad (5)$$

Using the momenta for final states, the decay width becomes

$$\Gamma(h^0 \rightarrow f\bar{f}) = \frac{1}{8\pi} \frac{p}{m_h^2} |M|^2 = \frac{m_h m_f^2}{8v^2} \left(1 - \frac{4m_f^2}{m_h^2}\right)^{3/2} \quad (6)$$

The  $h^0 \rightarrow W^+W^-, Z^0Z^0$  decay channels

In this section, we describe the amplitude for  $h^0 \rightarrow W^+W^-$  as

$$iM(h^0 \rightarrow W^+W^-) = \frac{ig^{\mu\nu} g^2 v}{2} \epsilon_\mu^*(p_1) \epsilon_\nu^*(p_2) \quad (7)$$

and the square amplitude reads

$$\begin{aligned} \Sigma |M|^2 &= \frac{g^4 v^2}{4} \left( g_{\mu\nu} \frac{p_{1\mu} p_{1\nu}}{m_W^2} \right) \left( g^{\mu\nu} \frac{p_{2\mu} p_{2\nu}}{m_W^2} \right) \\ &= \frac{\pi\alpha}{\sin^2 \theta_w} \frac{m_h^4}{m_W^2} \left( 1 - \frac{4m_W^2}{m_h^2} + \frac{12m_W^4}{m_h^2} \right) \end{aligned} \quad (8)$$

Finally, the decay width becomes

$$\begin{aligned} \Gamma(h^0 \rightarrow W^+W^-) &= \frac{1}{8\pi} \frac{p_1}{m_h^2} |M|^2 \\ &= \frac{\alpha m_h^3}{16\pi m_W^2 \sin^2 \theta_w} (1 - 4\tau_W^{-1} + 12\tau_W^{-2}) (1 - 4\tau_W^{-1})^{1/2} \end{aligned} \quad (9)$$

where  $\tau_W = (m_h/m_W)^2$ . In order to compute for  $h^0 \rightarrow Z^0Z^0$ , we just swap the masses of the vector boson and add an extra factor of half and arrive at

$$\Gamma(h^0 \rightarrow Z^0Z^0) = \frac{\alpha m_h^3}{32\pi m_Z^2 \sin^2 \theta_w} (1 - 4\tau_Z^{-1} + 12\tau_Z^{-2}) (1 - 4\tau_Z^{-1})^{1/2} \quad (10)$$

with  $\tau_Z = (m_h/m_Z)^2$ .

The  $h^0 \rightarrow gg$  decay channel

The amplitude takes the form

$$\begin{aligned} iM(h^0 \rightarrow gg) &= -\frac{im_q}{v} (ig_s)^2 \epsilon_\mu^*(p_1) \epsilon_\nu^*(p_2) \text{tr}(t^a t^b) \\ &\times \int \frac{d^d p}{(2\pi)^d} \left\{ (-1) \text{tr} \left[ \gamma^\mu \frac{i}{\not{q} - m_q} \gamma^\nu \frac{i}{\not{q} + \not{p}_2 - m_q} \frac{i}{\not{q} - \not{p}_1 - m_q} \right] \right. \\ &\left. + (-1) \text{tr} \left[ \gamma^\nu \frac{i}{\not{q} - m_q} \gamma^\mu \frac{i}{\not{q} + \not{p}_1 - m_q} \frac{i}{\not{q} - \not{p}_2 - m_q} \right] \right\} \end{aligned} \quad (11)$$

From Equation 11, we can simplify the trace as

$$\text{tr} \left[ \gamma^\nu \frac{i}{\not{q} - m_q} \gamma^\mu \frac{i}{\not{q} + \not{p}_2 - m_q} \frac{i}{\not{q} - \not{p}_1 - m_q} \right] = \frac{-i \text{tr}[(\not{q} + m_q)(\not{q} + \not{p}_2 - m_q)(\not{q} - \not{p}_1 - m_q)]}{(q^2 - m_q^2)[(q + p_2)^2 - m_q^2][(q - p_1)^2 - m_q^2]}$$

$$= -2i \int_0^1 dx \int_0^{1-x} dy \frac{N^{\mu\nu}}{(q'^2 - \Delta)^3} \quad (12)$$

where

$$q'_\mu = q_\mu - xp_{1\mu} + yp_{2\mu} \quad (13)$$

$$\Delta = m^2 - x(1-x)p_1^2 - y(1-y)p_2^2 - 2xy p_1 \cdot p_2 = m^2 - xym^2_h \quad (14)$$

$$N^{\mu\nu} = 4m \left( p_1^\nu p_2^\mu - p_1^\mu p_2^\nu + 2p_2^\nu q^\mu - 2p_1^\mu q^\nu + 4q^\mu q^\nu + (m_q^2 - p_1 \cdot p_2 - q^2) \eta^{\mu\nu} \right) \quad (15)$$

After rigorous computations, the final decay width becomes

$$\Gamma(h^0 \rightarrow gg) = \left( \frac{\alpha m_h}{8 \sin^2 \theta_w} \right) \cdot \frac{m_h^2}{m_W^2} \cdot \frac{\alpha_s^2}{9\pi^2} \cdot \left| I_f(\tau_q) \right|^2 \quad (16)$$

Generally, for  $N_q$  quarks, the decay width takes the form

$$\Gamma(h^0 \rightarrow gg) = \left( \frac{\alpha m_h}{8 \sin^2 \theta_w} \right) \cdot \frac{m_h^2}{m_W^2} \cdot \frac{\alpha_s^2}{9\pi^2} \cdot \left| \sum_f I_f(\tau_q) \right|^2 \quad (17)$$

The cross section for  $gg \rightarrow h^0$

By using the results in section 4 as well comparing Equations 2 and 3, we deduce that

$$\sigma(gg \rightarrow h^0) = \frac{\pi^2}{8m_h} \delta(\hat{s} - m_h^2) \Gamma(h^0 \rightarrow gg) \quad (18)$$

By using Equation 17, we obtain

$$\sigma(gg \rightarrow h^0) = \frac{\alpha \alpha_s^2}{576 \sin^2 \theta_w} \cdot \frac{m_h^2}{m_W^2} \left| \sum_f I_f(\tau_q) \right|^2 \delta(\hat{s} - m_h^2) \quad (19)$$

Finally, the gluon-gluon fusion cross section at proton level becomes

$$\sigma_{GGF}(p(P_1)p(P_2) \rightarrow h^0) = \int_0^1 dx_1 \int_0^1 dx_2 f_g(x_1) f_g(x_2) \sigma(g(x_1 P_1)g(x_2 P_2) \rightarrow h^0)$$

$$= \int dM^2 Y \left| \frac{\partial(x_1, x_2)}{\partial(M^2, Y)} \right| f_g(x_1) f_g(x_2) \sigma(g(x_1 P_1)g(x_2 P_2) \rightarrow h^0)$$

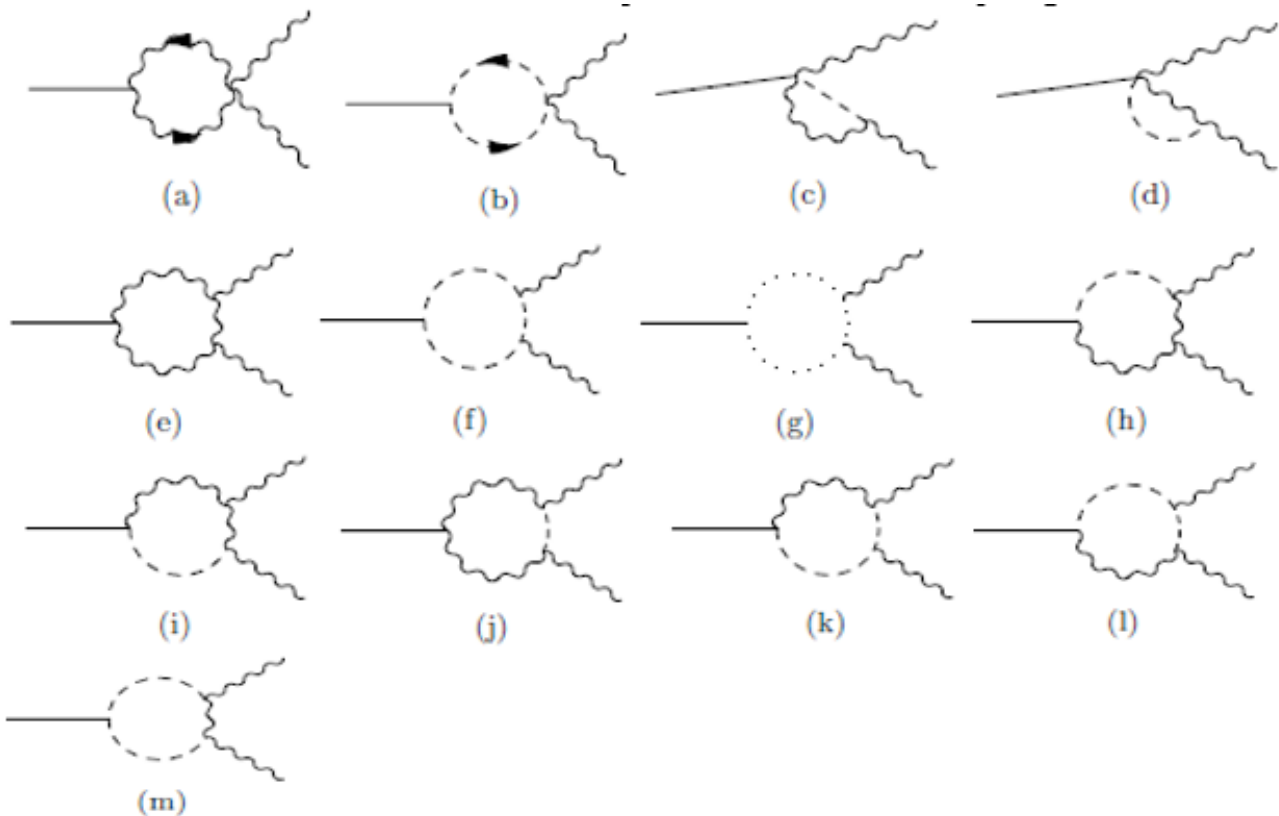
$$= \int dM^2 Y \frac{1}{M^2} x_1 f_g(x_1) x_2 f_g(x_2) \sigma(g(x_1 P_1)g(x_2 P_2) \rightarrow h^0) \quad (20)$$

The  $h^0 \rightarrow 2\gamma$  **decay channel**

In this decay channel, the contribution is as a result of both the  $W$  boson and the fermion loop. It is easier to compute for the later contribution and our beginning point is to pick the result in Equation 17. By including the internal fermion electric charge to our result we arrive at

$$iM(h^0 \rightarrow 2\gamma) = \left( \frac{\alpha m_h}{8 \sin^2 \theta_w} \right) \cdot \frac{m_h^2}{m_W^2} \cdot \frac{\alpha_s^2}{18\pi^2} \cdot \left| \sum_f Q_f^2 N_c(f) I_f(\tau_f) \right|^2 \quad (21)$$

For the  $W$  boson contribution, we have to compute for all the thirteen loop diagrams.



Thus, we have

$$iM^{(a)} = \frac{1}{2} \frac{ig\rho\sigma g^2 v}{2} (-ie^2) (2\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}) \epsilon_{\mu}^* (p_1) \epsilon_{\nu}^* (p_2) \times \int \frac{d^d q}{(2\pi)^d} D_W(q) D_W(k-q) = -\frac{2i}{(4\pi)^{d/2}} \frac{e^2 m_W^2}{v} \epsilon^*(p_1) \cdot \epsilon^*(p_2) (d-1) \Gamma(2-d/2) \times \int_0^1 \frac{dx}{[m_W^2 - x(1-x)m_h^2]^{2-d/2}} \quad (22)$$

$$iM^{(b)} = \frac{1}{2} (-2i\lambda v) (2ie^2) \epsilon_{\mu}^* (p_1) \cdot \epsilon_{\nu}^* (p_2) \int \frac{d^d q}{(2\pi)^d} D_s(q) D_s(k-q) = -\frac{i}{(4\pi)^{d/2}} \frac{e^2 m_W^2}{v} \epsilon^*(p_1) \cdot \epsilon^*(p_2) \Gamma(2-d/2) \times \int_0^1 \frac{dx}{[m_W^2 - x(1-x)m_h^2]^{2-d/2}} \quad (23)$$

$$iM^{(c)} = iM^{(d)} = \frac{ig^2 \sin\theta_w}{2} \cdot \frac{ig^2 v \sin\theta_w}{2} \frac{1}{2} \epsilon_{\mu}^* (p_1) \cdot \epsilon_{\nu}^* (p_2) \int \frac{d^d q}{(2\pi)^d} D_s(q) D_W(p_2-q) = -\frac{i}{(4\pi)^{d/2}} \frac{e^2 m_W^2}{v} \epsilon^*(p_1) \cdot \epsilon^*(p_2) \Gamma(2-d/2) \frac{1}{(m_W^2)^{2-d/2}} \quad (24)$$

$$iM^{(e)} = \frac{ig^2 v}{2} (-ie)^2 \eta_{\rho\sigma} \epsilon_{\mu}^* (p_1) \epsilon_{\nu}^* (p_2) \int \frac{d^d q}{(2\pi)^d} D_W(q) D_W(q-p_1) D_W(q-p_2) \times [\eta^{\rho\lambda} (2q-p_1)^{\mu} + \eta^{\mu\rho} (2p_1-q)^{\lambda} - \eta^{\lambda\mu} (p_1+q)^{\rho}] \times [\eta_{\lambda}^{\sigma} (2q-p_2)^{\nu} - \eta_{\lambda}^{\nu} (q-p_2)^{\sigma} - \eta^{\sigma\nu} (2p_2+q)^{\lambda}] = \frac{i}{(4\pi)^{d/2}} \frac{e^2 m_W^2}{v} \epsilon^*(p_1) \cdot \epsilon^*(p_2) \left[ \int dxdy \frac{(\pi-x-y+4xy)m_h^2}{m_W^2 - xy m_h^2} + 6(d-1)\Gamma(2-d/2) \int_0^1 \frac{dxdy}{(m_W^2 - xy m_h^2)^{2-d/2}} \right] \quad (25)$$

$$iM^{(f)} = (-2i\lambda v) (-ie)^2 \epsilon_{\mu}^* (p_1) \epsilon_{\nu}^* (p_2) \int \frac{d^d q}{(2\pi)^d} (2q-p_1)^{\mu} (2q-p_2)^{\nu} \times D_s(q) D_s(q-p_1) D_s(q+p_2) = \frac{i}{(4\pi)^{d/2}} \frac{e^2 m_h^2}{v} \epsilon^*(p_1) \cdot \epsilon^*(p_2) \Gamma(2-d/2) \int \frac{2dxdy}{(m_W^2 - xy m_h^2)^{2-d/2}} \quad (26)$$

$$iM^{(g)} = \left( -\frac{im_W^2}{v} \right) (ie)^2 \epsilon_{\mu}^* (p_1) \epsilon_{\nu}^* (p_2) \int \frac{d^d q}{(2\pi)^d} (-1)(q-p_1)^{\mu} q^{\nu} \times D_s(q) D_s(q-p_1) D_s(q+p_2) = -\frac{i}{(4\pi)^{d/2}} \frac{e^2 m_W^2}{v} \epsilon^*(p_1) \cdot \epsilon^*(p_2) \Gamma(2-d/2) \int \frac{dxdy}{(m_W^2 - xy m_h^2)^{2-d/2}} \quad (27)$$

$$iM^{(h)} = iM^{(i)} = \frac{ig^{\mu\lambda} g^2 v \sin\theta_w}{2} (-ie)^2 \epsilon_{\mu}^* (p_1) \epsilon_{\nu}^* (p_2) \int \frac{d^d q}{(2\pi)^d} (q-p_1-k)^{\sigma} \times [\eta_{\sigma\lambda} (2q+p_2)^{\nu} - \eta_{\lambda}^{\nu} (q-p_2)^{\sigma} - \eta_{\sigma}^{\nu} (2p_2+q)_{\lambda}]$$

$$\times D_W(q) D_s(q-p_1) D_W(q+p_2) = \frac{i}{(4\pi)^{d/2}} \frac{e^2 m_W^2}{v} \epsilon^*(p_1) \cdot \epsilon^*(p_2) \left[ \int dxdy \frac{(1-x)(1+y)m_h^2}{m_W^2 - xy m_h^2} - \frac{1}{2} (\pi d - 1) \Gamma(2-2/d) \int \frac{dxdy}{(m_W^2 - xy m_h^2)^{2-2/d}} \right] \quad (28)$$

$$iM^{(j)} = \frac{ig^2 v}{2} \left( \frac{ig^2 v \sin\theta_w}{2} \right)^2 \epsilon^*(p_1) \cdot \epsilon^*(p_2) \times \int \frac{d^d q}{(2\pi)^d} D_s(q) D_W(q-p_1) D_W(q+p_2) = \frac{i}{(4\pi)^{d/2}} \frac{e^2 m_W^2}{v} \epsilon^*(p_1) \cdot \epsilon^*(p_2) \int dxdy \frac{2m_W^2}{m_W^2 - xy m_h^2} \quad (29)$$

$$iM^{(k)} = iM^{(l)} = \frac{ig^2 v \sin\theta_w}{2} (-ie)^2 \epsilon_{\mu}^* (p_1) \epsilon_{\nu}^* (p_2) \times \int \frac{d^d q}{(2\pi)^d} (p_1+2p_2+q)^{\mu} (2q+p_2)^{\nu} D_s(q) D_W(q-p_1) D_W(q+p_2) = \frac{i}{(4\pi)^{d/2}} \frac{e^2 m_W^2}{v} \epsilon^*(p_1) \cdot \epsilon^*(p_2) \Gamma(2-2/d) \int \frac{dxdy}{(m_W^2 - xy m_h^2)^{2-d/2}} \quad (30)$$

$$iM^{(m)} = (-2i\lambda v) \left( \frac{ig^2 v \sin\theta_w}{2} \right)^2 \epsilon^*(p_1) \cdot \epsilon^*(p_2) \times \int \frac{d^d q}{(2\pi)^d} D_W(q) D_s(q-p_1) D_s(q+p_2) = \frac{i}{(4\pi)^{d/2}} \frac{e^2 m_W^2}{v} \epsilon^*(p_1) \cdot \epsilon^*(p_2) \int dxdy \frac{m_W^2}{m_W^2 - xy m_h^2} \quad (31)$$

## Results

Our results for the total decay width, the cross section as well as the branching ratios can be plotted to show in the figures.

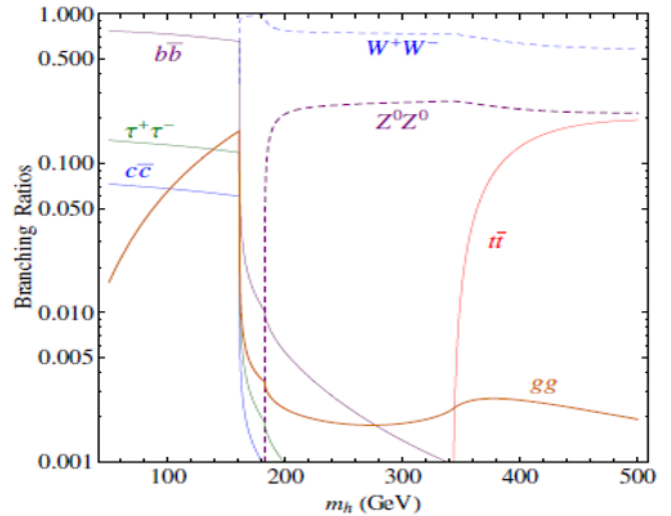


Figure 1 Branching ratios vs. Higgs mass.

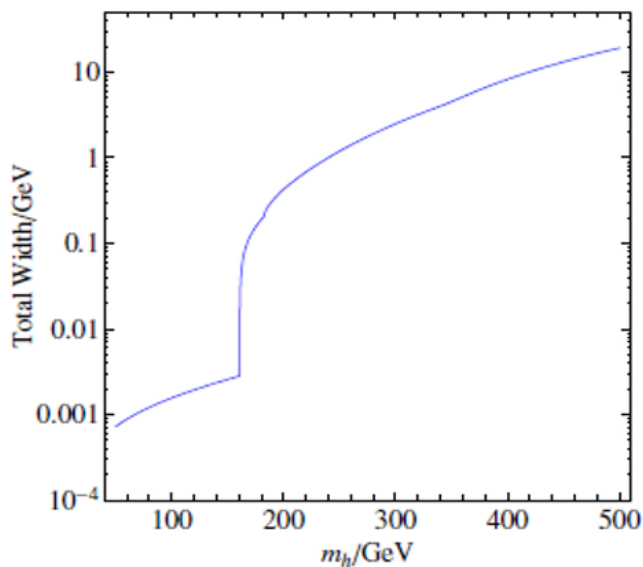


Figure 2 Total width vs. Higgs mass.

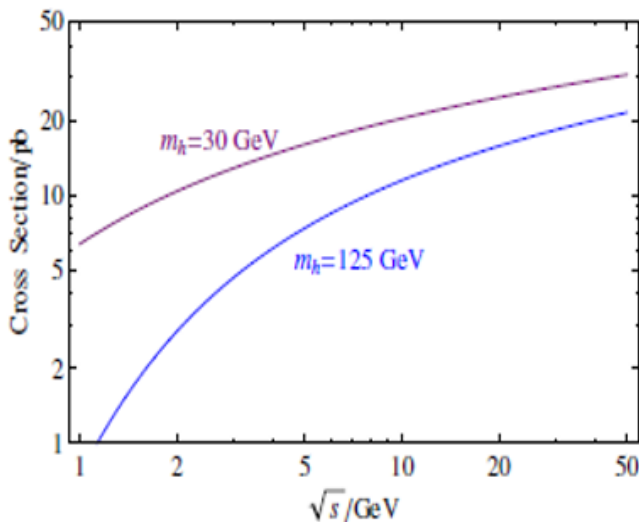


Figure 3 Cross section vs. center-of-mass energy.

## Conclusion

Not only does the Higgs boson give mass to quarks but also undergoes decay via a number of channels from which a few have been discussed in this paper. Experimental discovery of these decay modes have given room for unknown potential particles that may

contribute mass to be discovered in future experiments. Studying the Higgs boson tells us that it is capable of interacting with other particles like quarks and give them mass.

## Acknowledgments

None.

## Conflicts of Interest

None.

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