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Relativistic life time of spinning black holes

Abstract

The present research paper discusses the relativistic study of the life time of black holes using the model $\Gamma = 2.098\left(\frac{M}{M_{\odot}}\right)^3 \times 10^{67}$ years as proposed by Stephen Hawking and concludes that the relativistic life time of spinning black holes is dependent of mass as well as spinning velocity of black holes.

Keywords: Life time, Relativistic life time and Spinning velocity.

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Introduction

The creation of black hole takes place after supernova explosion due to extremely gravitational contraction of the core part of the dying star and has limited life time.[1] This limit of life time is not small, but very long. The black hole absorbs everything and emits nothing as per classical concept, but quantum mechanically, there is a possibility to produce a pair production of particles at the event horizon of black hole in which one is able to tunnel the gravitational barrier and other escapes from the horizon of black hole and can radiate or evaporate particles.[2,3] Hawking introduced what is now called Hawking radiation as the effective black body radiation from a black hole in terms of the 4th power of the black hole temperature and the Stefan-Boltzmann constant.[3,4] The statistical analysis of lifetime and temperature of the black holes have been studied existing in X-ray binaries and active galactic nuclei.[1] The model for life time of black holes as given by formula $\Gamma = 2.098\left(\frac{M}{M_{\oplus}}\right)^3 \times 10^{67}$ years is transformed in terms of Chandrasekhar limit [M_{ch}] and calculated their values of different black holes existing in XRBs and AGN.[5]

The present work discusses the relativistic study of the life time of black holes using the model $\Gamma = 2.098\left(\frac{M}{M_{\oplus}}\right)^3 \times 10^{67}$ years as proposed by Stephen Hawking.

Theoretical discussion

The evaporation time (in year) of a black hole of mass M in terms of fundamental parameters is given by the following equation:[6]

$$\Gamma = \frac{5120\pi G^2 M^3}{\hbar c^4} \tag{1}$$

In terms of solar mass, the above equation can be written as.[7]

$$\Gamma = 2.098\left(\frac{M}{M_{\odot}}\right)^3 \times 10^{67} \tag{2}$$

Some of the black holes have their spinning velocity from 50% to 99% of the velocity of light[8] and hence the mass of black hole does not remain constant, but varies with velocity as per Einstein's special theory of relativity as.[9]

$$M = \frac{M_0}{\sqrt{1-v^2/c^2}} \tag{3}$$

Where M_0 is the rest mass and v be the spinning velocity of black holes.

or

$$M = M_0(1 - v^2/c^2)^{-1/2} \tag{4}$$

$$M = M_0\left[1+\frac{1}{2}(v/c)^2+\frac{3}{8}(v/c)^4+\frac{5}{16}(v/c)^6+........\right] \tag{5}$$

Since, v< c, hence

$$\frac{v}{c} < 1,\ \left(\frac{v}{c}\right)^2 << 1,\ \left(\frac{v}{c}\right)^4 <<< 1,\ \left(\frac{v}{c}\right)^6 <<<< 1 \text{ and so on} \tag{6}$$

Hence, it is clear that the terms of higher power of v/c in equation (5) can be neglected and finally, we have

$$M = M_0\left[1+\frac{1}{2}(v/c)^2\right] \tag{7}$$

Putting the above value in the equation (2), we have

$$\Gamma_{Rel} = 2.098\left(\frac{M_0}{M_{\odot}}\right)^3 \times \left\{1+\frac{1}{2}\left(\frac{v}{c}\right)^2\right\}^3 \times 10^{67} \tag{8}$$

Now

Expanding the term $\left\{1+\frac{1}{2}\left(\frac{v}{c}\right)^2\right\}^3$ by using the formula, we have

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$\left\{1+\frac{1}{2}\left(\frac{v}{c}\right)^2\right\}^3 = 1+\frac{3}{2}\left(\frac{v}{c}\right)^2+\frac{3}{4}\left(\frac{v}{c}\right)^4+\frac{1}{8}\left(\frac{v}{c}\right)^6$$

Using the condition(6), the terms of higher power are neglected, we have

$$\left\{1+\frac{1}{2}\left(\frac{v}{c}\right)^2\right\}^3 = 1+\frac{3}{2}\left(\frac{v}{c}\right)^2 \tag{9}$$

Putting the above value in the equation(8), we ha

$$\Gamma_{Rel} = 2.098\left(\frac{M_0}{M_{\oplus}}\right)^3 \times \left\{1+\frac{3}{2}\left(\frac{v}{c}\right)^2\right\} \times 10^{67} \tag{10}$$

The above equation is known as relativistic life time of spinning black holes with spinning velocity v and can be written as:

$$\Gamma_{Rel} = 2.098\left(\frac{M_0}{M_{\oplus}}\right)^3 10^{67} + 2.098\left(\frac{M_0}{M_{\oplus}}\right)^3 \frac{3}{2}\left(\frac{v^2}{c^2}\right)10^{67} \tag{11}$$

$$\Gamma_{Rel} = 2.098\left(\frac{M_0}{M_\oplus}\right)^3 10^{67} + 3.147\left(\frac{M_0}{M_\oplus}\right)^3\left(\frac{v^2}{c^2}\right)10^{67} \quad (12)$$

The equation(12) consists of two parts:

It is clear from above equation that the first part is independent of velocity and exactly the same to the life time due to non-relativistic effect, while the second part depends on velocity and rest mass. This gives additional effect due to relativity. This concludes that the relativistic life time is greater than the non-relativistic life time.

Let us consider the equation (10) for the study of relativistic life time of spinning black holes having spinning velocity 50% to 100% of the velocity of light.

For the spinning black holes having the velocity 50% of the velocity of light (v/c=1/2), the relativistic life time is given by the following equation

$$\Gamma_{Rel} = 2.098\left(\frac{M_0}{M_\oplus}\right)^3 \times\left\{1+\frac{3}{8}\right\}\times 10^{67} \quad (13)$$

$$\Gamma_{Rel} = 2.098\left(\frac{M_0}{M_\oplus}\right)^3 10^{67}\left(\frac{11}{8}\right) \quad (14)$$

$$\Gamma_{Rel} = \left(\frac{11}{8}\right)\Gamma \quad (15)$$

The above equation shows that the life time (ΔT) is increased by 37.5% to that of the black holes due to non-relativistic effect and the life time of the black hole becomes 137% due to relativistic effect (Γ_{Rel}).

$$\Gamma_{Rel} = 2.8847\left(\frac{M_0}{M_\odot}\right)^3 \times 10^{67} \quad (16)$$

For the spinning black holes having the velocity 60% of the velocity of light, the relativistic life time is given by the following equation.

$$\Gamma_{Rel} = 2.098\left(\frac{M_0}{M_\oplus}\right)^3 \times\left\{1+\frac{27}{50}\right\}\times 10^{67} \quad (17)$$

$$\Gamma_{Rel} = 2.098\left(\frac{M_0}{M_\oplus}\right)^3 10^{67}\left(\frac{77}{50}\right) \quad (18)$$

Graph 1

$$\Gamma_{Rel} = \left(\frac{77}{50}\right)\Gamma \quad (19)$$

The above equation shows that the life time ($\Delta\Gamma$) is increased by 54% to that of the black holes due to non-relativistic effect and the life time of the black hole becomes 154% due to relativistic effect (Γ_{Rel}).

Where M_0 = Rest mass of spinning black holes and $M_\oplus$ = Mass of the sun used in the present work.

Similarly for the velocity from 70% to 100% of the velocity of light, the relativistic life times are calculated and tabulated in the following table, we have

Table 1 Relativistic life time of spinning black holes

S No.	The velocity of spinning black holes of the velocity of light (in %)	Relativistic life time of spinning black holes (Γ_{rel} in %)	The life time of spinning black holes increased due to relativistic effect to that of Non-relativistic effect (ΔT in %)
1	50	137.0	37.5
2	60	154.0	54.5
3	70	173.5	73.5
4	75	184.4	84.4
5	80	196.0	96.0
6	85	208.3	108.3
7	90	221.5	121.5
8	95	235.0	135.0
9	99	247.0	147.0
10	100	250.0	150.0

For the black holes having maximum rate of spinning velocity (v=c), equation (10) becomes as follows:

$$\Gamma_{Rel} = 2.098\left(\frac{M_0}{M_\oplus}\right)^3 \times\left\{1+\frac{3}{2}\right\}\times 10^{67} \quad (20)$$

For the black holes having mass equal to the mass of our sun, the above equation becomes

$$\Gamma_{Rel} = 5.245\times 10^{67} \text{ years} \quad (21)$$

Fit 1: Linear, Equation, Y = 2.298303346 X +15.88641098, NDPU = 10, $\bar{X}$ = 0.55, $\bar{Y}$ = 0.126962, RLSS = 120.058, RSS = 13387.2, COD = 0.991112, Residual mean square, sigma-hat-squared = 15.0072.

Fit.2: Polynomial, Y=0.01457514983X^2+0.06853315927X+97.24990752,, Degree=2, NDPU = 10, $\bar{X}$ = 80.4, $\bar{Y}$ = 200.67, RLSS = 0.175161, RSS = 13387.2, COD = 0.999987,

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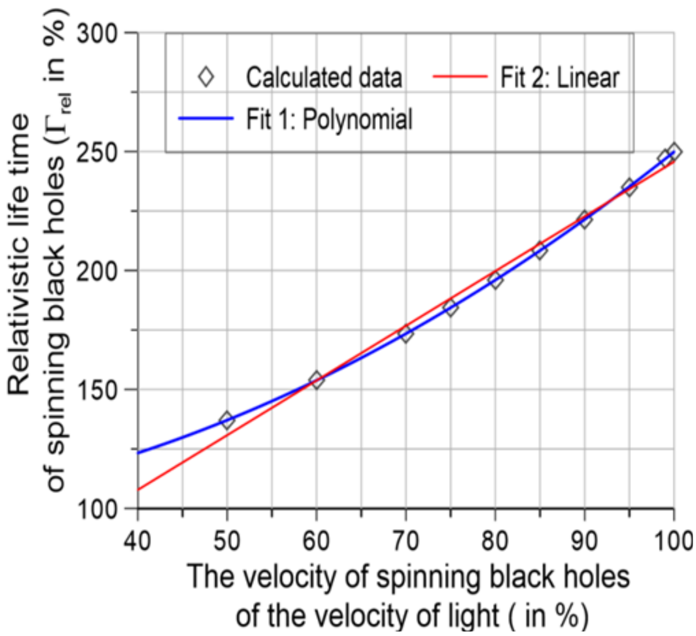

Figure 1 The figure shows the graph between the % speed of spinning BHs of velocity of light and their corresponding % relativistic life time of spinning black holes.

Graph 2:

Fit 3: Linear, Equation, Y = 2.28828125 X -83.2078125, NDPU = 10, $\bar{X}$ = 80.4, $\bar{Y}$ = 100.77, RLSS = 126.617, RSS = 13270.7, COD = 0.990549, Residual mean square, sigma-hat-squared = 15.8271.

Fit.4: Polynomial, Y=0.01496181145X^2 - 0.0006421241055X+0.314162157, Degree=2, NDPU = 10, $\bar{X}$ = 80.4, $\bar{Y}$ = 100.77, RLSS = 0.289353,, COD = 0.999987.

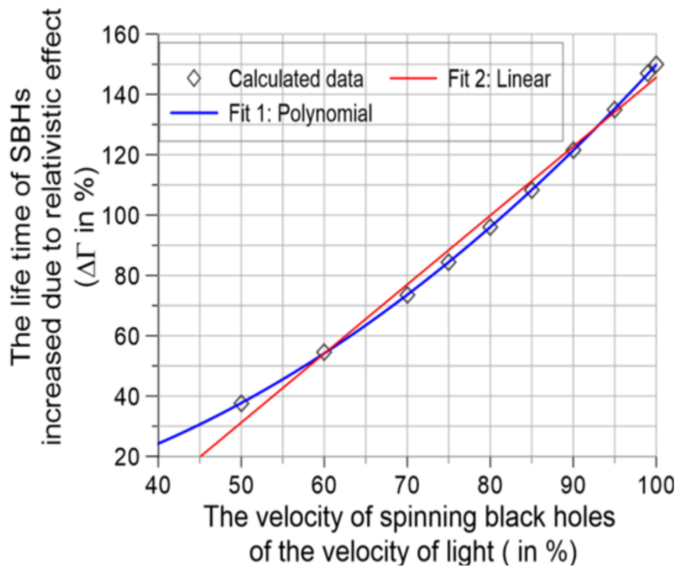

Figure 2 The figure shows the graph of the percentage velocity of spinning black holes of the velocity of light and their corresponding life time of Spinning Black Holes increased due to relativistic effect.

Results and discussion

In the present work, we have started our work with the model for life time of black holes represented by the equation (1). In the present model, we have applied the variation of mass with velocity to get the relativistic effect of the life time of spinning black holes. It is clear from the equation (2), the life time of the black hole about some multiple of 10^{67} years, which is enough long period. After applying the relativistic effect on the life time of the spinning black holes, we obtain the equation (12) for the relativistic life time of the black hole which is greater than the life time of the black holes due to non-relativistic effect. The life time for the both cases, are so long period that the age of the universe is very small in compared with the life of the black holes. In a particular case of black hole having mass equal to the sun, the life time is too long.

From the observation of Figure 1 of the percentage of spinning velocity of BHs of speed of light along x-axis and the corresponding relativistic life time of spinning black holes along y-axis, it is clear that the variation of relativistic life time of spinning black holes with % velocity (v/c) is followed by the linear equation represented by $y = mx + c_1$ and polynomial equation of degree two represented by $y = ax^2 + bx + c_2$ where m=2.298303346, c_1=15.88641098, a=0.01457514983, b=0.06853315927, c_2=97.24990752 with slope 2.298303346 and fitting accuracy equal to 0.991112 of linear equation showing that the proposed model is very good fit for the data and of 99.11% accuracy of or proposed model, it is explained to get predicting outcome, while for polynomial nature of degree 2 of graph for the same model gives fitting accuracy equal to 0.999987 showing the best fitting for the data and of 99.99% accuracy of proposed model, which is explained to get predicting outcome.

From the observation of Figure 2 of the percentage of spinning velocity of BHs of speed of light along x-axis and the corresponding relativistic life time of the SBHs increased(dT) due to the RE along y-axis, we see that the changing of relativistic life time of spinning black holes with % velocity (v/c) is followed by the linear equation represented by $y = mx - c_1$ and polynomial equation of degree two represented by $y = ax^2 + bx + c_2$ where m=2.28828125, c_1=83.2078125, a=0.01496181145, b=0.0006421241055, c_2=0.314162157 with slope 2.28828125 and fitting accuracy equal to 0.990549 of linear equation showing that the proposed model is very good fit for the data and of 99.05% accuracy of proposed model. It is explained to get predicting outcome, For polynomial nature of degree 2 of graph for the same model gives fitting accuracy equal to 0.999987 showing the best fitting for the data and of 99.99% accuracy of proposed model.

Conclusion

The life time of the black holes due to relativistic effect increases with increasing the spinning rate of black holes and it is twice time greater than to that the life time of the black holes due to Non-relativistic effect for the spinning velocity of black holes equal to the velocity of light and mass equal to the sun.

The non-relativistic life time of spinning black holes is dependent of mass, but independent on spinning rate of black holes. The relativistic life time of spinning black holes is dependent of mass as well as spinning rate of black holes.

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