Review Article

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Future math teachers 'knowledge about fair games

Abstract

This work reports the results of a study whose objective was to analyze the didactic-mathematical knowledge of future mathematics teachers when solving an unequal game. The sample was made up of 24 future professors from a Chilean university. Task-based interviews were used to collect the information. The results show that future teachers have adequate intuitive ideas about expected value, prioritize the classical approach to solve situations and determine if the game is fair or not, taking into account both the probability of winning or losing the game, and the value that is gained or lost, using few solution strategies, which makes it easier for them to combine their statistical thinking with its teachableness. It concludes on the need to improve the teachability of probability, in both disciplinary and didactic aspects, suggesting support for teachers, both in training and in practice.

Keywords: probability and statistics, teachers in training, expected value, fair games, teachable probability.

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Introduction

Despite the fact that data, variation and chance are omnipresent in human life, in academic contexts, because it is considered "part of the necessary cultural heritage for every citizen, being valued by various educational and political agents as a basic component to function effectively in the information society",[1] it is common to find conflicts related to the mathematical management of situations that involve these topics.[2-4] However, the need that people in communities have to interpret reality and its phenomena, and to build probabilistic models to measure uncertainty,[5] make probability and statistics an instrument that enables them to information management, which allows them to make assertive decisions.[6] In this sense, the processing and analysis of information become a source of signifiers and the basis for interpreting results that can be very useful in people's community life, helping them to be increasingly critical[2,7] and therefore better decision-making. The foregoing reveals the need to strengthen the training of those who guide the teaching and learning processes on probability and statistics,[4] in such a way that it guarantees an adequate enculturation of the communities in these topics. However, many teachers have never studied applied statistics or been involved in data analysis activities.[2,8] Likewise, they have not received training on probability and statistics that allows them to carry out suitable teaching in the classroom, which could lead them to teach erroneous concepts.[9] However, [10]reports that teachers show interest in probability and statistics, but at the same time, they are concerned about the lack of knowledge and experience in these topics. This lack of knowledge or experience has meant that many of these teachers do not achieve a level of understanding that allows them to explore the different meanings that underlie the solution of everyday probability problems, or that help them understand the errors that, in the development of their teaching activity, commit their students, and often themselves.[10] That is, a training is necessary in which teachers can understand what students know, as well as what they need to know and, therefore, give support guidelines that challenge their students to acquire new knowledge,[11] and at the same time, are aware of their own learning needs that lead them to implement actions to overcome them. In relation to the above, "there is a general consensus that teacher training is an essential element for the quality of education in the community where the teacher develops his professional practice".[12] This leads to the assumption that if the central elements of statistical thought are data systems, variation, chance, and uncertainty, then the instructional processes related to their teaching must aim to provide students with the necessary skills to to be able to notice, recognize, explain and treat them.[10] What emphasizes the importance of a solid formation in probability and statistics by future mathematics teachers, if a solid formation is wanted in the students that they will guide in the future,[11] since the inadequate development of these formative processes, can cause limitations in the teacher's mathematical knowledge, and inhibit the comprehension processes of their students. In this framework, probability and decision-making is at the base of the justification of the probability curriculum of the State of Chile,[13] in this sense, the work with probabilit

y cannot be only descriptive, but rather allow the decision making. One aspect that Elicer and Carrasco[13] show that the basis for the construction of ideas about probability, is in the determination of whether a game is fair or not. Fact that was at the base of the epistolary exchange between Pascal and Fermat. Then their teaching becomes central and for this it is fundamental in the knowledge of the teacher. Therefore, in this work, the objective was to analyze the didactic-mathematical knowledge that future secondary school teachers show when solving problem situations of unfair games. Emphasis was placed on the semiotic activity of the participants when solving probability problems, which could help them minimize conceptual errors and difficulties.

The improvement in the knowledge that teachers build on probability and statistics, as well as the tools and resources that improve their teachability, requires the ability to learn from their own experiences and to adapt their practices to the conditions of the sociocultural context where the study takes place. teaching and learning process,[14] as a natural process, in their development. For Da Ponte[12] the process of training probability and statistics teachers needs special support, which allows them to interact with peers, share and develop joint experiences, articulate topics of mutual interest, share resources and continue to develop professionally after graduation. Title. The competencies suggested by Contreras[14] and Da Ponte[12] are adaptive, that is, they allow the teacher to learn from their own practice as they develop professionally. This is a crucial aspect in the training of a teacher, who seems to be training with current needs, to meet future needs. The training of a teacher, which leads him to acquire adaptive skills, requires work with problems, which leads him to identify them in any context, to adapt or create them according to the learning needs of the communities where the processes of teaching and learning, and to share their experiences with peers.[15] The creation of joint activities would make the knowledge public, in such a way that all colleagues in the area can use it. The activities created jointly should lead the learner to integrate social processes, producing multiple representations of the mathematical objects studied, connecting these representations with each other and with conceptual elements and the sociocultural context, so that the student can understand the concepts in the most comprehensive possible. 1.1 Records and representations in statistics A representation of a mathematical object is any concrete configuration that is made of it,[16] but since none of them describes it in its entirety, and each one can have different forms, it is necessary to produce and connect the greater number of representations, to facilitate a better conceptual image of the studied object. In addition, the use of multiple representations can facilitate the development of different ideas and processes in students, as well as the assignment and restriction of meanings, and these processes as a whole promote a deep and comprehensive understanding of the object being studied (Elia et al., 2007). Therefore, the use of multiple forms of representation and the connections that can be made between them in learning mathematics is very relevant.[16] 1.2 Probability and statistics in the Chilean curriculum The tendency of school statistics curricula in many parts of the world is for chance and data to be introduced in parallel and progressively in the teaching of mathematics, without being considered an independent area, where the ability to establish in students is promoted. connections between the mathematical concepts studied, with other areas of knowledge, and with everyday life experiences.[11] In the case of the Chilean curriculum, the study of data and probability has been incorporated as one of the fundamental areas,[17] where in the treatment given to probability, work with everyday situations predominates. that lead to concepts such as 'possible', 'sure' or 'impossible', among others, to then continue with a frequentist approach to probability[5] with little articulation with data management, rather regarding games. Likewise, it is reported in said curriculum "firstly, the absence of activities associated with the study of probability and, secondly, an adequate and gradual approach towards the construction of certain basic notions of statistics".[18] Under these conditions, an attempt has been made to establish a continuum in basic education, where both the understanding of ideas of chance and probability, present in everyday situations, and the calculation of simple probabilities through experimentation and application of theorems are gradually reached.[19]

The interest in the study of probability and statistics has generated in the Chilean curriculum both new social demands and research interests[5,19,20] which have led to curricular rethinking on the way of teaching this area.[19] Among the suggestions that have emerged, as a result of said investigations, is that of Vásquez,[5] who suggests that whoever guides the teaching processes of probability, implements probabilistic tasks around the resolution of problems, in which uncertainty is present, which allow their students to explore and reflect on chance and probability, and thus build new knowledge, which stimulates in them the development of their probabilistic reasoning. For their part, Batanero et al.[6] suggest that, in order to learn probability, the apprentice must have the opportunity to solve various probabilistic situations associated with the various meanings of probability: intuitive, frequency, classical, subjective and axiomatic. According to Vásquez,[5] the trend is for the study of probability to be incorporated from the first years of schooling, and for it to be done progressively and continuously throughout the school mathematics curriculum. Thus, it is expected to develop in students a solid probabilistic knowledge, which facilitates the understanding and communication of different types of information present in many situations of daily life. That is, they develop critical thinking. This trend has also permeated teacher training programs in pedagogy for the teaching of mathematics, which have seen the need to adjust both to social demands and to curricular trends in the world today,[19] to fulfill its purpose of training someone, to guide the understanding of others. 2 METHODOLOGICAL ASPECTS The study is part of a qualitative and exploratory methodology, in which attention is focused on identifying the didactic-mathematical knowledge,[21,22] considered necessary for future teachers of mathematics, they can develop their professional practice.[23] 2.1 Context and participants in this study, 24 teachers in training from a Pedagogy program in secondary education in mathematics from a university in southern Chile participated. Said participants were between the ages of 21 and 30 and, at the time of data collection, were finishing the sixth semester, had taken the subjects offered by the program on probability and statistics (Chance and probabilities, Probabilities and inferential statistics), and they were studying Didactics of probability and statistics. In addition, they had already completed two of the four pedagogical internships offered in the program and they only needed to do the professional internship. The program lasts for 9 semesters. 2.2 Instruments and techniques to collect information The information was collected using semi-structured interviews based on the tasks that the participants solved,[24] which are appropriate to know the ideas and mathematical procedures that the students use,[25] since they facilitate an interaction with the solver, during, or immediately after, solving the task, thus evidencing their knowledge, their behavior and reasoning in solving said task.[26] In this way, the structure of the interviews were the same questionnaires that each teacher in training was solving, incorporating questions such as why did you do that in that way? What are you basing yourself on to do it? Or explain how you did it.

Six questionnaires were used, which were validated by experts and adjusted according to their suggestions. In the questionnaires, problems were raised with inequitable games, with a single prize and a single player, where the player may or may not obtain a prize, in the event of one occurring between the two events of a random experiment.[27] In each game, the future teachers are asked to determine if the game is fair or not, and if it is not, to find a way to make it fair. We agree with Guerrero et al.[27] in which the chosen context, with games of chance, is propitious for the joint study of probability, discrete random variables and the concept of mathematical expectation. For their part, Elicer and Carrasco[13] highlight, in light of the correspondence review between Pascal and Fermat, that probability arises from projective decision-making, in the effort to establish fair participation, particularly in an interrupted game. In this way, this type of problems related to fair play, offers an educational context that was the origin of the theory of probabilities. The situations were adapted from exercises available on multiple Internet pages (for example: http://cb.mty.itesm.mx/probabilidad/materiales/ma1006-02-02.pdf;

https://l.exam-10. com/ekonomika/20606/index.html;

https://www.clame.org.mx/documentos/alme%2011.pdf).

Table 1 below shows two of the situations used to collect the information, which are used to give examples in this report.

The first item of activity 1, puts the teachers in training to use their common and extended knowledge of the content, when solving a proposed task, being able to show different solution strategies, and use previous or more advanced knowledge to solve it. While the second item seeks to generate alternatives that facilitate the transition from common knowledge of the content, to knowledge on the mathematical horizon[21] or expanded knowledge of the conten,[22,24] which lead them to propose different solution strategies and connect the elements of the different representations produced, until they find a way to make the game fair. The whole task as a whole, seeks to project teachers in a brainstorming activity, mobilizing their mathematical knowledge on the subject and projecting its teachability. This type of activity, according to Afeltra et al.[3] allows teachers to recognize or project possible difficulties with the solution of the task and, consequently, realize the need to design teaching strategies that help their future students to minimize them. The information was collected in the second semester of 2019, in five work sessions, in the development of the Didactics of probability and statistics class, and the indications to solve the questionnaires were given in writing. However, some clarifications were made to resolve doubts that arose when solving the tasks, the meetings were videotaped, and the striking events that resulted from the interaction with the future teachers, or between them, were recorded in a field diary. For the analysis of the information, indicators and analysis categories are determined, which are shown in Table 2.

Table 1 Example of the situations that were worked on

Situation	Basic questions for which they were investigated
Situation 1. In a game with dice in which two players participate (we will call one the player and the other the bank), where each player has only $10,000 to play with, the following rules are set: a) Only the player rolls the dice b) Every time the player rolls a sum of seven on the upper faces of the two dice, the bank must pay $3,000. c) Every time the player rolls any amount other than seven he must pay the bank $1,000. d) The results of each move are recorded on a game sheet and how much money the player and the bank have at the end of each roll of the dice. e) The player with the most money at the end of the game is the winner, if one of the players goes bankrupt, the game is over and the other player is the winner.	a. Is the game fair? Why? b. If you conclude that the game is not fair then propose a way to make it fair.
Situation 2. A man has time to play roulette at most five times. In each game he bets $10,000, that is, he wins or loses $10,000 in each game. He starts with $20,000 and will stop playing before the fifth game if he loses all of his money or if he wins $30,000 (if he completes $50,000). If at the end of the game the man has won $30,000, that value is doubled, otherwise he loses everything and must pay $20,000 more.	a. Is the game fair? Why? b. If you conclude that the game is not fair then propose a way to make it fair.

Game sheet (Situation 1):

	shots	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	No. 9	No. …
	Addition 7	si	si	si	si	si	si	si	si	si	si…
		no	no	no	no	no	no	no	no	no	no…
Player	$10.000										
Banco	$10.000										

Source: self made

Table 2 Categories of analysis and indicators or criteria used for the analysis

Analysis Categories	Criteria for analysis
Solution given to the task	a. Identification and use of probability and prize, which facilitate solving the problem b. Construction of the sample space and its use to solve the task
Establishment of connections between the elements of the representations when solving the task	a. Use given to the elements produced b. Type of strategy used to return the game fair
conception of probability	a) Type of meaning given to probability, used when solving the task b) Interpretation given to the expected value

The categories of analysis made sense, to the extent that they facilitated projecting the analysis that was expected to be carried out, and as guides to make the corresponding adjustments, to the unexpected alternatives that arose in the development of the process. Thus, in the solution that the future teachers gave to each proposed task, it was projected that they elaborate the sample space, find the probability of occurrence of the corresponding event, and use it to find the expected value, and return the game fair; establishing connections between the different representations that they managed to produce, by using the different meanings of probability. 2.3 Treatment and analysis of information Both oral and written information was processed using the content analysis technique,[28] grouping basic ideas by thematic criteria, attending to both the notions of greatest frequency of use[29] and the categories of analysis. Both the conceptual errors committed by the trainee teachers in solving the task, as well as their progress in their training process, were analyzed. Then the results were compared with the theoretical sources consulted, and their description was made.

Results and analysis of the information

In this section, the results and analysis of the work in general are presented, but examples of only two of the situations worked are given. As requested in each activity, the preservice teachers focused on two aspects: answering if the game is fair or not, and after justifying why it is not fair, looking for ways to make it fair. Regarding the use of the different meanings of probability to demonstrate the different conceptions and the errors that the future teachers made when resolving each situation, it is noted, the use of the intuitive approach, in the enthusiasm that it produced in them, that they worked with games, like rolling dice, and then making inferences about which sums occur most frequently. In this sense, we agree with Alvarado et al.,[20] regarding the influence that sociocultural contexts can have on informal ideas of probability, as well as the need to confront the previous conceptions of students, with formal probabilistic concepts, in favor of the objectives sought by the school, that help to minimize the limitations of the teaching of probability. However, the expected work was not glimpsed, regarding the making of inferences about which sums appear more frequently in the table, and their use in estimating the hope of winning the game.[6] In the answers communicated by this group of teachers in training, when facing the tasks that involve games with dice, it can be seen that they first dedicated themselves to throwing the dice and with the results of the sums of the upper faces, they filled out the corresponding table. More than an action based on those probability meanings that, as compulsory school graduates, they should have achieved, an action is observed that responds to an intuitive approach to frequentist probability, which led them to complete the table, without using this information, to answer the questions that were asked. Despite the fact that the throw of the dice is one of the typical contexts to introduce ideas of probability, this group of trainee teachers, as in Batanero et al.[6] seems to have generated counterintuitive ideas about the representativeness of the sample and the fairness of the game. That is, their previous experiences with the throwing of a single die led them to consider, initially, that all the sums had the same probability of occurrence, and that games, like most of those seen in sociocultural contexts where inhabit are fair, however, with the interaction and the step by step of the interviews, they were correcting these misconceptions. After filling out the corresponding table, without using the information contained therein to find the probability of occurrence of any of the events, they started to use the classical approach, where it seems that it was easier for them to carry out the procedures and operations to obtain the answers, which allowed them to make a decision[20] about the fairness of the game. The change in approach from the frequentist to the classic, carried out by these teachers in training, could be due to the ease of identifying and using, in a traditional way, in the classical approach, inferences about the probability of occurrence of an event; however, Afeltra et al.,[3] state that it is necessary to handle different approaches to probability at the same time, since it is necessary for students to become aware of the intrinsic limits between these approaches. In fact, according to these authors, the game with dice allows to introduce the Law of large numbers in a significant way, and in the frequentist approach, there are strong tools to do so. In relation to the excessive use of the classic approach, Vásquez and Alsina,[19] warn that the mechanical and thoughtless resolution of exercises, through formulas that are applied, but are not understood, can often lead to the fact that, when trying to teach probability, students end up learning only arithmetic. Trying to answer whether the game is fair or not, the solutions given by this group of trainee teachers, although they bear a certain similarity, have some very differentiating aspects. 66.6% intuitively used the concept of mathematical expectation, but did not report its use until reporting the decision. All the future teachers identified the mathematical expectation as the concept to use to obtain elements that would allow them to compare the results of the game and make a decision about whether the game is fair or not. Initially they built the sample space, and only 8.3% elaborated the probability distribution, and although they were not asked to elaborate it, those who did, showed greater ease and clarity in their answers.[27] In the particular case of situation 1, the sample space is widely known and the future teachers built it without major difficulties, as can be seen in the P4 manuscript, shown in Figure 1. In addition, by direct observation, it was possible to appreciate, that no teacher-in-training attempted to find the relative frequency of the noted sums. However, they soon inferred that continuing to roll the dice and recording the sums was not the most appropriate way to reach an optimal solution to this situation. It is as if the intuitive experience linked to the game had come into conflict with the formal handling of the situation,[7] and then they began to build the corresponding sample space, and contrary to what was reported by Mohamed,[15] none of the participants had difficulty finding it. After building the sample space, they proceeded to find the mathematical expectation for the player: they counted the results of sum seven and wrote the probability for both this sum and its complement. Subsequently, they multiplied the probability of obtaining a sum seven with the value that the player gains for drawing it, and from this they subtracted the probability of the complement of sum seven, multiplied by the value that the player loses for not drawing seven, obtaining -330, as shown by P4 in his manuscript (Figure 1).

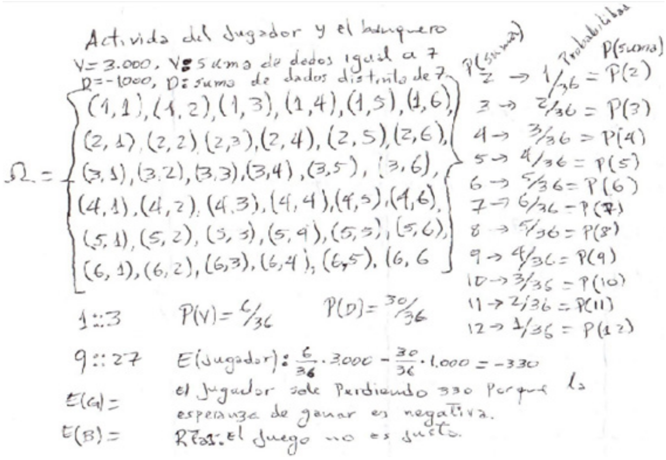

Figure 1 Response given by P4 to the activity proposed in situation 1.

Below is a fragment of the interview conducted with P4 when resolving situation 1 (Player vs. bank): Interviewer: Show me what you did to resolve the situation Q4: The first thing I did was I started rolling the dice Interviewer: How many times did you roll the dice? Q4: Ten times, as indicated by the table that was given to us Interviewer: Where does the table indicate that there are only ten rolls of the dice? P4: Here – and he pointed to the tenth square of the table, where the ellipses are- wait, there are not ten, the ellipses indicate that I can continue shooting indefinitely. But I only shot ten times Interviewer: What did you use the results of rolling the dice for? P4: With the results of the throw of the dice I did the sums and filled in the table Interviewer: According to the results you entered in the table, is the game fair or unfair? P4: Just by analyzing the table I can't know if the game is fair or not, because I would lack probability information. Interviewer: But I see that you advanced. Tell me how you did it? Q4: First I found the sample space and then the probabilities that I wrote here- shows your manuscript, the one in Figure 1- Interviewer: According to the results you obtained, is the game fair or unfair? P4: The game is not fair because the expectation that the bank will win is greater than the expectation that the player will win. Interviewer: How did you arrive at this answer? Q4: I calculated the expectation for the player and it gave me -330. What I did was that I looked for the probability that the sum would give seven and I multiplied the result by the value that the player wins with this sum, and I did the same with the corresponding value to be paid by the player for not throwing sum seven. Then I subtracted the two results and it gave me -330. Interviewer: What meaning does the minus sign that has the value of mathematical expectation have for you? P4: I interpret it as the player loses 330, that is to say, that the hope of winning is negative means that the player, at the end of the game, will lose 330 pesos. Like P4, the other students recorded less than ten rolls of the dice. This could be motivated by the structure of the table that was presented to them, which had only ten cells. This could also cause no one to try to find the probability from this information and it was only used to calculate the sums of the experimental values with which they filled the table. A reflection on the use of the information contained in the table was not appreciated, and after completing it, the future teachers began to build the sample space, and to calculate the expectation, which allowed them to decide whether the game was fair or not. Once they identified that the game is not fair, they proposed several alternatives to make it fair, thus, 54.2% changed the amounts to be won, and 37.5% modified the probability values, and only P1 and P7 contemplated both alternatives, in addition, the values used when changing the amounts varied from one teacher to another, it seems that they were satisfied with the first solution found. This aspect could originate from some deficiency in their expanded content knowledge[22,24] or Horizon content knowledge,[21] which is limiting them in the production of different alternative solutions to the same task, since they seem to focus on the development of skills, without being able to combine their ability to think statistically,[2] with the teachability of this discipline. Below is a fragment of the interview conducted with P1 when resolving situation 1 (Player vs. bank): Interviewer: How would you go about making the game fair? I mean, how would you make the game fair?

P1: What the bank receives is changed, instead of \$1,000, make it \$600 and then it becomes fair. Interviewer: How do you know that with those values the game becomes fair? P1: Because the mathematical expectation gives me zero. Interviewer: Did you find other ways to fair the game? P1: Yes, it is just looking for all the combinations of sums where the numerators of the probabilities have a sum of 36, like this one that I used here 9::27, and thus, in general, when finding the ratio where the sum of the two terms is 36, it is enough to appropriately change the value of the prize, either for the bank or for the player. Interviewer: It is not clear what you are saying, please explain it with another example P1: Look, in the case that the player plays with a probability of sum 3 or 5, it gives us 6/36, that of 9 or 11 also does not matter, then the bank would play with a probability of 30/36, since 6 +30 =36, so if the player wins \$5,000 on that sum and the bank wins the same \$1,000, the game would be fair since $\frac{6\times 5.000}{36}-\frac{30\times 1.000}{36}=0.$

Figure 2

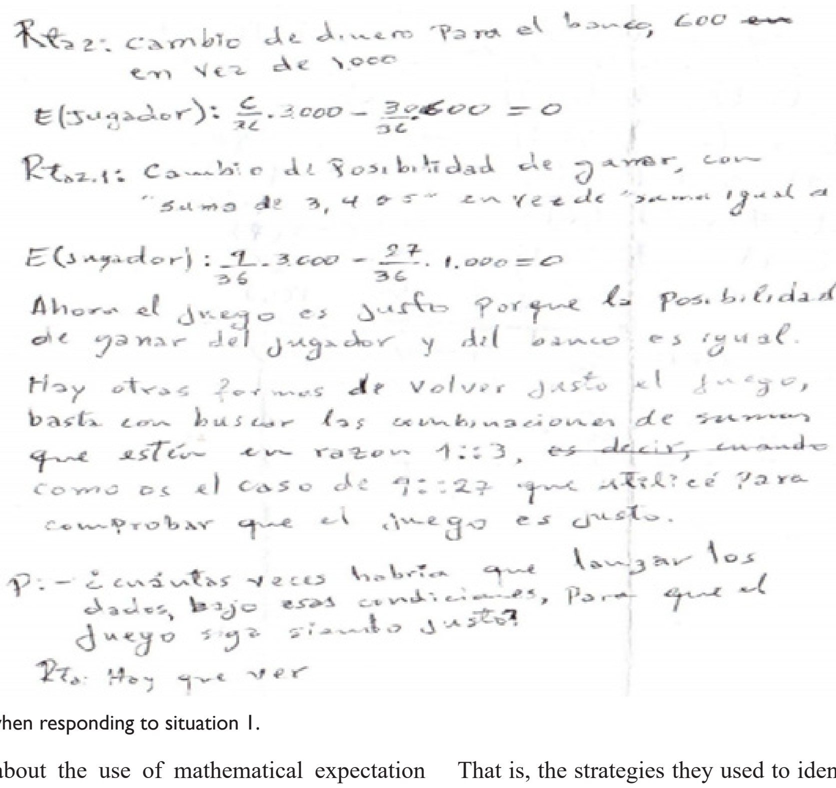

Figure 2 P1's manuscript when responding to situation 1.

The intuitive ideas about the use of mathematical expectation used by P1, and in general, by this group of teachers in training, are quite adequate, since they took into account both the probability of winning or losing the player, as well as the value that the player wins or loses. the player loses, to decide if the game was fair or not.[27,30] That is, the strategies they used to identify if the game is fair or not, when comparing the probabilities of winning or losing, using the corresponding prizes, were adequate, the difficulties arose was trying to make the game fair. Well, contrary to the solution given by P1, the difficulties were related to the calculation of the probabilities of winning, since they found solutions with non-composite basic probabilities, but in the case of the probability of a sum, they did not conceive that they should use two or more point probabilities to form the overall probability of the event. Furthermore, similar to what was reported by Guerrero et al.,[27] some of the future teachers seem to have no clarity about concepts such as union and interception of events, for which reason they used additive strategies instead of multiplicative or vice versa. For the case of situation 2 the sample space is S = {GGG, GPGGG, GPGPG, GPPGG, GPGPP, GPPGP, GGPGG, GGPPG, GGPPP, GPPP, GGPGP, PP, PGGGG, PGGPG, PGGPP, PGGGP, PGPGG, PGPPG, PGPGP, PGPP}, where G corresponds to the event the man wins a game, and P the man loses. In this game, in which there was no space for experimentation, and the phenomenon was not known to the future teachers, the construction of the sample space was difficult, so that only two of them (P1 and P7) managed to find it, the rest, while some had elements left over, because they repeated them, others lacked them, so their responses to this item were not appropriate. The difficulties in achieving the sample space led to difficulties in solving the rest of the task, and as in Batanero et al.[6] produced incorrect calculations of the player probabilities, which did not allow them to make an adequate analysis of the game. Despite these, the procedures carried out to find the expected value and decide if the game was fair or not, were adequate, invalidated because the probabilities used were incorrect, since without the sample space it is difficult to obtain the probability of occurrence of this type of event. events. Below is a fragment of the interview conducted with P7 when solving situation 2 (roulette player).

Interviewer: Show me how you resolved the situation P7: Actually, to get to this sample space, I have had to do it three times, and I know that this one is good. Interviewer: How do you know that this sample space is the correct one? Q7: Because I already verified it using the addition rule Interviewer: How did you do it? P7: Look, you are asked to select 3 out of 5 or 2 out of 5, and what the addiction rule suggests, for this is:

$\binom{5}{3}+\binom{5}{2}$ =10+10 =20 so the game can be developed in 20 ways, and since I already have the number of ways the game can be developed, then I started looking for them one by one, checking that no possibility was repeated, or that none was missing, and it gave me this – he pointed to the sample space in his manuscript shown in Figure 3 – Interviewer: Why did you divide the sample space into two parts? That is, why did you separate these events from the others? P7: It really is only one, but here I placed the cases in which the man wins – he points in his manuscript where he says he wins, in figure 3 – and here the cases in which he loses – he goes back and points in his manuscript, but now where it says lose – Interviewer: But in some events like {GGPGP}, the man doesn't lose, does he? P7: Not really, but he has to return what he wins, the money he started with, plus 20,000 pesos, due to the conditions of the game, so in the end he does lose. Interviewer: Is the game fair or not for you? P7: It is not fair, because if these cases occur {GGG, GGPGG, GPGGG, PGGGG}, the man at the end of the game wins 30,000, and since they are doubled it would be 60,000 pesos. But in these other cases, although in some he wins 10,000 pesos, he does not complete 50,000, so he loses everything and would have to pay 20,000 more. Interviewer: But why do you say that the game is not fair? Q7: Because when finding the mathematical expectation for the player it gives me negative, look:

$$\frac{4\times 60.000}{20}-\frac{16\times 40.000}{20}=--20.000$$

Interviewer: And how do you interpret that result? P7: The negative sign indicates that the player loses 20,000 pesos. Interviewer: And how would you go about fairing the game? P7: For the man to come out winning $30,000 as initially planned, it is only to give him $130,000 more as a prize for winning, without changing the rest, and thus the game becomes fair, look:

$$\frac{4\times 160.000}{20}-\frac{16\times 40.000}{20}=0$$

Figure 3 P7's manuscript when responding to situation 2, "man playing roulette.

Interviewer: Did you look at any other way to make the game fair? P7: Yes, but it's not really that there are many options like in dice games, because modifying the probabilities only leaves four options: win $30,000, win $10,000, lose $10,000 or lose $20,000, so it would be putting together an equation where the sums of the numerators of the probabilities give 20, and then, the player's hope of winning is given the prize if he wins, and the remaining, the penalty for losing, is given the unknown and equaling zero and solve, you get the value that makes the game fair.

As can be seen in the solution of P7, it was decisive for him to find the number of elements in the sample space, to find it and thus be able to determine if the game is fair or not; and only P1 and P7 used equations to find an unknown. Regarding the solutions of this group of teachers to this situation, we agree with Alvardo et al.,[20] in which the lack of previous experiences with this type of activities, could inhibit in them, the conception of informal ideas of probability, which led them to confront their intuitive daily beliefs with the probabilistic concepts used, and provide adequate solutions to the situation. Likewise, when comparing the solutions given to situations that involve games with dice, with those given to situations where the future teachers had not had previous experiences in achieving a sample space of these characteristics, it can be seen that the experience with this This type of situation affected their intuition and conceptual development, which did not make it easier for teachers in training to associate the conceptual elements studied here with lived experiences. This shows that experiences, which generate intuitive reasoning about probability, can favor their understanding,[31] if favorable intuitions are generated, or inhibit it, if they are unfavorable. Likewise, we agree with Batanero et al.,[10] in that this type of experiences that involve the probability of occurrence of an event, and that generate intuitions unfavorable to its understanding, can be difficult to teach, since they demand much more effort on the part of the student. However, "the analysis of comprehension difficulties about probabilistic reasoning when faced with a task that involves uncertainty",[20] is an aspect of great interest in Didactics of Probability, since erroneous intuitions they can lead to misanalyses, which in turn lead to misconceptions

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that are very difficult to deal with. Therefore, if you want future mathematics teachers to explore sociocultural contexts, where they can identify and use for didactic purposes, situations in which there is uncertainty, which lead them to make appropriate decisions for their teaching, it is necessary to put them to analyze the contradictions that may result from their own beliefs and conceptions, with formal aspects of probability, which help to enhance their probabilistic reasoning,[30] as well as in the ways of teaching it. In addition, interaction with peers is always important in this type of activity, so collaborative work between teachers in training could be a complementary alternative to what was said above.

Final thoughts

The task of a mathematics teacher helping his students to understand the meaning or to develop their mathematical knowledge, based on experimentation and using their own reasoning, seems to be more crucial in probability than in other fields, since in probability there are some aspects that they facilitate epistemic conflicts because they produce ambiguities and counterintuitive ideas.[3] However, these conflicts, whether of linguistic, semiotic or other origin, can play a fundamental role in the development of probability learning, since they lead students to make mistakes, and these are essential in any learning process. But said errors are useful in the process if whoever guides it is well-founded to treat them properly, using them for the benefit of the learner.[12] The training processes of teachers in initial training on probability and statistics require ingenious ways of working that allow the integration of their previous conceptions, the teaching trends of the discipline and the educational needs of their future students, which can be integrated into the curricula of educational institutions,[4] in such a way that the meanings of institutional, intended and implemented of probability, are as homogeneous as possible. In addition, if you want a process of these characteristics to flow, you should think about including practicing teachers in the training processes of future teachers, in the end they are the ones who implement the curricula and therefore serve the bulk of the population, so They could give suggestions on the grassroots needs of the communities in this area.

Conclusion

In their solutions, the future teachers prioritized the classical approach, and although in some solutions they obtained data, with which they could make inferences using the frequentist approach, their solutions were oriented to the formal calculation of a priori probabilities, obviating the need to use different approaches to probability, which make it easier for their future students to become aware of their intrinsic limits. The presence of correct intuitive ideas about expected value is evident in this group of future teachers,[30] since they take into account both the probability of winning the game and the value that is won or lost, to decide whether a game is fair or not. In addition, they use appropriate strategies to compare the probabilities of winning or losing a game, but have difficulty transforming an unfair game into a fair one. His difficulties are related to the calculation of compound probabilities, showing difficulties in concepts such as union and interception of events. Likewise, they showed limitations to find the sample space in games with which they had not had previous experiences, which made it difficult to solve the task. The interviews based on the tasks that the future teachers solved, acted as intervention elements, because, the step by step of the questions, made them change their misconceptions, leading them to more appropriate solutions. The solution strategies used by this group of future teachers were very limited, to make the game fair, they focused on the development of skills, without evidencing their ability to combine their statistical thinking with the teachability of this discipline. It is concluded about the need to improve the teaching of probability, starting with supporting the training of teachers, both in training and in practice, in both disciplinary and didactic aspects, which lead them to improve their teaching processes,[3,18] and acquire specific skills for the analysis of the didactic processes that they themselves develop.[1] Finally, it is considered that the validity of the results may be limiting the representativeness of the sample, since its size is small; however, the probabilistic thinking of the participants seems to be quite homogeneous, so with a small sample one could have a reliable representativeness. Likewise, it is considered interesting to project other works in this line of research, taking into account the need for constant training of practicing teachers and future mathematics teachers.

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Conflicts of interest

None.

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