Mini Review

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Structure formation from stability criteria

Abstract

In the short article we show that in view of the simple stability criteria an approximate estimation of the sizes and masses of planets, stars and galaxies may be predicted. Although no new result emerges, it might help to understand structure formation from the very first principle, which is essentially the stability criterion.

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Introduction

The fact that atoms are stable, initiated formulation of quantum mechanics, at the very beginning. Likewise, we observe that the planets, the stars the galaxies, all are stable objects as well. Therefore, stability criterion is the first principle that should be studied to understand why these objects do not encounter catastrophic collapse due to self-gravitational attraction. Newtonian dynamics suffices to study stability criterion of these objects. A galaxy is a gravitationally bound system containing about 10^{11} stars of different types, each of which is having 10^{11} cm. radius on an average. The size of a galaxy is of the order of 10^{23} cm and its mass is around 10^{45} gm. We show that it is possible to apprehend the existence of stars and galaxies from fundamental physical considerations. Referring to commonly used symbols (such as m_e, m_p being the masses of electron and proton respectively, ϵ_0 is the permittivity of the vacuum, etc.) one of the fundamental physical aspects is the ratio of the electromagnetic (e-m) and the gravitational forces:

$$\frac{F_e}{F_G} = \frac{e^2}{4\pi\epsilon_0 G m_P^2} = \frac{e^2}{4\pi\epsilon_0 \hbar c \alpha_G} = \frac{\alpha}{\alpha_G} \simeq 10^{36}. \tag{1}$$

In analogy to the e-m fine structure constant $\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} = \frac{1}{137} \simeq 10^{-2}$, we dub the dimensionless quantity $\alpha_G = \frac{Gm_p^2}{\hbar c} \simeq 10^{-38}$ as the gravitational fine structure constant, since it essentially measures the gravitational attraction between two electrons. This implies that gravity is much weaker than e-m interaction and therefore at small scales, physical processes are dominated by-and-large by e-m interaction only. By small scale we mean the atomic scale of dimension

$$a_0 = \frac{4\pi\epsilon_0 \hbar c}{m_e e^2}\left(\frac{\hbar}{c}\right) = (m_e\alpha)^{-1}\left(\frac{\hbar}{c}\right) \simeq 10^{-10}\, m, \tag{2}$$

the binding energy $E_b \simeq m_e c^2\alpha^2$ and typical densities of matter $\rho = A\left(\frac{m_p}{\frac{4}{3}\pi a_0^3}\right)$, where A is the atomic mass.

A. Formation of planets:

Let us now consider a spherical body made up of N hydrogen atoms having mass $M = Nm_p$ and radius $R = a_0 N^{\frac{1}{3}}$. Gravitational potential and the Binding energy of such a body are $E_G \simeq \frac{GM^2}{R} \simeq \frac{GN^2 m_p^2}{R}$, and $E_0 \simeq N\alpha^2 m_e c^2$ respectively. Since gravity is attractive, we have introduced binding energy instead of the electrostatic repulsion, to get rid of the sign from both sides. This means we are simply considering the numerical values. Such a body remains in stable equilibrium provided $E_0 > E_G$. Now substituting the above expression of R, and that of a_0 (Eqn. 2) as well, we can express E_G as,

$$E_G \simeq N^{\frac{5}{3}}\frac{Gm_p^2}{a_0} = \frac{N^{\frac{5}{3}} G m_p^2 \alpha^2 m_e}{\alpha} \times \frac{c}{\hbar}. \tag{3}$$

Now, for $N = N_{max}$, stability of the spherical body under consideration requires $E_{G(max)} < E_{0(max)}$, i.e.

$$N_{max}^{\frac{5}{3}}\frac{Gm_p^2}{\alpha}\alpha^2 m_e \times \frac{c}{\hbar} < N_{max}\alpha^2 m_e c^2,$$

$$\text{Or,}\quad N_{max} < \left(\frac{\alpha\hbar c}{Gm_p^2}\right)^{\frac{3}{2}} = \left(\frac{\alpha}{\alpha_G}\right)^{\frac{3}{2}} \simeq 10^{36\times\frac{3}{2}} = 10^{54}. \tag{4}$$

Such a limiting value of a spherical body has radius $R = N_{max}^{\frac{1}{3}} a_0 \simeq \sqrt{\frac{\alpha}{\alpha_G}}\, a_0 \simeq 10^{10}\, cm$, having a maximum (critical) mass of $M_c \simeq N_{max} m_p \simeq 10^{30}\, gm$. This clearly corresponds to the radius and the mass of Jupiter, the largest planet of our solar system. Hence at this end we find that, stability criteria result in the maximum mass and size of a planet.

B. Formation of stars:

Next, if we want to go beyond and consider large bodies having mass $M > M_c$, we need to consider several physical processes, in addition. Firstly, we need to consider the evolution of a gaseous sphere of hydrogen (say), with $N > N_{max}$. For such a system, gravity dominates and in the steady state, Virial theorem implies: $|K| \simeq |U|$, where, $K = NkT$ and $|U| \simeq \frac{GM^2}{R}$ stand for the kinetic and the potential energies of the system respectively, and k being the Boltzmann constant. Thus $kT \simeq \frac{GMm_P}{R}$, where we have substituted $M = Nm_p$. It is to be noted that for $N > N_{max}$, kT is higher than the typical binding energy of solids and hence it is reasonable to consider the associated matter to be in plasma state, with electrons moving freely around the nuclei. However, when the body contracts under the action of its own gravitational force, its temperature increases according to the relation: $NkT \simeq \frac{GM^2}{R}$. It appears that the temperature increases without bound as the sphere shrinks to $R \to 0$, which is uncanny. To resolve the issue, let us express the temperature in terms of the inter-nuclear separation ($d \simeq RN^{-\frac{1}{3}}$).

Since $NkT \simeq \frac{GM^2}{R} = \frac{GN^2m_p^2}{dN^{\frac{1}{3}}} = \frac{GN^{\frac{5}{3}}m_p^2}{d}$, so we find

$$kT = N^{\frac{2}{3}}\left(\frac{Gm_p^2}{d}\right). \quad (5)$$

Therefore, again it appears that the temperature increases unboundedly as inter-nuclear separation (d) decreases. But this is not quite correct, since as d decreases, electrons in atoms are confined progressively to smaller and smaller region of space. As a result, electron degeneracy pressure generates, which starts contributing. For such electrons being confined within 'd' region, $\frac{p^2}{m_e} \simeq \frac{\hbar^2}{m_e d^2}$. In view of which we can write, $kT + \frac{\hbar^2}{m_e d^2} \simeq N^{\frac{2}{3}}\frac{Gm_p^2}{d}$ and so we arrive at,

$$kT\ (d) = N^{\frac{2}{3}}\left(\frac{Gm_p^2}{d}\right) - \frac{\hbar^2}{m_e d^2}. \quad (6)$$

Clearly, the temperature $T(d)$ reaches its maximum value $T = T_{(max)} = (2k)^{-1}N^{\frac{4}{3}}m_e\alpha_G^2c^2$, at a minimum value of the inter-nuclear separation, $d_{\min} = 2N^{-\frac{3}{2}}(m_e\alpha_G)^{-1}\left(\frac{\hbar}{c}\right)$. In the process, degeneracy pressure forbids an unbounded increase of the temperature. It is important to mention that the degeneracy term dominates at higher density. If T_{max} is high enough to trigger nuclear reaction at the centre of the spherical cloud, then contraction halts. If not, the body cools in a very short time-scale and forms a planet-like object. Note that the temperature required to ignite nuclear reaction is $T_N \simeq \eta k^{-1}\alpha^2 m_p c^2$, where, η is a parameter which depends on the details of nuclear reaction. The condition $T_{max} > T_N$ simplifies to

$$N > (2\eta)^{\frac{3}{4}}\left(\frac{m_p}{m_e}\right)^{\frac{3}{4}}\left(\frac{\alpha}{\alpha_G}\right)^{\frac{3}{2}} \simeq 10^{57}\times(0.2)^{\frac{3}{4}}. \quad (7)$$

This means $Nm_p = M > M_*$, and hence,

$$M_* = (2\eta)^{\frac{3}{4}}\left(\frac{m_p}{m_e}\right)^{\frac{3}{4}}\left(\frac{\alpha}{\alpha_G}\right)^{\frac{3}{2}} m_p \simeq 10^{29}\ kg,\ \text{taking}\ \eta = 0.1. \quad (8)$$

Once nuclear reaction starts, gravitational collapse halts, and the body becomes luminous, since energy generated through nuclear reaction is radiated away by the body. Such objects are identified as stars. Investigating the largest known Arches cluster by Hubble space telescope, the upper limit of the mass of a star has been reported to be $M \le 150 M_\odot$.[1] In fact, above this limit, stars run out into instabilities, due to pair production and also by crossing the Eddington limit (the upper limit of a mass of a star at which the radiation pressure balances the inward gravitational force of the star), to be precise. Stability of such a body therefore requires $M \le 150 M_\odot$. Note that $M_\odot = 1.989\times10^{30}kg, M_\odot = 6.96\times10^8 m$, stand for the solar mass and radius respectively. On the contrary, European Southern Observatory press release reported,[2] AB Doradus C, a faint companion to AB Doradus A, is the smallest known star $(0.09 M_\odot)$ currently undergoing nuclear fusion in its core,[3,4] which is supposedly the critical mass required to ignite nuclear fuel. Gas clouds which are less massive end up as brown dwarfs which occupy a poorly defined grey area between stars and gas giants.[4,5]

C. Formation of galaxies:

Finally, let us consider gas clouds having $M \gg M_\odot$, for which a region of size λ has thermal energy, $E_{th} \simeq \frac{\rho\lambda^3}{m_p}kT$, while the gravitational potential energy is $E_G \simeq G\left(\rho\lambda^3\right)^2\lambda^{-1}$. Since stability configuration of such an object requires $E_{th} > E_G$, so, $kT \ge G\rho\lambda^2 m_p$. However, this condition may be violated if the object cools rapidly. In that case it will have a tendency to fragment into smaller bodies $(\lambda \ll R)$, each of which would satisfy the above condition $E_{th} > E_G$, and evolve separately. To be precise, as the cloud collapses, its density increases, and since the collapse is isothermal, an increment of density implies that the Jeans mass of the cloud also falls. So small pieces of the cloud start to collapse of their own. A rising density also implies a declining free fall time. So these small dense clumps collapse further than the overall cloud. Thus instead of one gigantic cloud undergoing monolithic collapse, the cloud fragments into small collapsing pieces. Such collapse halts when mass of each fragment reaches the mass of an average star. Hence we are required to explore the dominant cooling processes.

For systems having temperature $kT \simeq \frac{GMm_p}{R}$, which is much greater than ionization potential $\alpha^2 m_e c^2$, the dominant cooing process is Bremsstrahlung. It is important to mention that the cooling time due to Bremsstrahlung process is given by,

$$t_{cooling} \simeq \left(n\alpha\sigma_T\right)^{-1}\left(\frac{kT}{m_e c^2}\right)^{\frac{1}{2}} = \frac{3}{8\pi}\left[\frac{m_e^2}{\alpha^3\hbar^2 n}\left(\frac{kT}{m_e}\right)^{\frac{1}{2}}\right], \quad (9)$$

where, $\sigma_T = \frac{8\pi}{3}\left(\frac{q^2}{4\pi\epsilon_0 m_e c^2}\right)^2 = \frac{8\pi}{3}\left(\frac{\alpha\hbar}{m_e c}\right)^2$ is the Thomson scattering cross-section for electron, and n is the number density of charged particles. On the contrary, the time scale for gravitational collapse is,

$$t_{gravcollapse} = \sqrt{\frac{2R_G^3}{GM}}, \quad (10)$$

Now the condition required for efficient cooling of the object is, $t_{cooling} < t_{gravcollapse}$, which leads to the condition $R < R_G$, where,

$$R_G \simeq 2\sqrt{2}\,\alpha^3\,\alpha_G^{-1}\,m_e^{-1}\left(\frac{m_p}{m_e}\right)^{\frac{1}{2}}\left(\frac{\hbar}{c}\right) \simeq 2.54\times10^{23}\,cm \cong 80\,Kpc \quad (11)$$

in the limit. To derive the above relation, we have used the expression $n = \frac{N}{\frac{4}{3\pi}R_G^3}$, with $Nm_p = M$, and introduced the unit Kpc (Kiloparsec), where, $1Kpc \simeq 3.08567758\times10^{21}\,cm$. The condition that $kT > \alpha^2 m_e c^2$ at $R = R_G$ implies:

$$R_G \simeq \frac{GM_G m_p}{\alpha^2 m_e c^2} = \sqrt{2}\pi\alpha^3\,\alpha_G^{-1}\,m_e^{-1}\left(\frac{m_p}{m_e}\right)^{\frac{1}{2}}\left(\frac{\hbar}{c}\right) \quad (12)$$

Hence, one ends up with

$$M_G = \sqrt{2}\,\pi\alpha^5\alpha_G^{-1}\left(\frac{\hbar c}{Gm_p^2}\right)\left(\frac{m_p}{m_e}\right)^{\frac{1}{2}} m_p = \sqrt{2}\pi\alpha^5\alpha_G^{-2}\left(\frac{m_p}{m_e}\right)^{\frac{1}{2}} m_p \simeq 10^{11}\,M_\odot \quad (13)$$

which is the average mass of a galaxy. Thus, gaseous clouds with $M \ge M_G$ and $R \le R\_G$ cools due to Bremsstrahlung process and fragments into smaller objects. These smaller fragments evolve independently and end up forming luminous stars or non-luminous planets. Systems with $R > R_G$ evolves quasi-statically with $T \propto R^{-1}$, untill the condition $R < R_G$, is reached. In a nutshell, in view of the above analysis, we expect that gravitationally bound objects may contain 10^{11} or even more stars. These objects are identified as galaxies.

Conclusion

Inflation, which was initiated in the very early universe $(10^{-32\pm6} s)$ is a very short phase of rapid expansion (faster than the speed of light) of the universe. At the end of inflation, particles are created which coalesce and heats up the universe. Thus hot Big-Bang starts. Inflation left behind the seeds of perturbation, which grew to form the structures - the stars, the galaxies, the cluster of galaxies, that we observe in the sky. Present article does not deal with the origin of structure formation. However, why the structures having different sizes and masses exist in the sky, is a natural question. The answer may be found in a host of text books of Astrophysics. However, the technique adopted in the standard text books is much involved. In this manuscript, we show that in view of Newtonian dynamics, it is possible to explain the formation of structures.

The requirement of the stability of atoms laid the foundation of quantum mechanics. Likewise, we know that a covalent bond formed by two hydrogen atoms is stable. Thus stability criterion is the first fundamental principle for which we exist. In this short note, we have tried to discuss why planets with large sizes and masses of the order of a mediocre star cannot exist in nature, and what causes stars and galaxies to encounter catastrophic gravitational collapse.

In a nutshell, during inflation the seeds of perturbations went outside the horizon and freeze. As inflation halts, these seeds enter the horizon, and over-dense regions accumulate masses from nearby under-dense regions and contract. At some stage, the temperature becomes high enough to trigger nuclear reaction, which halts further collapse due to radiation energy, and in the process stars are formed. A larger dense region of space of gaseous cloud fragments. These fragments collapse faster than the large region, to form hundreds of billions of such stars, which altogether take the shape of galaxies. It is important to mention that there was a dark age in the early cosmos, and the first star twinkled some 100 million years after inflationary phase, commonly said to be after hot Big-Bang. First galaxy, almost took a billon year to form after the Big-Bang.

I would finally like to specifically mention that this article should not to be treated as a fundamental or original research, since it does not explore any new result. Rather it is essentially meant for the graduate students and else, for quick understanding of the issue.

Acknowledgments

None.

Conflicts of Interest

None.

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