

A revisited study of the cosmic matter recombination in expanding universes

Abstract

It is generally believed by cosmologists that the universe back in its past was hot, and matter was completely ionized and in thermal equilibrium, while in the present era matter, due to a strong cosmic temperature decrease, has recombined to neutral atoms. As we argue here, contrary to general assumption, a non-equilibrium state predominates at this phase of the recombination and the usually used Saha-Eggert theorem hence is inapplicable. In a preceding paper, we had already derived a specific kinetic transport equation which describes the distribution function of cosmic baryon gas (i.e. hydrogen atoms) just after cosmic matter recombination. We could solve the relevant kinetic transport equation for that period and found the gas distribution function $f(v, t)$ as function of the particle velocity v and of the cosmic time t . However here, in this paper, we shall go one important step further back in the cosmic evolution and do study in more detail, how in successive steps the recombination of cosmic electrons and protons did actually occur. We clearly show that matter and radiation in the phase of cosmic recombination is not anymore in a thermodynamic equilibrium state, since matter and radiation do cool off in different forms, and a thermodynamic situation predominates where protons, electrons and photons have different temperatures, and collision-based energy transfer processes operate between them. Hence cosmic recombination, thought to have occurred about 400000 years after the Big-Bang, does not take place starting from a thermodynamic equilibrium state as generally presumed, and hence the standard Saha-Eggert assumptions and predictions on the ionization degree of cosmic matter as function of the system temperature can not be used. We follow in detail the processional track how the cosmic radiation and the cosmic matter behave in this critical non-equilibrium phase and show that the recombination of cosmic matter actually occurs, though there is a yet unrespected tendency that freely moving particles in an expanding universe become heated by the action of the differential Hubble drifts which, in a first glance, should impede their recombination.

Keywords: cosmic expansion, recombination, hydrogen gas, kinetic transport equation

Introduction

The thermodynamic state at the cosmic recombination point. In standard cosmology it is generally assumed that at the beginning of the cosmic evolution matter was at high temperature and in a fully ionized state, i.e. in a plasmatic phase with electrons and protons in unbound states (i.e. the plasma universe!).^{1,2} Cosmic photons in their number density n_ν ,³ if correctly derived from their redshifts, were strongly dominant by a factor of 10^9 compared to particle number densities, like electron or proton densities n_e or n_p .⁴ Due to the strong thermodynamic coupling between photons, electrons and protons at these pre-recombination phase, the temperatures of all these species were identical, i.e. $T_\nu = T_e = T_p$, i.e. all species did belong to the same thermodynamic system, and consequently “thermodynamic equilibrium” would be a perfect characterisation of this state. But in an expanding universe matter densities will systematically decrease, and the strengths of thermodynamic couplings, i.e. energy exchanges between electrons, protons and photons, become weaker and weaker, and temperatures consequently decouple from each other.⁵⁻⁹

In case thermodynamic equilibrium between electrons, protons and photons can be assumed, the degree ξ of ionisation can be calculated with the help of the Saha-Eggert equation.^{10,11} In principle the actual degree of ionization $\xi(T)$ is then obtained from the minimum of the Gibbs potential $G = G(\xi)$ by the request: $dG/d\xi = 0$!, where Gibbs potential is given by $G(\xi) = U(\xi) + P(\xi) \cdot V_0 + T_0 \cdot S(\xi)$, - U, P, S

denoting the internal energy, the total pressure, and the total entropy of the system, T_0 and V_0 being the total volume of the system and the common equilibrium temperature with $T_0 = T_\nu = T_e = T_p$. But the whole of that classic Saha-Eggert theorem is based on the fundamental assumption.

Thermodynamic equilibrium! If the latter is not guaranteed, and if temperatures T_ν, T_e, T_p are different, then this theorem is not applicable.

In the following part of the paper we shall, however, clearly demonstrate that the equilibrium state is perturbed as soon as the energetic coupling between photons, electrons and protons becomes weaker, as it unavoidably occurs during the ongoing cosmic expansion due to permanent density decreases. Even if a Maxwellian distribution would have prevailed at the entrance to the collision-free cosmic expansion phase, it would not have continued to exist for later times as already shown in Fahr.¹² After the recombination phase when electrons and protons should recombine to H-atoms, and photons start propagating through cosmic space practically without further interaction with matter, thereby establishing the cosmic radiation background, the CMB, the thermodynamic contact between matter and radiation at the following cosmic time is stopped. Both behave in principle independent of each other, in first order only reacting to the fact of the cosmic scale expansion. For this reason the initial Maxwellian atom distribution function does not persist in an expanding universe over times of the ongoing collision-free expansion. Herewith the preliminary aspects of the ongoing evolution

have been touched and now in some more explicit considerations and calculations this thermodynamical point will be taken under a more microscopic view.

The kinetic transport equation of cosmic electrons and protons

In Fahr¹² the mathematical procedure has been derived to describe the physical and thermodynamical behaviour of a cosmic baryon gas, i.e. essentially of the H -atom gas just after the process of recombination of cosmic electrons and protons at and following the recombination phase of cosmic matter - roughly about 400000 years after the Big Bang. The relevant kinetic transport equation has been derived therein as given by Equ.(1) for this cosmic situation. This equation in its original form, however, has not been solved there by the associated kinetic distribution function $f(v, t)$. Only the velocity moments of this function $f(v, t)$, like the density $n(t)$ and the pressure $P(t)$, could be precisely derived as function of the cosmic time t . With this knowledge of the exact form of the kinetic transport equation and the knowledge of the moments $n(t)$ and $P(t)$ as functions of cosmic time one could be seduced to now become more ambitious and find out more about the kinetic situation of the cosmic gas under these conditions just after or at the recombination era.

What kind of distribution function $f(v, t > t_0)$ and especially what kind of temporal change of it should be expected for that period? To answer this question we want to approach the problem here a little bit from an other direction and want to use here a new independent way to access this kinetic problem, namely to use a slightly different kinetic transport equation compared to that used by Fahr,¹² however nevertheless treating the identical cosmophysical situation as already envisioned there. Starting from a kinetic transport equation used by Fahr¹³ for a plasma physical scenario which, however, for the purposes here is directly transferable as an analogon, since only of importance in both cases are the two terms for a temporal derivative of $f(v, t)$ and for the particle redistribution in velocity space under collision-free conditions, in our case here due to the up to now completely unrespected Hubble-induced velocity space drift $\dot{v}_H = \dot{v}_H(v)$ of the particles, i.e. the electrons or the protons. With these two terms the kinetic transport equations would then attain the following, surprisingly simple form describing the temporal change of the distribution function, both of the protons as well as of the electrons, as due to the spherical Hubble drift $\dot{v}_H(v) = -v \cdot H$ of the particles on spherical shells in velocity space:

$$\frac{\partial f_e(v, t)}{\partial t} = \frac{1}{v^2} \frac{\partial}{\partial v} [v^2 \dot{v}_{eH} f_e(v, t)]$$

and

$$\frac{\partial f_p(v, t)}{\partial t} = \frac{1}{v^2} \frac{\partial}{\partial v} [v^2 \dot{v}_{pH} f_p(v, t)]$$

where the terms on the left side denote the explicit temporal change of the distribution functions $f_{e,p}(v, t)$ and the terms on the right side describe the temporal change of the distribution function $f_{ep}(t, v)$ under the Hubble-induced velocity drift migration $\dot{v}_{eH} = \dot{v}_{pH} = -v \cdot H$, quite analogous to the velocity space drift which was formulated as due to wave-particle-induced velocity

diffusion for a completely different, but analogously operating plasma-physical scenario in Fahr.¹³

The above transport equations would adequately regulate the kinetics of the electrons and protons, if all other interaction processes like elastic collisions between protons and electrons, or Thomson scattering processes between cosmic photons and electrons could be excluded and would in fact predict electron and proton temperatures to increase in an expanding Hubble universe.¹⁴ If those latter interaction processes for the cosmic era of interest, however, cannot be excluded, one needs additional terms for an adequate description a) for the energetic coupling between protons and electrons and b) for the coupling between electrons and photons, in the upper transport equations like those given in their basic forms by Sunyaev and Zel'dovich,⁷ or later by Fahr and Loch,⁸ which, in their case, do lead to the following enlarged system of equations:

$$\frac{\partial f_e(v, t)}{\partial t} = \frac{1}{v^2} \frac{\partial}{\partial v} [v^2 \dot{v}_{eH} f_e(v, t)] + \frac{f_e - f_p}{\tau_{e,p}} + \frac{f_e}{\tau_{e,v}} \frac{[(m_e v^2 / 2) - K T_e(t)]}{K [T_e(t) - T_v(t)]}$$

and

$$\frac{\partial f_p(v, t)}{\partial t} = \frac{1}{v^2} \frac{\partial}{\partial v} [v^2 \dot{v}_{pH} f_p(v, t)] + \frac{f_e - f_p}{\tau_{e,p}}$$

Hereby the quantities $T_e = T_e(t)$ and $T_v = T_v(t)$ represent the actual, time-dependent electron and photon temperatures, $\tau_{e,p}$ and $\tau_{e,v}$ denote typical electron-proton and electron-photon energy exchange periods which are given by:

$$\tau_{e,v} = \frac{T_v - T_e}{\partial T_e} = \frac{3 m_e c}{8 \sigma_{Th} \alpha T_v^4}$$

with $\sigma_{Th} = (8\pi/3)(e^2 / m_e c^2)^2 = 0.66 \cdot 10^{-24} \text{ cm}^2$ denoting the Thomson photon-electron scattering cross section and α being the Stefan-Boltzmann constant, and furthermore according to Spitzer¹⁵

$$\tau_{e,p} = 11.4 \sqrt{\frac{m_p}{m_e}} \frac{(K T_e)^{3/2}}{\delta_{ee} n_e \bar{E}}$$

where Λ is the Coulomb logarithm, and δ_{ee} denotes the mean energy transfer rate in electron-electron collisions.

The interesting thing now is that if photons, electrons and protons are only embedded in the cosmic expansion, without mutual interactions, then the cosmic photons are redshifted with time, and their temperature T_v is permanently reduced according to $T_v = T_{v0} (R_o / R)^{1/3}$,¹⁶ but also see alternative views by Fahr and Heyl,^{17,18} while, to the contrary, proton and electron temperatures purely reflecting the effect of the Hubble migration, as shown in Fahr,¹⁴ both are increasing, thus creating along the standard view evidently a strange, even escalating NLTE-situation with $T_v \ll T_{p,e}$. This NLTE situation would probably not allow at all the recombination of electrons and protons to neutral H-atoms, suggesting that the recombination should not take place at all, and consequently keeping

intergalactic matter impenetrable for stellar light till the present days. This latter point, however, is in contradiction to the present cosmic fact, since we are at present clearly seeing distant stars and galaxies!

To start a study of these multiple interactions, we are permitted to assume that photons are by far the dominant species by their number density (factor 10^9 with respect to electrons¹⁵). Hence the energetic interactions between photons and electrons energetically is a one-way interaction, communicating simply the lower temperature of the photon field to the electrons, i.e. cooling electrons. Photons thus are only subject to the cosmic expansion and are redshifted by the Hubble expansion, however, practically remaining untouched by the electrons. This then leads to the fact that the cosmic photon temperature is falling off with the scale of the universe by Goenner, Fahr and Zoennchen.^{16,19}

$$T_v(t) = T_{vo} \cdot (R_0 / R(t))$$

The upper relation serves now as the initially fixed temperature scale for our following considerations: We assume that we safely know that the cosmic photon field, independent on all other interactions, is cooling down by cosmic photon redshifting according to $T_v(t) = T_{vo} \cdot (R_0 / R(t))$, electrons by their effective, thermal coupling to photons on one hand are cooled by the cosmic photon field, on the other hand they are heated being freely exposed to the Hubble expansion.¹⁴ where by the outcome clearly depends on the actually prevailing cosmic coupling coefficients.

Starting from a mass density $\rho_0 = 10^{-31} \text{ g/cm}^3$ of the present universe, which converts to an actual proton density of $n_0 = 6.3 \cdot 10^{-6} \text{ cm}^{-3}$, and looking back in cosmic time thus leads us to a proton density n_r at the recombination era, assumed to have occurred at a cosmic redshift of $z = (R_0 / R(t_r)) + 1 = 10^3$, of:

$$\Delta \in_2 = \frac{d}{dt} (KT_e(t)) = KT_{e0} (-2)(1 - H(t - t_0))^{-3} (-H) = 2H \cdot KT_{e0} (1 - H(t - t_0))^{-3}$$

Average energy loss of electrons to the photons

This then evidently means that the electrons in this phase either would increase or decrease their temperature $T_e(t)$, dependent on whether $\Delta \in_2 < \Delta \in_1$, i.e. whether $\Delta \in_2$ is larger or smaller than $\Delta \in_1$, which means:

$$2H \cdot KT_{e0} (1 - H(t - t_0))^{-3} \leq \nu_e \cdot (KT_e - KT_v)$$

The above relations, when putting in temperature values for electrons and photons, then delivers the following relation:

$$2H \cdot T_{e0} (1 - H(t - t_0))^{-3} \leq \nu_e \cdot \left(\frac{T_{e0}}{(1 - H(t - t_0))^2} - T_{vo} \frac{R_0}{R(t)} \right)$$

and reminding that $T_{e0} = T_{vo}$ will then lead to:

$$2H \cdot (1 - H(t - t_0))^{-3} \leq \nu_e \cdot \left(\frac{1}{(1 - H(t - t_0))^2} - \frac{R_0}{R(t)} \right)$$

$$n_r = n_0 \cdot (R_0 / R(t_r))^3 = 6.3 \cdot 10^3 \cdot \text{cm}^{-3}$$

The frequency ν_{Th} of Thomson scattering processes between photons and electrons at this recombination era would thus be given by:

$$\nu_{Th} = [\sigma_{Th} \cdot n_e \cdot n_v \cdot c]_r = 262 \cdot \text{cm}^{-3} \text{s}^{-1}$$

or meaning that each electron undergoes a Thomson collision with a CMB photon with an average frequency of $\nu_e = \nu_{Th} / n_{e,r} = 0.042 \cdot \text{s}^{-1}$.

Assuming that in average such collisions transfer an energy of $(KT_e - KT_v)$ from the colliding electron to the photon field, would then express the fact that the electrons via Thomson scattering lose an energy per time of

$$\Delta \in_1 = \nu_e \cdot (KT_e - KT_v)$$

Average energy loss of electrons to the photons

This energy exchange rate should then be compared with the average energy gain of an electron per time due to being kinetically influenced at free motions in space by the Hubble expansion. In Fahr¹⁴ as a yet unrespected, new point it has been shown that without the influence of collisions the electron temperature in an expanding universe increases according to:

$$T_e(t) = \frac{T_{e0}}{(1 - H(t - t_0))^2}$$

meaning that the average energy gain per time of an electron due to the Hubble migration is given by; $\Delta \mathcal{E}_2$

and with the Hubble parameter at the recombination phase given by $H = H_o = \dot{R}_0 / R_0$, and keeping $\Delta t = t - t_0$ small enough, so that $\Delta t \cdot H_0 \ll 1$, will then lead to the relation:

$$2H_0 \cdot (1 + 3H_0(t - t_0)) \leq \nu_e \cdot \left(1 + 2H_0(t - t_0) - \frac{R_0}{R_0(1 + H_0(t - t_0))} \right)$$

or

$$2H_0 \cdot (1 + 3H_0(t - t_0)) \leq \nu_e \cdot 3H_0(t - t_0)$$

or:

$$(2/3) \cdot (1 + 3H_0(t - t_0)) \leq \nu_e \cdot (t - t_0)$$

In case, the left side is larger than the right one, - electrons are heated, in case of the opposite, - electrons are cooled!

Let us assume here the time $(t - t_0) = 0.1\tau_0 = 0.1 / H_0$, then we obtain:

$$(2/3) \cdot (1 + 0.3) \leq \nu_e \cdot 0.1 / H_0$$

delivering the request:

$$4.66 \cdot H_0 \leqslant v_e$$

If the CMB redshift can be assumed with $z_{CMB} = 1000$, then we have $v_e = 0.042 s^{-1}$, and the above relation requires \leqslant :

$$H_0 \leqslant v_e / 4.66 = 9 \cdot 10^{-3} s^{-1}$$

Assuming for the value $H_0 = H_{Today}$ we then obtain

$$2.33 \cdot 10^{-18} \ll 9 \cdot 10^{-3}$$

That means that the left side is much smaller than the right side, unless the Hubble parameter at the recombination era would have been about a factor $\xi = 10^{15}$ larger than the present-day Hubble parameter H_{today} . Otherwise one would in all cases have the left side smaller, i.e. the electrons would be systematically cooled by the photons. But when they are cooled and Coulomb collision are effective enough, then these electrons cannot be impeded from recombining with protons leaving neutral H -atoms for the rest of the cosmic evolution. However, drastically different from earlier approaches now these collision-less, interaction-free neutral atoms are solely subject to the effect of the Hubble-migration which slowly leads to a gas temperature increase again. Whether or not this heated cosmic gas will then later be able to form larger massive complexes like stars and galaxies thus needs to be answered from quite a new basis!

Conclusion

In an aforesaid paper¹⁴ it had been shown that after a recombination of cosmic matter, the remaining hydrogen atoms as a collision-free gas will be solely subject to the cosmic scale expansion of the universe, and, astonishingly, due to the kinetic action of the Hubble drift on the whole gas population the thermal spread, i.e. the temperature of this population, will increase, though embedded in an expanding universe. This latter effect, however, only takes place in this form after the recombination of electrons and protons to neutral gas atoms, since before this occurs, electrons, instead of being heated, are much stronger cooled by Thomson scatter processes with the cosmic photon radiation field, which itself is strongly redshifted and unavoidably cooled by the expansion of the universe. The cooled electrons on their side are then also effectively cooling the protons by Coulomb collisions, since it turns out that in the first collision-dominated phase this cooling by the cooled electrons is dominant over the Hubble-heating in the expanding universe. As we could show in this paper the electron cooling by Thomson scatter processes with the cooling CMB radiation field is much more effective compared to the Hubble-induced heating. Hence one can conclude that the recombination of electrons and protons is not impeded by the electron heating. However, when finally the recombination is finished and only neutral gases are left, then these neutral gases, when exclusively being subject to the Hubble expansion, will then again start being heated despite the expansion of the scale of the universe, since increasing at free motions in space their thermal spreads with cosmic time. The question then remains whether or not this heated cosmic gas would, or would not, impede cosmic matter in the expanding universe from the evolution to larger structures of cosmic matter in form of stars and galaxies, the so-called cornerstones of the present-day universe.

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Conflicts of interest

The author declares there is no conflict of interest.

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