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N-dimensional plane symmetric solutions in *f(R,T)* theory of gravity

Abstract

In this Paper, we have evaluated the N-dimensional plane-symmetric space times solutions in $f(R,T)$ theory of gravity. For this purpose we use the more general class of $f(R,T)$ model, i.e., $f(R,T) = f_1(R) + f_2(T)$. Here, also we make the assumption that $f(R) \alpha f_0 R^q$, where f_0 and q are arbitrary constants. To find the solutions, we assume the dust case with $p = 0$. The field equations are solved by assuming exponential and power law forms of metric coeffcient. Moreover, we have evaluated the energy densities and corresponding functions of $f(R,T)$ model.

Keywords: Modified gravity, $f(R,T)$ theory of gravity, Plane symmetric solutions

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Introduction

Our universe is going through an accelerating expansion phase shown by the results obtained from different observations and experiments. General view is that this expansion is due to an enigmatic force dubbed as dark energy carrying large amount of negative pressure. In order to study its complete features, researchers have proposed alternative approaches to general relativity (GR) either by modifying the geometric part or matter part of the Einstein Hilbert action. The modified theories of gravity such as Gauss-Bonnet theory, Brans-Dick theory, String theory, Scalar tensor theory of gravity, $f(R)$ theory and $f(R,T)$ theory of gravity have extend GR. These theories can simply be obtained by applying the Einstein-Hilbert action. The $f(R)$ theory of gravity is the modification of GR, where f is an arbitrary function of the Ricci scalar R Hollenstein et al.[1] examined the exact solution of static spherically symmetric space times linked to non-linear electrodynamics in $f(R)$ modified theory of gravity. In metric $f(R)$ theory of gravity, cylindrically symmetric vacuum solutions is suggested by Azadi[2] and his co-workers.[2] In $f(R)$ gravity, plane symmetric solutions is explored by Sharif and Shamir.[3] Amendola et al.[4] discovered the circumstances subordinate which dark energy $f(R)$ models are cosmologically suitable. Locally Rotationally Symmetric vacuum solutions in $f(R)$ gravity is explored by Jamil and Sadia.6

Obviously $f(R)$ theory of gravity is the simplest generalization of GR but still generally there are some $f(R)$ models which are not consistent with the solar system tests. Recently, a new modified theory has been found by Harko et al.[7] is called the $f(R,T)$ theory of gravity. It can be obtained new modified theory of gravity known as $f(R,T)$ theory of gravity which is by replacing the general function $f(R)$ with the scalar curvature R and the trace of energy-momentum tensor T in the Einstein-Hilbert Lagrangian of GR. The $f(R,T)$ theory of gravity is examined to be most attractive modified theory in all modified theories of gravity. Myrzakulov[8] discussed $f(R,T)$ gravity in which he gave point like Lagrangian. Adhav[9] explored the exact solutions of $f(R,T)$ field equations for locally rotationally symmetric Bianchi type I spacetime. Sharif et al.[10] studied the laws of thermodynamics in $f(R,T)$ theory of gravity. Houndjo[11] rebuilt $f(R,T)$ gravity by taking $f(R,T) = f_1(R) + f_2(T)$ in which he investigated that $f(R,T)$ gravity permitted transition of matter from dominated phase to an acceleration phase.

Shamir[12] investigated the solutions of Bianchi type-I in the context of gravity $f(R,T)$ and he explored two exact solutions by assuming constant deceleration parameter and the variation law of hubble parameter. Shamir and Raza[13] explored the cylindrically symmetric spacetimes solutions in the background of $f(R,T)$ gravity. Shamir[14] has also explored the locally rotationally symmetric Bianchi type-I cosmology in $f(R,T)$ gravity. He found the solution of modified field equations by using the assumption of expansion scalar θ proportional to shear scalar σ. In $f(R,T)$ theory of gravity, Adhav[15] Chaubey et al.[16] and Shamir et al.[17] explored different cosmological models. Amir et al.[18] studied spherically symmetric perfect fluid collapse in the frame work of $f(R,T)$ by considering the nonstatic spherically symmetric background in the interior regions and static spherically symmetric background in the exterior regions of the star.

Higher dimensional cosmological model play a vital role in many aspects of early stage of cosmological problems. The study of higher dimensional space- time provides an idea that our universe is much smaller at early stage of evolution as observed today. There is nothing in the equation of relativity which restrict them to four dimensions. Kaluza[19] and Klein[20] have done remarkable work by introducing an idea of higher dimension spacetime. Many researcher inspired to entered in to the field of higher dimension theory to explore knowledge of universe.

Lorentz and Petzold [21], Ibanez et.al [22] ,Khadekar and Gaikwad [23]have studied the multidimensional cosmological models in GR. Adhav et al [24] have studied the multidimensional cosmological models in modified theories of gravitations.[24] Samanta[25] investigated higher dimensional cosmological models filled with perfect fluid in $f(R,T)$ gravity. Mishra[26] studied the existence and behavior of solutions to some nonlinear integral equations. Vandana et al.[27] explored the duality relations for a class of a multi objective fractional programming problem involving support functions. Mishra[28] explained few problems on approximations of functions in Banach Spaces. Deepmala et al.[29] generalized the differential geometry by using deferential operators over modules and rings. Piscoran and his coauthors studied a Projective flatness of a new class of $(a,)(\alpha,\beta)$ -metrics.

Shamir et al.[6] explored the $n+1$ plane symmetric solutions in $f(R)$ gravity. In this paper, we extended their work in $f(R,T)$ theory of gravity. This paper contains the study of the solutions of the N-dimensional plane symmetric spacetime. The scheme of the paper is as follows: In next section, we discuss the field equation of $f(R,T)$ theory of gravity and section 3 contains the solutions of N-dimensional plan symmetric spacetime using the power law and exponential assumptions. Moreover, we have evaluated the energy densities and corresponding functions of $f(R,T)$ model in this section. The summary and final remarks are available in the last section.[29]

Field Equations in $f(R,T)$ theory of gravity

In this section, we formulate the field equations for $N=n+1$ dimensional plane symmetric spacetime in $f(R,T)$ gravity. The action of $f(R,T)$ gravity given by Harko et al.[7] for higher dimensional is generalized as

$$s=\frac{1}{2k}\int\sqrt{-g}f(R,T)d^{n+1}x+\int\sqrt{-g}L_{mtr}d^{n+1}x, \qquad (1)$$

Where $f(R,T)$ is taken as the arbitrary function of the scalar curvature R and of the trace T of the energy momentum tensor $T_{\alpha\beta}$. L_{mtr} represents the matter Lagrangian. By varying the action with respect to the metric tensor gαβ, one can obtain the following field equation

$$f_R(R,T)R_{\alpha\beta}-\frac{1}{2}f(R,T)g_{\alpha\beta}-(\nabla_\alpha\nabla_\beta-g_{\alpha\beta}\Box)f_R(R,T)=kT_{\alpha\beta}-f_T(R,T)(T_{\alpha\beta}+\Theta_{\alpha\beta}), \qquad (2)$$

here $fR(R,T)=\frac{\partial f(R,T)}{\partial R}$, $fT(R,T)=\frac{\partial f(R,T)}{\partial T}$ and $\Box=\nabla^\alpha\nabla_\alpha$. ∇_α represent the covariant derivative. The quantity $\Theta_{\alpha\beta}$ is given as

$$\Theta_{\alpha\beta}=-2T_{\alpha\beta}+g_{\alpha\beta}L_{mtr}-2g^{\mu\nu}\frac{\partial 2L_{mtr}}{\partial g^{\mu\nu}\partial g^{\alpha\beta}}, \qquad (3)$$

Harko et al.[7] proposed some functional forms of $f(R,T)$ that are given as

I. $f(R,T)=R+2\lambda T$, where λ is a constant

II. $f(R,T)=f_1(R)+f_2(T)$,

III. $f(R,T)=f_1(R)+f_2(R)f_3(T)$.

We consider more general class of $f(R,T)$ model, i.e., $f(R,T)=f_1(R)+f_2(T)$ for our study.

Consequently for this model, Eq.(2) takes the form[7]

$$f_{1R}(R)R_{\alpha\beta}-\frac{1}{2}f_1(R)g_{\alpha\beta}-(\nabla_\alpha\nabla_\beta-g_{\alpha\beta}\Box)f_{1R}(R)=kT_{\alpha\beta}+f_{2T}(T)T_{\alpha\beta}+\frac{1}{2}f_2(T)g_{\alpha\beta}, \qquad (4)$$

where $f_{1R}(R)=\frac{\partial}{\partial R}(f_1(R))$ and $f_{2T}(T)=\frac{\partial}{\partial T}(f_2(T))$. The stress energy tensor for dust case is given as

$$T_{\alpha\beta}=\rho\upsilon_\alpha\upsilon_\beta. \qquad (5)$$

where ρ is energy density. The contraction of the field equation (4) is given as,

$$Rf_{1R}(R)-2f_1(R)+3\Box f_{1R}(R)=kT+Tf_{2T}(T)+2f_2(T). \qquad (6)$$

From this, we obtain

$$f_1(R)=\frac{3\Box f_{1R}(R)+Rf_{1R}(R)-kT-Tf_{2T}(T)-2f_2(T)}{2}. \qquad (7)$$

Putting Eq.(7) in Eq.(4), we obtain

$$\frac{f_{1R}(R)R_{\alpha\beta}-\nabla_\alpha\nabla_\beta f_{1R}(R)-(k+f_{2T}(T))T_{\alpha,\beta}}{g_{\alpha\beta}}=\frac{1}{4}\times\left[Rf_{1R}(R)-\Box f_{1R}(R)-kT-Tf_{2T}(T)-4f_2(T)\right]. \qquad (8)$$

N-dimensional plane symmetric solutions

The line element representing N-dimensional plane symmetric spacetimes is given by.

$$ds^2=A(x)dt^2-C(x)dx^2-B(x)\Sigma_{i=2}^{n-1}dx_i^2 \qquad (9)$$

where A and B are arbitrary functions of x. For easiness, we take $C(x)=1$, so the above equation take the form

$$ds^2=A(x)dt^2-dx^2-B(x)\Sigma_{i=2}^{n-1}dx_i^2. \qquad (10)$$

For this line element Ricci scalar given as

$$R=\frac{1}{2}\left[\frac{2A''}{A}-\left(\frac{A'}{A}\right)^2+(n-2)\frac{A'B'}{AB}+2(n-2)\frac{B''}{B}+(n-2)(n-5)\frac{B'^2}{2B^2}\right], \qquad (11)$$

where prime shows the derivative with respect to x. Since the metric (10) depends only on x, we see that the equation (8)[30,31] is the set of differential equations for $f_{1R}(x), f_{2T}(x)$, A and B. The field equations can be written as

$$A_\alpha = \frac{f_{1R}(R) - \nabla_\alpha \nabla_\alpha (f_{1R}(R)) - (k + f_{2T}(T)) T_{\alpha\beta}}{g_{\alpha\beta}}. \quad (12)$$

Now put $f_{1R}(R) = F(R)$ and $f_{2T}(T) = \tilde{F}(T)$, Since A_α is just a notation for the traced quantity. Thus, $A_1 - A_0 = 0$ gives

$$(n-2)\frac{A'B'}{AB}F(R) - 2(n-2)\frac{B''}{B}F(R)(n-2)\frac{B'^2}{B^2}F(R) + \frac{2A'}{A}F'(R) - 4F''(R) - 4\left[k + F(T)\right]\rho = 0 \quad (13)$$

Similarly the subtraction of $A_2, A_2, A_3 .....A_{n-1}$ from A_0 then taking equal to zero yields a single independent equation of the for

$$(n-3)\frac{A'B'}{AB}F(R) - 2\frac{B''}{B}F(R) - (n-4)\frac{B'^2}{B^2}F(R) + \frac{2A''}{A}F(R)\frac{A'^2}{A^2}F(R) + 2\frac{A'}{A}F'(R) - 2\frac{B'}{B}F'(R)4\left[k + \tilde{F}(T)\right]\rho = 0. \quad (14)$$

In this way we have obtained two non-linear differential equations involving unknown variables $\rho, A, and\ B$. Because of the conservation of energy momentum tensor, we can find the solutions by assuming the metric coeffcient A=constant, i.e., A = 1. Therefore, above two equations now reduce to

$$-2(n-2)\frac{B''}{B}F(R) + (n-2)\frac{B'^2}{B^2}F(R) - 4F''(R) - 4\left[k + \tilde{F}(T)\right]\rho = 0. \quad (15)$$

$$-2\frac{B''}{B}F(R) - (n-4)\frac{B'^2}{B^2}F(R) - 2\frac{B'}{B} - 2\frac{B'}{B}F'(R) - 4\left[k + \tilde{F}(T)\right]\rho = 0. \quad (16)$$

Subtracting above two equations, we obtain

$$-2(n-3)\frac{B''}{B}F(R) + 2(n-3)\frac{B'^2}{B^2}F(R) - 4F''(R) + 2\frac{B'}{B} = 0 \quad (17)$$

which implies

$$(n-3)\frac{B''}{B}F(R) - (n-3)\frac{B'^2}{B^2}F(R) + 2\frac{F''(R)}{F} - \frac{F'B'}{FB} = 0. \quad (18)$$

Now we follow the approach of Nojiri and Odintsov[35] and make the assumption $F(R) \alpha f_0 R^q$, which implies that $F(R) = f_0 R^q$, where f_0 and q are arbitrary real constants. Thus Eq.(18) takes the form

$$2q(q-1)\frac{R'^2}{R^2} + q\left(\frac{R''}{R} - \frac{B'R'}{BR}\right) + (n-3)\left(\frac{B''}{B} - \frac{B'^2}{B^2}\right) = 0. \quad (19)$$

Further, we will solve the last equation by using following. two assumptions

I. Exponential Law assumption

II. Power Law assumption.

Exponential solution

In this case we assume that, $B = c_1 e^{c2x}$, for the constraint $q(2q-1) = 0$, solution of the metric takes the form

$$ds^2 = dt^2 - dx^2 - c_1 e^{c2x} \sum_{i=2}^{n-1} dx_i^2. \quad (20)$$

Here we have two choices from the constraint.

CASE (A-I)

For this case, by substituting $q = 0$, we obtain $F(R) = f_0$ and we get

$$\frac{\partial}{\partial R} f_1(R) = f_0. \quad (21)$$

Integrating above equation, we obtained

$$f_1(R) = f_0 R + c_3, \quad (22)$$

where c_3 is the constant of integration. Hence $F(R)$ turn out to be

$$f(R,T) = f_0 R + f_2(T) + +c_3, \quad (23)$$

For this case, the trace of energy-momentum T and energy density ρ are given

$$\rho = -\frac{(n-2)c_2^2(f_0 R + c_3)}{4\left[k + \tilde{F}(T)\right]} = T. \quad (24)$$

Also the Ricci scalar R is given as

$$R = \frac{(n-1)(n-2)c_2^2}{2} \neq 0. \quad (25)$$

CASE (A-II)

In this case, substituting $q = \frac{-1}{2}$, we get

$$F(R) = f_0 R^{-\frac{1}{2}}, \quad (26)$$

which implies

$$\frac{\partial}{\partial R} f_0 R^{-\frac{1}{2}}. \quad (27)$$

Integrating Eq.(27), we get

$$F_1(R) = 2 f_0 R^{\frac{1}{2}} + c_4 \quad (28)$$

where c_4 is the constant of integration. So $F(R,T)$ becomes

$$F(R,T) = 2 f_0 \sqrt{R} + f_2(T) + c_4. \quad (29)$$

For this case, the trace of energy-momentum T and energy density ρ are given as

$$\rho = -\frac{(n-2)c_2^2\left(f_0 R^{\frac{1}{2}} + c_4\right)}{4\left[k+\tilde{F}(T)\right]} = T. \qquad (30)$$

Also the Ricci scalar R is given as

$$R = \frac{(n-1)(n-2)c_2^2}{2} \neq 0. \qquad (31)$$

Power law solutions

For this case, we assume that $B\,\alpha\,x^{\omega}$, where ω is any real number. We substitute $B(x) = c_5 x^{\omega}$ in Eq.(18), where c_5 is an arbitrary constant. Now solving equation (19) by taking $\omega = 2$, we get

$$4q^2 + 4q - (n-3) = 0. \qquad (32)$$

The solution of the metric takes the form

$$ds^2 = dt^2 - dx^2 - c_5 x^{\omega} \sum_{i=2}^{n-1} dx_i^2. \qquad (33)$$

The solution of Eq.(32) gives, $q = \frac{-1\pm\sqrt{n-2}}{2}$. Here, we have two case for two different roots of the Eq.(32).

CASE (B-I)

For $q = \frac{-1\pm\sqrt{n-2}}{2}$, we have

$$F(R) = f_0 R^{\frac{-1+\sqrt{n-2}}{2}}, \qquad (34)$$

$$\frac{\partial}{\partial R} f_1(R) = f_0 R^{\frac{-1+\sqrt{n-2}}{2}}. \qquad (35)$$

Integrating Eq.(35) , we get

$$f_1(R) = \frac{2f_0}{1+\sqrt{n-2}} R^{\frac{1+\sqrt{n-2}}{2}} + c_6, \qquad (36)$$

which implies

$$f_1(R) = \frac{2f_0}{1+\sqrt{n-2}}\left(\frac{2}{x^2}\right)^{\frac{1+\sqrt{n-2}}{2}} + c_6, \qquad (37)$$

where c_6 is the constant of integration. Hence $f(R,T)$ takes the form

$$f(R,T) = \frac{2f_0}{1+\sqrt{n-2}}\left(\frac{2}{x^2}\right)^{\frac{1+\sqrt{n-2}}{2}} + f_2(T) + c_6. \qquad (38)$$

For this case, the trace of energy-momentum T and energy density ρ are given as

$$\rho = \frac{-2+\sqrt{n-2}\left[(n-2)(n-3)\right]^{-1+\sqrt{n-2}}\left(n-2-\sqrt{n-2}\right)f_0}{x^{-1}+\sqrt{n-2}\left[k+\lambda\right]} = T. \qquad (39)$$

Also the Ricci scalar R can be given as

$$R = \frac{(n-2)\,\omega\left[\omega(n-1)-4\right]}{2x^2} \neq 0 \qquad (40)$$

CASE (B-II)

For $q = \frac{-1-\sqrt{n-2}}{2}$, we have

$$F(R) = f_0 R^{\frac{-1-\sqrt{n-2}}{2}}, \qquad (41)$$

$$\frac{\partial}{\partial R} f_1(R) = f_0 R^{\frac{-1-\sqrt{n-2}}{2}}. \qquad (42)$$

Integrating Eq.(42) , we get

$$F_1(R) = \frac{2f_0}{1-\sqrt{n-2}} R^{\frac{-1-\sqrt{n-2}}{2}} + c_7, \qquad (43)$$

which implies

$$F_1(R) = \frac{2f_0}{1-\sqrt{n-2}}\left(\frac{2}{x^2}\right)^{\frac{-1-\sqrt{n-2}}{2}} + c_7, \qquad (44)$$

where c_7 is the constant of integration, so $f(R,T)$ takes the form

$$f(R,T) = \frac{2f_0}{1-\sqrt{n-2}}\left(\frac{2}{x^2}\right)^{\frac{-1-\sqrt{n-2}}{2}} + f_2(T)c_7. \qquad (45)$$

For this case, the trace of energy-momentum T and energy density ρ are given as

$$\rho = \frac{-2^{-1-\sqrt{n-2}}\left[(n-2)(n-3)\right]^{-1-\sqrt{n-2}}\left(n-2+\sqrt{n-2}\right)f_0}{x^{-1-\sqrt{n-2}}\left[k+\lambda\right]} = T. \qquad (46)$$

While the Ricci scalar R turn out to be

$$R = \frac{(n-2)\,\omega\left[\omega(n-1)-4\right]}{2x^2} \neq 0. \qquad (47)$$

Summary and conclusion

We have evaluated the N-dimensional non-vacuum plane-symmetric solutions in the context of $f(R,T)$ theory of gravity. For this purpose, we solve the field equations by considering the metric representing the N-dimensional planesymmetric space-time in GR. We investigate the solutions by using the general class of $f(R,T)$ model, i.e., $f(R,T) = f_1(R) + f_2(T)$. Moreover, we find the solutions by making the assumption $f(R)\,\alpha\, f_0 R^q$, where f_0 and q are arbitrary constants. To find the solutions, we also assume the dust case with $p = 0$. Initially, the field equations look complicated and it was diffcult to solve these equations because they are highly non-linear. So the corresponding field equations are solved using exponential law forms given in Eq. (20) and power law assumption of metric coeffcient given in Eq.(33). In exponential forms there arises two cases, case (A-I): for $m = 0$, yields the function of Ricci scalar $f(R,T)$ given in equation (23) and case(A-II): for $m = -\frac{1}{2}$, yields the value of Ricci scalar function $f(R,T)$ given equation (29). The values of Ricci scalar R and matter density ρ are all evaluated in every case. However, the Riacci scalar is non-zero in both cases.

There also arises two cases in power law form, case(B-I): form $= \frac{-1+\sqrt{n-2}}{2}$, yields the function of Ricci scalar $f(R,T)$ given in Eq.(31) and case(B-II): for $m = \frac{-1-\sqrt{n-2}}{2}$, yields the function of Ricci scalar $f(R,T)$ given in equation (33). The values of Ricci scalar R and matter density ρ are all evaluated in every case. However, the Riacci scalar is non-zero in both cases. Moreover, the energy densities are evaluated in each case. Finally, we conclude that this work contain some information about the crucial issues of the localization of energy

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Table 1. Also this work provides the energy densities of different solutions with general classes $f(R,T)$ model which may be helpful to reduce the theoretical problems in cosmology. Thus, it is hoped that such types of solutions in the background of $f(R,T)$ gravity may explain the present phase of cosmic acceleration of our universe and may provide some attractive aspects of GR.[31,35]

Table I Solutions of the N-dimensional plane-symmetric spacetimes

CASE	SOLUTION
A	$ds^2 = dt^2 - dx^2 - c_1 e^{c2x} \sum_{i=2}^{n-1} dx_i^2.$
B	$ds^2 = dt^2 - dx^2 - c_5 x^{\omega} \sum_{i=2}^{n-1} dx_i^2.$

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None.

Conflicts of interest

The author declares there is no conflict of interest.

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