Review Article

On loop space-self avoiding string representations for Q.C.D(SU(∞))

Abstract

We present several clarifying comments on the loop space-self avoid string representation for \( Q.C.D(SU(\infty)) \) proposed by this author along last decades.

Keywords: Q.C.D self-avoiding strings representations, Q.C.D loop space

Introduction

We star our introduction by recalling the following highlights from eminent Physicists on the search for a mathematical formalism free from ambiguities in strong and weak strong nuclear forces theory

a) “Therefore conclusions based on the renormalization group arguments concerning Q.F.T summed to all orders are dangerous and must be viewed with due caution. So is it with all conclusions from Q.C.D.”

b) “Because of severe divergences, Yang-Mills theories cannot be consistently interpreted by conventional perturbation (LSZ) theory.”

c) “There are method and formulae in science, which serve as master-key to many apparently different problems. The resources of such things have to be refilled from time to time. In AM Polyakov’s opinion (and mine), we have to develop a formalism to sum over random surfaces and Riemann geometries (“Brownian Riemann Surfaces”).”

These comment are intend to clarify the concept of self-avoiding string representation for Q.C.D in the formalism of Loop Space Theory, mostly exposed in our previous works on the subject.

Revisiting the loop space formulation for Q.C.D.

Since its proposal several decades ago, the loop space formulation of Q.C.D has been an alternative to the well-known Feynman-diagramatics-perturbative formulation of well-defined quantum field theories.\(^{2,5-8}\)

In this section we intend to point out some the mathematical difficulties on applying the loop space formalism for Q.C.D or to any other quantum field theory with spinorial matter field.

Let us write the generaling functional of color singlet bilinear vectorial curesnts with the quark fermionic degrees integrated out

\[
Z[J_{\mu}] = \frac{1}{Z(0)} \langle \det \beta_\mu (A_\mu + J_\mu) \rangle_{SU(N)} \tag{1}
\]

Here \( \langle \cdot \rangle_{SU(N)} \) denotes the formal average of the quantum Yang-Mills fields on \( SU(N) \).

The next step is writing the above functional determinant as a continuous sum over the massless quark trajectories. In a plane wave (euclidean) spinor bases

\[
\begin{align*}
\langle x, \alpha \rangle &= e^{ip\cdot x} U^\dagger_{\alpha} (p) \quad \text{(2-a)}
\end{align*}
\]

\[
\begin{align*}
\langle y, \beta \rangle &= U^\dagger_{\beta} (p) e^{ipy} \quad \text{(2-b)}
\end{align*}
\]

One has the Feynman path integral formal expression for the above written quark determinant

\[
\begin{align*}
\mathcal{L}(n) &\langle \det D(A + J) \rangle = \left\{ \frac{d^D p}{T} \right\} \exp \left\{ iP_{\mu} (\sigma) \alpha \right\} \right\}
\end{align*}
\]

Unfortunately the path integral object eq.(3) remains not well understood from a mathematical point of view as far as this author knows.

However, one can use a formal path \( SU(N) \) valued variable change:

\[
P_{\mu} (\sigma) + A_\mu (X(\sigma)) + J_\mu (X(\sigma)) = \pi_{\mu} (\sigma) \tag{4}
\]

One thus gets the “less formal” and more mathematical expression palatable from a Theoretical Physics point of view for the fermion determinant

\[
\begin{align*}
\mathcal{L}(n) &\langle \det D(A_\mu + J_\mu) \rangle / \det D(0)
\end{align*}
\]

\[
\begin{align*}
= \frac{1}{T} \int_{SU(N)} \left\{ X(\alpha) D^\dagger (X(\sigma)) \right\}
\end{align*}
\]

\[
\begin{align*}
= \exp \left\{ iP_{\mu} (\sigma) \alpha \right\} \right\}
\]

\[
\begin{align*}
\exp \left\{ \frac{1}{T} \int_{SU(N)} \left\{ D^\dagger (X(\sigma)) \right\}
\end{align*}
\]

\[
\begin{align*}
\times \left\{ \exp \left\{ iP_{\mu} (\sigma) \alpha \right\} \right\}
\end{align*}
\]

\[
\begin{align*}
\times \left\{ \exp \left\{ \frac{1}{T} \int_{SU(N)} \left\{ D^\dagger (X(\sigma)) \right\}
\end{align*}
\]
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Unfortunately eq(4)-eq(5) still are somewhat mathematically formal and evaluations with them has never been performed in the literature, even if on the non-relativistic case it leads to the correct results. However it is worth to remain that on lattice, the chances for a fully mathematical rigorous and calculation scheme are greater than its version on the continuum $\mathbb{R}^4$. This vital matter will be presented elsewhere.

At this point, it is instructive to point out the supersymmetric path integral proposal for a spinning particles. However its suggested Wilson Loop necessarilly would involve the spin orbit coupling term with the strenght field which is loop length dependent. Explicitly:

$$w[\mathcal{A}] = \text{Tr}_{\mathcal{SU}(N)} \left( \text{Tr}_{\mathcal{Dirac}} \left( \mathcal{P}_{\mathcal{SU}(N)} \right) \right)$$

$$\exp\left( i\int_0^\alpha A_{\mu}(X(\sigma))\mu dX^\mu(\sigma) + \frac{1}{4} i\varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu}(X(\sigma)\mu dX^\rho(\sigma)) \right)$$

However to reformulate the ill defined quantum field theory of Q.C.D(SU(N)), one makes the hypothesis that the Wilson Loop on eq(5) is the correct collective variable to be represented through a random surface path integral at least on the lattice framework. We point out that this step can be regard as a correct “theoretical physics” argument at the deep infrared region, where the Dirac spin degrees of freedom are frozen. Note that on light of this hypothesis, the string path integratized the quantum Wilson Loop on eq(5) must be supersymetrized on the string ambient space-time to fully represent the spinning Wilson Loop eq(6). But mathematical proofs are required and have not been available since still there is not a mathematical theory for spinning Brownian motion at the present time.

But the whole point of finding string representations for Q.C.D(SU(N)) (or Q.C.D(SU(∞))) more precisely is to argue that Q.C.D as a mathematical object is an ill-defined object by itself. So all previous formulæ eq(1)-eq(6) are only suggestive at the continuum and could mathematically make sense (if any) only at lattice, where Q.C.D(SU(∞)) is well-defined. As a result one must search a string (where the Yang-Mills strenght condensate $\Phi_{\mu}[X_{\mu}(\sigma)]$) more precisely is to argue that this vital matter will be presented elsewhere.

Formal Euclidean Yang-Mills theory under the hypothesis of a non-zero Yang-Mills quantum average is granted to be factorized on the product of and to the Baryons excitations. All of this surely “sconisciuta terra”.

$\text{The “free” string path integral}$

In order to search solutions for the non linear “quadratic” loop space wave equation eq(7-b), one must give a correct meaning for the Wiener-Feynman sum under surfaces (bosonic Brownian surfaces), in place of the well known bosonic sum over Wiener-Feynman paths.

One fashionable proposal is due to AM Polyakov, it aloud it has been revealed to be clearly wrong (Appendix 2).

It is based on the Brink-Howe action, but added with a non-vanishing cosmological term

$$G(C) = \int d^{2\sigma} [g_{\alpha\beta}(\xi)] \left[ \exp\left( \frac{1}{2\pi\alpha} \int_{\sigma \leq \sigma} \left( \partial_{\mu} X^\mu(\sigma) - \partial_{\mu} X^\mu(\sigma) \right) \right) \right]$$

However in order for the classical solutions of eq(8) reproduces the Nambu-Goto area functional it is going to constraint the cosmological term to vanish (unless at the extrinsic space-time dimension $D=2$, a two restrictive dimensionality for the space time quantum dynamics).

It results that on the surface conformal gauge and for $D=26$ (or by introducing $N$ neutral fermions such that $N+D=26$), one obtains the expected gauge fixed propagator as a theory of free fields on the string domain parameter (otherwise the theory is non-renormalible, so ill-defined—see also Appendix 2).

$$G(C_{\alpha}) = \int d^{2\sigma} [g_{\alpha\beta}(\xi)] \left[ \exp\left( \frac{1}{2\pi\alpha} \int_{\sigma \leq \sigma} \left( \partial_{\mu} X^\mu(\sigma) - \partial_{\mu} X^\mu(\sigma) \right) \right) \right]$$

At this point it is argued that the following two-dimensional path integral, with a neutral set of $N=22$ fermions solve the Q.C.D(SU(∞)) loop wave equation(7-b) (with

$$\langle 0 | F^2 | 0 \rangle_{\mathcal{SU}(\infty)} = \frac{1}{\alpha^2} = 1$$

and $\xi = (\sigma, \tau)$).

$$\Phi_{\mathcal{SU}(\infty)} [C(\sigma), 0 \leq \sigma \leq 2\pi] = \int d\tau \left[ \text{Tr}_{\mathcal{SU}(\infty)} \left( \exp\left( \frac{1}{4} i\varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu}(X(\sigma))\mu dX^\rho(\sigma) \right) \right) \right]$$

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Here $\mathcal{I}^\mu(\vec{x}(\xi))$ is the normalized surface area tensor.\(^2\) 
\[
\mathcal{I}^\mu(\vec{x}(\xi)) = \frac{(e^{\mu\alpha\beta} \dddot{x}^\alpha \dddot{x}^\beta) \dddot{x}^\mu(\xi)}{\sqrt{2} \ln(\mathcal{I}(\xi))}
\]  
(12-a) 
\[
\mathcal{I}^\mu(\vec{x}(\xi)) \mathcal{I}^\mu(\vec{x}(\xi)) = 1
\]  
(12-b) 

At this point one could consider eq(11) as an interacting string path integral on the surface conformal gauge as done in eq(10).

Note that at this point it is somewhat irrelevant to consider the two-dimensional path integral as a path integral related to a random surface theory, even if this geometrical interpretation holds true on lattice, and necessary for theory’s unitarization afterwards Q.C.D is thus analytically solved through interpreting eq(11) as a string path integral extended to all surface genus (somewhat related the Mandelstam light-cone string path integral on euclidean space-time).

At this point of ours comments, we remark that eq(11) should be evaluated explicitly in terms of the loop boundary $C_{xx}$ and the loop parameter $\sigma$ and the string proper-time $A$.\(^3\) After this step, one expects that this “stringy” Wilson Loop is now well-defined and should replace the ill defined one given by eq(7-a) and averaged over the (ill defined) quantum Yang-Mills path integral. Namely

\[
\text{det} B_{\sigma}^{(A_{\sigma} + J_{\mu})} = \exp \left[ - \frac{1}{4} \int d\sigma \left( \mathcal{I}_C^{\mu}(\sigma) \mathcal{I}_C^{\mu}(\sigma) \right) \right]
\]

\[
\int \exp \left[ - \frac{1}{4} \int d\sigma \left( \mathcal{I}_C^{\mu}(\sigma) \mathcal{I}_C^{\mu}(\sigma) \right) \right] \left( \mathcal{I}^{2}\right)
\]

\[
\exp \left[ - \frac{1}{4} \int d\sigma \left( \mathcal{I}^{2}\right) \right] \Phi_{\sigma}(\mathcal{I}_{\sigma}(\sigma),0 \leq \sigma \leq t)
\]

We have thus that eq(13) should be the correct (string) definition of Q.C.D(SU(\(\infty\))) or possible Q.C.D(SU(N)) when the surface sum is defined for all possible topological genera and adjusted to the electroweak sector of Nuclear Forces.\(^4\)

It is thus suggested by eq(13) that correlation functions of the quark color singlet bilinear in the ill defined Lagrangean quantum field Q.C.D are well defined by the (on shell) string vertices averages scattering amplitudes associated to eq(11), producing as a result, the meson S-matrix and the determination of its physical spectrum (the quark color singlet bilinear in the ill defined Lagrangean quantum mechanics for Elementary Particle Physics).

So one must use the well defined possible Q.C.D string theory in place of the quantum field ill defined Q.C.D Lagrangean. Both coinciding only at lattice as well-defined mathematical Quantum Euclidean theories.

In this context of Q.C.D(SU(\(\infty\))) meson spectrum, one should point out that the numerical condition of vanishing of the conformal anomaly in the free string propagator eq(9) or in the proposed Q.C.D(SU(\(\infty\))) eq(11), 26=D, or 26=D-N respectively is replaced by taking the string two-dimension parameter Planck’s constant (\(\hbar \)) to vanishes, which means that \((\vec{c} \vec{x}^\alpha \partial_\alpha \vec{x}^\beta) \sim h_{\text{Planck}}^{\text{classical}} (x^{\mu}(\xi)) = \frac{(d\sigma^2) + (d\tau^2)}{\tau^2}

Work on the space-time supersymmetric version of eq(11) will appear elsewhere (Appendix 3,4).\(^6\)

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### Conflicts of interest

Authors declare there is no conflict of interest.

### References