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On loop space-self avoiding string representations for Q.C.D(SU(∞))

Abstract

We present several clarifying comments on the loop space-self avoid string representation for $Q.C.D(SU(\infty))$ proposed by this author along last decades.

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Introduction

We star our introduction by recalling the following highlights from eminent Physicists on the search for a mathematical formalism free from ambiguities in strong and weak strong nuclear forces theory

a) "Therefore conclusions based on the renormalization group arguments concerning Q.F.T summed to all orders are dangerous and must be viewed with due caution. So is it with all conclusions from Q.C.D."

b) "Because of severe divergences, Yang-Mills theories cannot be consistently interpreted by conventional perturbation (LSZ) theory."

c) "There are method and formulae in science, which serve as master-key to many apparently different problems. The resources of such things have to be refilled from time to time. In AM Polyakov's opinion (and mine), we have to develop a formalism to sum over random surfaces and Riemann geometries ("Brownian Riemann Surfaces")".

These comment are intend to clarify the concept of self-avoiding string representation for Q.C.D in the formalism of Loop Space Theory, mostly exposed in our previous works on the subject.[1–5] A word about this comment writing format. Since loop space $Q.C.D(SU(\infty))$ and self-avoiding string path integrals are a notoriously difficult mathematical methods subject to be exposed, we deliberately have transferred to 4 appendixes those more mathematical oriented arguments supporting the text main discussions.

Revisiting the loop space formulation for Q.C.D.

Since its proposal several decades ago, the loop space formulation of Q.C.D has been an alternative to the well-known Feynman-diagramatics-perturbative formulation of well-defined quantum field theories.[2,6–8]

In this section we intend to point out some the mathematical difficulties on applying the loop space formalism for Q.C.D or to any other quantum field theory with spinorial matter field.

Let us write the generaling functional of color singlet bilinear vectorial curents with the quark fermionic degrees integrated out

$$Z[J_\mu]=\frac{1}{Z(0)}\left\langle \det \not{D}(A_\mu+J_\mu)\right\rangle_{SU(N_c)} \qquad (1)$$

Here $\langle\ \rangle_{SU(N_c)}$ denotes the formal average of the quantum Yang-Mills fields on $SU(N_c)$.

The next step is writing the above functional determinat as a continuous sum over the massless quark trajectories. In a plane wave (euclidean) spinor bases

$$\langle x,\alpha\rangle = e^{ipx}U_\alpha^{(1)}(p) \qquad (2\text{-a})$$

$$\overline{\langle y,\beta\rangle} = \overline{U}_\beta^{(2)}e^{ipy} \qquad (2\text{-b})$$

One has the Feynman path integral formal expression for the above written quark determinant

$$\ell n(\det \not{D}(A+J)) = -\left\{\int_0^\infty \frac{dt}{t}\mathrm{Tr}(e^{-(\not{D}(A+J))})\right\}$$

$$=-\left\{\int_0^\infty \frac{dt}{t}\int d^Dx_\mu d^Dp_\mu\left\{\int_{X^\mu(0)=X^\mu(t)=x^\mu}D^F[X^\mu[(\sigma)]\times\int_{P^\mu(0)=P^\mu(T)=p^\mu}\right.\right.$$

$$\exp\left(i\int_0^t P_\mu(\sigma)\dot{X}^\mu(\sigma)d\sigma\right)$$

$$\mathbb{P}_{Dirac}\mathbb{P}_{SU(N)}\left[\exp\left(i\int_0^t\gamma^\mu\left((P_\mu(\sigma)+A_\mu(X(\sigma))+J_\mu(X(\sigma))\right)\right)\right]\Bigg\} \qquad (3)$$

Unfortunatelly the path integral object eq(3) remains not well understood from a mathematical point of view as far as this author knows.

However, one can use a formal path $SU(N)$ valued variable change:

$$P_\mu(\sigma)+A_\mu(X(\sigma))+J_\mu(X(\sigma))=\pi_\mu(\sigma) \qquad (4)$$

One thus gets the "less formal" and more mathematical expression palatable from a Theoretical Physics point of view for the fermion determinant

$$\ell n(\det D(A_\mu+J_\mu)/\det D(0))$$

$$=-\int_0^\infty\frac{dt}{t}[\int_{X^\mu(0)=X^\mu(t)}D^F[X(\sigma)]$$

$$\int_{\pi^\mu(0)=\pi^\mu(t)}D^F[\pi^\mu(\sigma)]\mathrm{Tr}_{SU(N)}\mathrm{Tr}_{Dirac}\mathbb{P}_{Dirac}\mathbb{P}_{SU(N)}$$

$$\times\{\exp\left(+i\int_0^t(\gamma^\mu\pi_\mu)(\sigma)d\sigma\right)$$

$$\times\mathrm{Tr}_{SU(N)}\left[\mathbb{P}_{SU(N)}\exp\left(-i\int_0^t(A^\mu(X(\sigma))-\pi^\mu(\sigma))dX_\mu(\sigma)\right)\right]$$

$$\times \exp\left(-i\int_0^t J_\mu(X(\sigma))dX^\mu(\sigma)\right)\Big\} \qquad (5)$$

Unfortunately eq(4)-eq(5) still are somewhat mathematically formal and evaluations with them has never been performed in the literature, even if on the non-relativistic case it leads to the correct results. However it is worth to remaind that on lattice, the chances for a fully mathematical rigorous and calculation scheme are greater than its version on the continuum $\mathbb{R}^D$. This vital matter will be presented elsewhere.

At this point, it is instructive to point out the supersymmetric path integral proposal for a spinning particles.[4,5] However its suggested Wilson Loop necessarilly would involves the spin orbit coupling term with the strenght field which is loop lenght dependent. Explicitly:

$$W^B[C_{xx}] = \mathrm{Tr}_{SU(N)}\Big\{\mathrm{Tr}_{\mathrm{Dirac}}(\mathbb{P}_{\mathrm{Dirac}}(\mathbb{P}_{SU(N)}$$

$$\exp[i\int_0^t (A_\mu(X(\sigma))\dot{X}^\mu(\sigma) + \frac{1}{4}i[\gamma^\mu,\gamma^\nu]F_{\mu\nu}(X(\sigma)d\sigma)])\Big\} \qquad (6)$$

However to reformulate the ill defined quantum field theory of *Q.C.D(SU(N))*, one makes the hypothesis that the Wilson Loop on eq(5) is the correct collective variable to be represented through a random surface path integral at least on the lattice framework.[1] We point out that this step can be regard as a correct "theoretical physics" argument at the deep infrared region, where the Dirac spin degrees of freedom are frozens. Note that on light of this hypothesis, the string path integral representing the quantum Wilson Loop on eq(5) must be supersymmetrized on the string ambient space-time to fully represent the spinning Wilson Loop eq(6).[4,5] But mathematical proofs are required and have not been available since still there is not a mathematical theory for spinning Brownian motion at the present time.[9]

But the whole point of finding string representations for *Q.C.D(SU(N))* (or $Q.C.D(SU(\infty))$) more precisely is to argue that Q.C.D as a mathematical object is an ill-defined object by itself. So all previous formulae eq(1)-eq(6) are only suggestive at the continuum and could mathematically make sense (if any) only at lattice, where $Q.C.D(SU(\infty))$ is well-defined. As a result one must search a string path integral from a formal point of view, heavily inspired on the formal objects eq(1)-eq(6). And Q.C.D should be fully replaced by the string path integral, which must reproduce lattice Q.C.D, when reformulated in the lattice[1] (the string path integral!). It is even expected that that Q.C.D string path integral on lattice is the calculational tool for Q.C.D evaluations. A final remark: *Q.C.D(SU(N))* string path integral is expected (but not proved yet!) to be the possible found $Q.C.D(SU(\infty))$ now endowed with all non trivial genus – Unitarization of the associated Q.C.D's Scattering Matrix). However, the determination of $N_c = 3$ must be make recourse to the flavor quark electro-weak sector and to the Baryons excitations. All of this surely "sconisciuta terra".[9]

So let us use scalar deep infrared $Q.C.D(SU(\infty))$ (where the Yang-Mills quantum average is granted to be factorized on the product of gauge invariant observables).

In this case, we have written the following loop wave string like wave equations eq(7-b) for formal Euclidean Yang-Mills theory under the hypothesis of a non-zero Yang-Mills strenght condensate.[10–12] Namely (Appendix 1)

$$\Phi_{N_c}[X_\mu(\sigma), 0\le\sigma\le 2\pi] = \frac{1}{N_c}[\mathrm{Tr}_{SU(N_c)}\{\mathbb{P}\left(\exp i\int_0^{2\pi} A_\mu(X(\sigma))dX^\mu(\sigma)\right)\}] \qquad (7\text{-a})$$

$$\int_0^{2\pi} d\bar{\sigma}\left[\left(\frac{\delta^2}{\delta X_\mu(\bar{\sigma})\delta X^\mu(\bar{\sigma})}\right) - \langle \mathrm{Tr}_{SU(\infty)}(F^2)\rangle\,|\dot{X}_\mu(\bar{\sigma})|^2\right]\Phi_\infty[X_\mu(\bar{\sigma})]$$

$$= (g^\infty)^2\{\int_0^{2\pi} d\sigma\int_0^{2\pi} d\bar{\sigma}\,\delta^{(D)}(X_\mu(\bar{\sigma}) - X_\mu(\sigma))\times(\dot{X}_\mu(\sigma)\dot{X}_\mu(\bar{\sigma}))$$

$$\Phi_\infty[X_\mu(\tilde{\sigma}); 0\le\tilde{\sigma}\le\bar{\sigma}]\Phi_\infty[X_\mu(\tilde{\sigma}); \bar{\sigma}\le\tilde{\sigma}\le 2\pi]\} \qquad (7\text{-b})$$

The "free" string path integral

In order to search solutions for the non linear "quadratic" loop space wave equation eq(7-b), one must give a correct meaning for the Wiener-Feynman sum under surfaces (bosonic Brownian surfaces), in place of the well known bosonic sum over Wiener-Feynman paths.

One fashionable proposal is due to AM Polyakov,[8] altought it has been revealed to be clearly wrong (Appendix 2).

It is based on the Brink-Howe action, but added with a non-vanishing cosmological term

$$G(C) = \int d_\mu^{cov}[g_{ab}(\xi)\int_{\partial X^\mu\underbrace{(\sigma,t)}_{\xi}=C^\mu(\sigma)} d^{cov}[X_\mu(\xi)]$$

$$\exp\{\frac{1}{\pi\alpha'}\int_{\mathbb{D}}\left(\frac{1}{2}\sqrt{g}\,g^{ab}\partial_a X^\mu\partial_b X_\mu\right)(\xi)d^2\xi - \mu_0^2\int_{\mathbb{D}}(\sqrt{g})(\xi)d^2\xi\} \qquad (8)$$

However in order for the classical solutions of eq(8) reproduces the Nambu-Goto area functional it is going to constraint the cosmological term to vanish (unless at the extrinsic space-time dimension D=2, a two restrictive dimensionality for the space time quantum dynamics).

The full correct meaning of eq(8) was however written in full in reference.[12]

$$G(C_{xx}) = \int d_\mu^{cov}[g_{ab}(\xi)]\int_{\partial X^\mu(\sigma,\zeta)=C^\mu_{xx}(\sigma)} d^{cov}[X^\mu(\xi)]$$

$$\exp\left\{-\frac{1}{2}\int_{\mathbb{D}}(\sqrt{g}\,g^{ab}\partial_a X^\mu\partial_b X_\mu)\right\}$$

$$\exp\left\{-\left(\frac{1}{2\pi\alpha'}\right)\int_{\mathbb{D}}(\sqrt{g})(\xi)d^2\xi\right\} \qquad (9)$$

$$\delta_{cov}^{(F)}\left(g_{ab}(\xi) - (\partial_a X^\mu\partial_b X_\mu)(\xi)\right)$$

It results that on the surface conformal gauge and for D=26 (or by introducing N neutral fermions such that N+D=26), one obtains the expected gauge fixed propagator as a theory of free fields on the string domain parameter (otherwise the theory is non-renormalible, so ill-defined – see also Appendix 2).

$$G(C_{xx}] = \int_{\partial X^\mu(\xi)=C^\mu_{xx}} D^F(X^\mu(\xi)]\times\delta^{(F)}((\partial_+X^\mu)^2 + (\partial_-X^\mu)^2)$$

$$\exp\left[-\frac{1}{2\pi\alpha'}\int_{\mathbb{D}}((\partial_+X^\mu)(\partial_-X_\mu))(\xi^+,\xi^-)d^2\xi\right] \qquad (10)$$

At this point it is argued that the following two-dimensional path integral,[1,11,13] with a neutral set of N=22 fermions solve the $Q.C.D(SU(\infty))$ loop wave equationeq(7-b) (with $\langle 0|F^2|0\rangle_{SU(\infty)} = \frac{1}{\pi\alpha'} = 1$ and $\xi = (\sigma,\tau)$).

$$\Phi_{SU(\infty)}\left[C_\mu(\sigma), 0\le\sigma\le t\right] = \int_0^\infty dA\Big\{\int_{\partial X^\mu(\xi)=C^\mu} D^F[X^\mu(\xi)]$$

$$\exp\left[-\frac{1}{2}\int_0^A d\tau\int_0^t d\sigma(\partial X^\mu)^2(\sigma,\tau)\right]\int(D\psi^i D\overline{\psi}^i)(\xi)$$

$$\exp\left[+\int_0^A d\tau\int_0^t d\sigma(\sqrt{h(X)}\overline{\psi}^i\partial_h\psi^i)(\sigma,\tau)\right] \qquad (11)$$

$$\exp[-\lambda_0^2\int_0^A d\tau\int_0^A d\tau'\int_0^t d\sigma\int_0^t d\sigma'(\sqrt{h(X)}\overline{\psi}^i\psi_i)(\sigma,\tau)$$

$$(\sqrt{h(X)}\overline{\psi}^i\psi_i)(\sigma',\tau')\mathfrak{I}^{\mu\nu}(X(\xi)\delta^{(D)}(X(\xi) - X(\xi'))\mathfrak{I}_{\mu\nu}(X(\xi'))]\}$$

[1]computer

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Here $\Im^{\mu\nu}(X(\xi))$ is the normalized surface area tensor.[2]

$$\Im^{\mu\nu}(X(\xi)) = \frac{(\varepsilon^{ab}\partial_a X^\mu \partial_b X^\nu)(\xi)}{\sqrt{2}\sqrt{h(X(\xi))}} \qquad (12\text{-a})$$

$$\Im^{\mu\nu}(X(\xi))\Im_{\mu\nu}(X(\xi)) = 1 \qquad (12\text{-b})$$

At this point one could consider eq(11) as an interacting string path integral on the surface conformal gauge as done in eq(10).

Note that at this point it is somewhat irrelevant to consider the two-dimensional path integral as a path integral related to a random surface theory, even if this geometrical interpretation holds true on lattice, and necessary for theory's unitarization afterwards Q.C.D is thus analitically solved through interpreting eq(11) as a string path integral extended to all surface genus (somewhat related the Mandelstam light-cone string path integral on euclidean space-time).

At this point of ours comments, we remark that eq(11) should be evaluated explicitly in terms of the loop boundary C_{xx} and the loop parameter σ and the string proper-time A.[11] After this step, one expects that this "stringy" Wilson Loop is now well-defined and should replace the ill defined one given by eq(7-a) and averaged over the (ill defined) quantum euclidean Yang-Mills path integral. Namely

$$\left\langle \frac{\det \not{D}(A_\mu + J_\mu)}{\det \not{D}(0)} \right\rangle_{SU(\infty)} = \exp\left\{ -\int_0^\infty \frac{dt}{t} \int d^\nu x \left\{ \int_{C^\mu_{xx}(\sigma)]} D^F[C^\mu_{xx}(\sigma)] \right.\right.$$

$$\int_{\pi^\mu(0)=\pi^\mu(t)} D^F[\pi^\mu(\sigma)] \exp\left(i\int_0^t \pi^\mu(\sigma)\dot{X}_\mu(\sigma)d\sigma \right)$$

$$Tr_{\text{Dirac}}\left[\mathbb{P}_{\text{Dirac}}\{\exp\left(-i\int_0^t (\gamma^\mu\pi_\mu)(\sigma)d\sigma\right)\right]$$

$$\left.\left.\exp\left(-i\int_0^t J_\mu(X(\sigma))dX^\mu(\sigma)\right) \times \Phi_{SU(\infty)}[C^\mu_{xx}(\sigma), 0 \le \sigma \le t] \right\} \right\} \quad (13)$$

We have thus that eq(13) should be the correct (string) definition of $Q.C.D(SU(\infty))$ or possible $Q.C.D(SU(N_c))$ when the surface sum is defined for all possible topological genera and adjusted to the eletroweak sector of Nuclear Forces.[11]

It is thus suggested by eq(13) that correlation functions of the quarks color singlet bilinear in the ill defined Lagrangean quantum field Q.C.D are well defined by the (on shell) string vertexs averages scattering amplitudes associated to eq(11), producing as a result, the meson S-matrix and the determination of it physical spectrum (the meson mass spectrum through the Regge dual model predictions. The most relevant basic problem on extending successfully quantum mechanics for Elementary Particle Physics).

So one must use the possible well-defined Q.C.D string theory in place of the quantum field ill defined Q.C.D Lagrangean. Both coinciding only at lattice as well-defined mathematical Quantum Euclidean theories.

In this context of $Q.C.D(SU(\infty))$ meson spectrum, one should point out that the numerical condition of vanishing of the conformal anomaly in the free string propagator eq(9) or in the proposed $Q.C.D(SU(\infty))$ eq(11), 26=D, or 26=D+N respectively is replaced by taking the string two-dimension parameter Planck's constant $(\hbar)^(2)$ to vanishes, which means that $(\partial_+ X^\mu \partial_- X_\mu)(\xi) \sim h_{ab}^{classical}(X^\mu(\xi)) = \frac{(d\sigma^2)+(d\tau)^2}{\tau^2}$.[15]

Work on the space-time supersymmetric version of eq(11) will appear elsewhere (Appendix 3,4).[16–19]

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Conflicts of interest

Authors declare there is no conflict of interest.

References

1. Luiz CL Botelho. String wave equations in Polyakov's path integral framework. *Journal of Mathematical Physics*. 1989;30(9):2160.
2. AA Migdal. QCD=Fermi string theory. *Nucl Phys B*. 1981;189(2):253−294.
3. AM Polyakov. Confining strings. *Nucl Phys B*. 1997;486(1−2):23−33.
4. Luiz CL Botelho. A simple renormalization scheme in random surface theory. *Modern Phys Letters B*. 1999;13(6−7):203−207.
5. AI Karanikas, CN Ktorides. Surface self-intersections and instantons. *Phys Lett B*. 1990;235(1−2):90−96.
6. Luiz CL Botelho. A fermionic loop wave equation for quantum chromodynamics at Nc = +∞. *Phys Letters B*. 1986;169(4):428−431.
7. SG Rajeev. Quantum electrodynamics as a supersymmetric theory of Loops. *Annals of Physics*. 1987;173:249−276.
8. AM Polyakov. *Gauge Field and Strings*. Switzerland: Harwood Academic; 1987. p. 301.
9. Luiz CL Botelho. A note on Feynman–Kac path integral representations for scalar wave motions. *Random Operators and Stochastic Equations*. 2013;21(3):271.
10. Luiz CL Botelho. Studies on Polyakov and Nambu–Goto random surface path integrals on QCD(SU(∞)) : Interquark potential and phenomenological scattering amplitudes. *International Journal of Modern Physics A*. 2013;32:1750030.
11. Luiz CL Botelho. QCD(SU(∞)) as a model of infinite-dimensional constant gauge field configurations. *International Journal of Modern Physics A*. 2017;(6−7):1750031.
12. Luiz CL Botelho. Covariant path integral for Nambu-Goto string theory. *Phys Rev D*. 1975;49:1975.
13. Luiz CL Botelho. Critical String Wave Equations and the QCD(U(N_c)) String. (Some Comments). *International Journal of Theoretical Physics*. 2009;48:2715.
14. Luiz CL Botelho. *Methods of Bosonic and Fermionic Path Integrals Representations - Continuous Random Geometry in Quantum Field Theory*. UK: Nova Science Publishers; 2009. p. 336.
15. Luiz CL Botelho. *Lecture Notes in Topics in Path Integrals and String Representations*. Singapore: World Scientific Publishing; 2017. p. 244.
16. Lars V Ahlfors. *Complex Analysis*. 3rd edn. New York: McGraw-Hill International Editions; 1979. p. 347.
17. SG Mikhlin. *Integral Equations*. UK: Pergamon Press; 1957.

[2]Firstly one must imposes the "Pauli-Fermi" conditions on the possible Q.C.D string surface $\Im^{\mu\nu}(X(\sigma,\tau))\Im_{\mu\nu}(X(\sigma',\tau)) = 0$ for $\sigma \neq \sigma'$. For trivial self intersection points $(\sigma,\tau) = (\sigma',\tau')$ one has formally the result $\delta^{(D)}(X_\mu(\xi) - X_\mu(\xi')) = \frac{\delta^{(2)}(\xi-\xi')\delta_\varepsilon^{(D-2)}(0)}{2^{D/2}\cdot h^{D/8}(X(\xi))}$. So one can expect that for D=4, the self avoiding term (responsible for the $Q.C.D(SU(\infty))$ string be a full interacting string theory) reduces to a U(11) Gross-Neveu 2D model on the string surface.