Review Article

On the dependence of the speed of light in vacuum on temperature

Abstract

It is shown that the interaction of the electromagnetic field with the vacuum of the electron-positron field gives rise to dependence of the speed of light propagation on the radiation temperature. Estimates show that in the modern epoch, even at very high temperatures, such for example which exist in the star interiors, the temperature-dependent correction to the speed of light proves to be extremely small. But in the cosmological model of the hot Universe, in the first instances after the Big Bang the temperature was so high that the speed of light exceeded its present value by many orders of magnitude. The effect of dependence of the speed of light on temperature must be important for understanding the early evolution of the Universe.

Keywords: speed of light, electromagnetic field, electron-positron vacuum, temperature, model of the hot Universe

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Introduction

Maxwell's classical equations in vacuum are linear and contain a fundamental constant of the velocity dimension which has the meaning of the speed of propagation of electromagnetic waves. Nevertheless within the framework of quantum electrodynamics the interaction of the electromagnetic field with the vacuum of the electron-positron field leads to interaction of photons with each other.[1] Consequently, the equations of electromagnetic field become nonlinear. Although this nonlinearity and the effects of scattering of light on light are as a rule negligibly small, they can lead to qualitatively new phenomena, in particular, as shown in this work, to dependence of the speed of light of the equilibrium radiation on temperature.

It should be noted that the dependence of the speed of propagation of light in a material medium on temperature is a natural effect, because the dielectric permittivity of a medium depends on thermodynamic variables and, in particular, on temperature. In the case under consideration in this work, at issue is the dependence of the speed of light on temperature in vacuum, by which the complex nature of the physical vacuum is manifested.

The performed estimates show that the considered effect is extremely small at temperatures which can be realized at modern conditions, but it should be quite important in the first instances of evolution of the Universe after the Big Bang when temperatures were anomalously high in comparison with temperatures of the modern epoch. In this early stage of evolution the speed of light had to exceed the present one by many orders of magnitude. We calculate the thermodynamic characteristics of the equilibrium radiation with taking into account the dependence of the speed of light on temperature.

Self-consistent description of the nonlinear electromagnetic field

In order to describe the equilibrium electromagnetic field we will use the self-consistent field model in the version which was developed for the nonrelativistic Fermi and Bose systems.[2–4] With regard to the relativistic field models, this approach was applied in. The influence of the nonlinear effects on sound propagation in a solid was investigated within the framework of this approach in works.[7,8] Usually the Hamiltonian of a considered many-particle system can be represented as a sum of the Hamiltonian of noninteracting particles and the operator of their interactions. The effectiveness of perturbation theory considerably depends on the successful choice of the main approximation. When using perturbation theory, most often one chooses the Hamiltonian of noninteracting particles as the main approximation and consideres the operator of their interaction as perturbation. Such a decomposition while studying many-particle systems proves, as a rule, to be unsuccessful, and to obtain physically correct results one needs to sum an infinite number of terms.[9] With decreasing temperature the contribution of the kinetic energy into the total energy of a system decreases and the interaction energy between particles is coming to the foreground, as it no longer can be considered as a small correction to the kinetic energy. Moreover, the neglect of interaction in the main approximation does not allow to study effectively the phase transitions. It is possible, however, to reformulate perturbation theory to take the interaction into account approximately already in the main approximation by the self-consistent field method.[2–6] Accounting for the phonon-phonon interaction in a solid in the continuum Debye model[7,8] by means of this method leads to renormalization of the speed of sound and arising of its dependence on temperature. In this work a similar approach is applied for analysis of influence of the photon-photon interaction on light propagation in vacuum.

The energy density of the electromagnetic field can be presented as a sum of two terms

$$w = w_0 + w_I, \tag{1}$$

where the first term which is quadratic in the electric and magnetic field intensities

$$w_0 = \frac{E^2 + H^2}{8\pi} \tag{2}$$

determines the energy of the noninteracting electromagnetic field, and the second term

$$w_I = 2D\Big[3E^2E^2 - H^2H^2 - \big(E^2H^2 + H^2E^2\big)\Big] + 2mm + 7D\Big[(EH)^2 + (HE)^2\Big] \tag{3}$$

describes the interaction between photons due to creation of virtual electron-positron pairs.[1] The constant in (3) can be calculated by the methods of quantum electrodynamics[1] and in Gaussian units $D \equiv \eta \dfrac{\hbar^3}{m^4c^5}$, where the dimensionless coefficient $\eta \equiv \dfrac{\alpha^2}{45(4\pi)^2} \approx 7.5\cdot 10^{-9}$

$\alpha = e^2/\hbar c \approx 1/137$ is the fine-structure constant, m is the electron mass. The given coefficients contain the constant c having the dimension of speed, which we will call the "bare" speed of light. For estimation of coefficients the value of this speed was taken equal to the observed speed of light, though, as will be shown, it somewhat differs from the observed speed of light at zero temperature. It is convenient to write the coefficient in formula (3) through the Compton wavelength of an electron $= \hbar/mc$ in the form $D = \eta \frac{\lambda\!\!\!^{-3}}{mc^2}$. It is interesting to estimate the value of the ratio of energies w_I/w_0. This quantity is equal to the ratio of the field energy contained in the volume $\lambda\!\!\!^{-3}$ to the rest energy of an electron. In addition, this ratio should be multiplied by the small dimensionless coefficient η. For the magnetic field intensity of the order of $H \sim 10^6$ Gs we have $w_I/w_0 \sim 10^{-20}$, so that the contribution of interaction into the total field energy is indeed extremely small.

Let us proceed to the description of the electromagnetic filed in terms of the Fourier components of the fields, using the expansion of the fields in plane waves

$$E(r,t) = \sum_k E_k(t)e^{ikr}, \quad H(r,t) = \sum_k H_k(t)e^{ikr}. \qquad (4)$$

Then the full Hamiltonian of the field in the volume V, in accordance with (1), is a sum of the free Hamiltonian and the interaction Hamiltonian

$$H = H_0 + H_I, \qquad (5)$$

where

$$H_0 = \frac{V}{8\pi}\sum_k \left(E_k^+ E_k + H_k^+ H_k\right), \qquad (6)$$

$$\begin{aligned} H_I = {} & 2VD \times 2mm \\ & \times \sum_{\{k_i\}} \{3\left(E_{k_1}^+ E_{k_2}\right)\left(E_{k_3}^+ E_{k_4}\right) - \left(H_{k_1}^+ H_{k_2}\right)\left(H_{k_3}^+ H_{k_4}\right) - 0mm \\ & - \left(E_{k_1}^+ E_{k_2}\right)\left(H_{k_3}^+ H_{k_4}\right) - \left(H_{k_4}^+ H_{k_3}\right)\left(E_{k_2}^+ E_{k_1}\right)\} \times 2mm \\ & \times \Delta(k_1 - k_2 + k_3 - k_4) + 3.5mm \\ & + 7VD\sum_{\{k_i\}} \{\left(E_{k_1}^+ H_{k_2}\right)\left(E_{k_3}^+ H_{k_4}\right) + \left(H_{k_4}^+ E_{k_3}\right)\left(H_{k_2}^+ E_{k_1}\right)\} \times 0mm \\ & \times \Delta(k_1 - k_2 + k_3 - k_4). \end{aligned} \qquad (7)$$

Here $\Delta(k) = 1$ if $k = 0$ and $\Delta(k) = 0$ if $k \neq 0$. In (6) and (7) we can pass to the operators of creation a_{kj}^+ and annihilation a_{kj} of photons, using representations of the operators of the Fourier components of the fields:

$$\begin{aligned} E_k &= -i\sqrt{\frac{2\pi\hbar\omega_k}{V}}\sum_j (a_{kj}^+ - a_{-kj})e_j(k), 2mm \\ H_k &= ic\sqrt{\frac{2\pi\hbar}{V\omega_k}}\sum_j (a_{kj}^+ + a_{-kj})[k \times e_j(k)], \end{aligned} \qquad (8)$$

where $\omega_k = ck$, and the polarization vectors $e_j(k)$ satisfy the conditions of orthonormality and completeness:

$$e_{j_1}^*(k)e_{j_2}(k) = \delta_{j_1 j_2}, \ \sum_j e_j^{\alpha *}(k)e_j^{\alpha'}(k) = \delta_{\alpha\alpha'} - \frac{k_\alpha k_{\alpha'}}{k^2}, \qquad (9)$$

as well as the conditions

$$k e_j(k) = 0, \qquad e_j^*(-k) = e_j(k). \qquad (10)$$

The free Hamiltonian of the field (6) is reduced to a sum of the Hamiltonians of harmonic oscillators

$$H_0 = \sum_{k,j}\hbar\omega_k\left(a_{kj}^+ a_{kj} + \frac{1}{2}\right). \qquad (11)$$

The electromagnetic field with account of the nonlinear effects is characterized by the full Hamiltonian (5). In order to account for interaction in a many-particle system, usually one chooses the Hamiltonian of noninteracting particles as the main approximation and considers the interaction Hamiltonian as perturbation (in our case, those are H_0 (11) and H_I (7)). Such choice, as remarked above, is not optimal, because the effects caused by interaction are totally disregarded in the main approximation. Although the interaction is small in the considered case, it can lead, as we will see, to qualitatively new effects. It is known from the self-consistent approach for description of many-particles systems that accounting for the interaction effects in the main approximation leads to a change in the dispersion law of the initial particles and, thereby, we pass from the representation of free particles to the language of collective excitations – quasiparticles.

It is natural to consider that also in the case studied here the interaction effects will lead to renormalization of the "bare" speed of light c entering into the Hamiltonian. Taking into account this consideration, let us decompose the full Hamiltonian (5) into the main part and the perturbation in a different way, that is

$$H = H_S + H_C, \qquad (12)$$

where the self-consistent (or approximating) Hamiltonian is chosen in the form similar to the free Hamiltonian (11), but with the speed of light $\tilde{c}$ being renormalized due to the photon-photon interaction:

$$H_S = \sum_{k,j}\hbar\tilde{\omega}_k a_{kj}^+ a_{kj} + E_0, \qquad (13)$$

where $\tilde{\omega}_k = \tilde{c}k$. The correlation Hamiltonian describing the interaction between the renormalized or "dressed" photons is chosen from the condition that the full Hamiltonian should be unchanged:

$$H_C = \sum_{k,j}\hbar(\omega_k - \tilde{\omega}_k)a_{kj}^+ a_{kj} + \sum_k \hbar\omega_k - E_0 + H_I. \qquad (14)$$

This Hamiltonian describes the interaction between photons propagating with the renormalized speed of light, which we will not consider. Formulas (13), (14) contain the non-operator term E_0, taking account of which proves to be important for correct formulation of the self-consistent field model. Let us choose it from the consideration that the approximating Hamiltonian (13) should be maximally close to the exact Hamiltonian. This means we have to require that the quantity $I \equiv |\langle H - H_S\rangle| = |\langle H_C\rangle|$ should be minimal, that is equal to zero. From here we obtain the conditions being natural for the self-consistent field theory:

$$\langle H\rangle = \langle H_S\rangle, \quad \langle H_C\rangle = 0. \qquad (15)$$

The averaging is performed by means of the statistical operator

$$\rho = \exp\beta(F - H_S), \qquad (16)$$

Where F is the free energy, $\beta = 1/T$ is the inverse temperature. The condition (15) allows to determine the non-operator part of the Hamiltonian (13):

$$E_0 = 2(c - \tilde{c})\sum_k \hbar k f_k + \sum_k \hbar ck + \langle H_I\rangle, \qquad (17)$$

where the distribution function of the renormalized photons has the Planck form

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$$f_k = \langle a_{kj}^{+} a_{kj} \rangle = \frac{1}{\exp(\beta\hbar\tilde{\omega}_k) - 1} \tag{18}$$

and does not depend on the polarization index. From the normalization condition for the statistical operator (16) $\mathrm{Sp}\rho = 1$ it follows the expression for the free energy of radiation

$$F = 2(c - \tilde{c})\sum_k \hbar k f_k + \sum_k \hbar c k + \langle H_I \rangle + 2mm + 2T\sum_k \ln\left(1 - e^{-\beta\hbar\tilde{\omega}_k}\right). \tag{19}$$

With neglect of the photon-photon interaction and zero fluctuations, from formula (19), of course, there follow the usual formulas of the thermodynamics of blackbody radiation.[9,10] It is natural to require that in the used approximation with the Hamiltonian (13) and the free energy (19), like in the case of a gas of noninteracting photons, the thermodynamic relations should hold. Since the introduced renormalized speed $\tilde{c}$ itself can, in principle, depend on thermodynamic variables, then in order for the thermodynamic relations to hold the following condition should be satisfied:

$$\frac{\partial F}{\partial \tilde{c}} = 0. \tag{20}$$

From this condition and formula (19) it follows the relation which determines the renormalized speed:

$$\tilde{c} - c = \frac{\dfrac{\partial \langle H_I \rangle}{\partial \tilde{c}}}{2\dfrac{\partial}{\partial \tilde{c}}\sum_k \hbar k f_k}. \tag{21}$$

Since formula (21) contains the temperature-dependent distribution function (18) then, naturally, also the speed of light $\tilde{c} = \tilde{c}(T)$ is a function of temperature. Thus, we have to calculate the average of the interaction Hamiltonian $\langle H_I \rangle$. Here, as in the theory of phonons in solids,[7,8] divergent integrals appear. While describing phonons within the continuum model it is natural to cut off such integrals at the wave number, which equals the inverse average distance between particles or, at integration over frequencies, at the Debye frequency. In the case of photons, we will cut off divergent integrals at some wave number k_m, the choice of which is discussed a little later. With this in mind, the calculation of the average of the interaction Hamiltonian (7) gives

$$\langle H_I \rangle = \frac{1312V}{15\pi^2} D\hbar^2 c^2 J\left(\frac{k_m^4}{4} + J\right), \tag{22}$$

where $J = 6\zeta(4)\left(\dfrac{T}{\hbar\tilde{c}}\right)^4$, $\zeta(4) = \pi^4/90 \approx 1.0823$ is the zeta function. Let $\sigma \equiv \tilde{c}/c$ be the ratio of the temperature-dependent speed of light to the "bare" speed of light. Considering that $\sum_k \hbar k f_k = \dfrac{V\hbar}{2\pi^2} J$, from (21) we get the equation for σ:

$$\sigma = 1 + \frac{328}{15} D\hbar c k_m^4 + \frac{328 \cdot 16}{5}\zeta(4) D\hbar c\left(\frac{T}{\hbar c}\right)^4 \frac{1}{\sigma^4}. \tag{23}$$

This implies that the ratio of the speed of light at zero temperature $\tilde{c}_0$ to the "bare" speed of light $\sigma_0 \equiv \tilde{c}_0 / c$ is determined by the formula:

$$\sigma_0 = 1 + \frac{328}{15} D\hbar c k_m^4. \tag{24}$$

It is the speed of light at zero temperature that is a directly measurable speed. As follows from (24), this speed does not coincide with the "bare" speed of light, which is caused by taking into account the interaction between photons. Because of the weakness of this interaction c and $\tilde{c}_0$ should differ very little and in the main approximation they could be considered equal, which would not affect further conclusions. Nevertheless, it is of certain interest to clarify in more detail the relation between c and $\tilde{c}_0$, which, as seen from (24), is essentially determined by the choice of the wave number k_m at which the cut-off of divergent integrals is carried out. We find this wave number from the condition $\hbar\tilde{c}_0 k_m = m\tilde{c}_0^2$, so that k_m is equal to the inverse Compton wavelength of an electron $k_m = m\tilde{c}_0/\hbar =_0^{-1}$. This condition implies that a real electron cannot be created from the energy of zero oscillations. A similar method of cutting off divergent integrals was employed, for example, by Bethe in the nonrelativistic calculation of the Lamb shift.[11] With such cut-off procedure, from (24) it follows

$$\sigma_0 = 1 + \chi\sigma_0^6, \tag{25}$$

where $\chi \equiv \dfrac{328}{15}\eta_0$, $\eta_0 \equiv \dfrac{\alpha_0^2}{45(4\pi)^2}$ and $\alpha_0 \equiv \dfrac{e^2}{\hbar\tilde{c}_0}$ is the fine-structure constant written through the observed speed of light. Formula (25) determines the ratio $\sigma_0 \equiv \tilde{c}_0/c$ through the observed fine-structure constant. With the help of it, the unobserved "bare" speed can be eliminated from Eq. (23). As a result, we come to the equation for the dimensionless quantity $\tilde{\sigma} \equiv \sigma/\sigma_0 = \tilde{c}/\tilde{c}_0$, which equals the ratio of the observed speeds of light at finite and at zero temperatures:

$$\tilde{\sigma}^5 - \tilde{\sigma}^4 = b\tau^4. \tag{26}$$

Here $b \equiv 3\chi\sigma_0^5 \approx 4.9 \cdot 10^{-7}$, $\tau \equiv T/T_0$ is the dimensionless temperature, and T_0 is characteristic temperature determined by the rest energy of an electron

$$m\tilde{c}_0^2 = 2[\zeta(4)]^{1/4} T_0, \tag{27}$$

so that $T_0 \approx 0.29 \cdot 10^{10}$ K. Thus, it follows from formula (26) that the speed of light rises with increasing temperature. As opposed to the "bare" photons with the dispersion law $\omega = ck$, the photons which speed is determined by the self-consistency Eq. (26) and depends on temperature have the dispersion law $\tilde{\omega} = \tilde{c}k$, and it is natural to call them "self-consistent" photons.

At $b\tau^4 \ll 1$ we have $\tilde{\sigma} \approx 1 + b\tau^4$. Since the coefficient b is very small, then the temperature dependence of the speed of light can manifest itself only at very high temperatures. For the observed relict radiation with the temperature $T = 2.73K$ we have $\tilde{\sigma} - 1 \approx 3.8 \cdot 10^{-43}$, so that the speed of light practically coincides with the speed of light at zero temperature. Inside stars, temperature can reach tens of millions degrees. For example, at the temperature inside the Sun that equals 15 million degrees, we have $\tilde{\sigma} - 1 \approx 3.4 \cdot 10^{-16}$. This means that the speed of light inside the Sun differs from the speed of light at zero temperature by the amount $\Delta\tilde{c} = \tilde{c} - \tilde{c}_0 \approx 10^{-5}$ cm/s. In order for the speed of light of the equilibrium radiation at a finite temperature to differ from the speed of light at zero temperature by one percent $\tilde{\sigma} = 1.01$, the temperature $T \approx 12T_0 \approx 3.5 \cdot 10^{10}$ K is required.

In the limit of very high temperatures $\tau \gg b^{-1/4} \approx 38$ we have

$$\tilde{\sigma} \approx b^{1/5}\tau^{4/5}. \tag{28}$$

Accounting for the dependence of the speed of light on temperature should be of principal importance in the very early stage of evolution of the Universe, when the dependence (28) could be valid. In the model of the hot Universe,[12] in the first instances after the Big Bang the temperature of the Universe was anomalously high in comparison with modern temperatures. As follows from the relations obtained above, also the speed of light was large in comparison with the present one. As the Universe was expanding and cooling the speed of light was decreasing and in the modern epoch it reached its value, practically equal to that of the speed of light at zero temperature. At the Planck temperature $T_p \approx 1.42\cdot 10^{32}\,\text{K} \approx 10^{19}\,\text{GeV}$ the speed of light $\tilde{c}_p$ had to exceed the present one by many orders of magnitude: $\tilde{c}_p/\tilde{c}_0 \approx 0.8\cdot 10^{17}$. The illustration of how the speed of light was varying as the Universe was cooling in the first instances after the Big Bang is given in Table 1.

Table 1 The value of the speed of light at different temperatures in the first instances after the Big Bang

t,s	***T, Gev***	**T,K**	$\hat{\tau} = T/T_0$	$\tilde{c}/\tilde{c}_0$
5.4×10^{-44}	1.2×10^{19}	1.42×10^{32}	4.9×10^{22}	0.8×10^{17}
10^{-39}	10^{16}	10^{29}	3.5×10^{19}	2.3×10^{14}
10^{-11}	100	10^{15}	3.5×10^{5}	1.5×10^{3}
10^{-5}	0.2	2×10^{12}	6.9×10^{2}	10
10^{-2}	10^{-2}	2×10^{11}	69	1.9
1.5	0.7×10^{-3}	0.8×10^{10}	2.8	1.00003

Thermodynamics of the equilibrium radiation of self-consistent photons

Let us give general formulas for the thermodynamic functions of a gas of self-consistent photons. The free energy (19), expressed through the observed speed of light, can be written in the form

$$\frac{F}{U_V} = 1 + 3\sigma_0\left(1 - \frac{4}{3}\tilde{\sigma}\right)\frac{\tau^4}{\tilde{\sigma}^4} + \frac{9}{2}\chi\sigma_0^6\frac{\tau^8}{\tilde{\sigma}^8}, \qquad (29)$$

where $U_V \equiv \frac{V}{8\pi^2}\frac{m\tilde{c}_0^2}{\hbar_0^3}\sigma_0^{-1}$ is the energy of zero oscillations. It is easy to verify that Eq. (26) follows from the condition $\frac{\partial}{\partial\tilde{\sigma}}\left(\frac{F}{U_V}\right) = 0$. This condition allows to calculate from the expression for the free energy (29) by the usual formulas the pressure $p = -\left(\frac{\partial F}{\partial V}\right)_T$ and the entropy $S = -\left(\frac{\partial F}{\partial T}\right)_V$, and due to fulfillment of this condition the temperature-dependent parameter $\tilde{\sigma}$ should not be differentiated. Considering (26), the formulas for the pressure and entropy can be written in the form

$$p = -\frac{U_V}{V}\left[1 + \frac{3\sigma_0}{2}\left(1 - \frac{5}{3}\tilde{\sigma}\right)\frac{\tau^4}{\tilde{\sigma}^4}\right], \qquad (30)$$

$$S = \frac{4\sigma_0 U_V}{T_0}\frac{\tau^3}{\tilde{\sigma}^3}. \qquad (31)$$

The total energy $E = F + TS$ is

$$E = U_V\left[1 + \frac{3}{2}\sigma_0(1+\tilde{\sigma})\frac{\tau^4}{\tilde{\sigma}^4}\right]. \qquad (32)$$

With neglect of the interaction between photons, when $\sigma_0 = \tilde{\sigma} = 1$ and without taking into account vacuum fluctuations formulas (29) – (32) turn into classical formulas of the theory of blackbody radiation.[10] In order to pass to this limit it is convenient to use the formula $\frac{U_V}{T_0^4} = V\frac{2\zeta(4)}{\pi^2\sigma_0\hbar^3\tilde{c}_0^3} = V\frac{\pi^2}{45\sigma_0\hbar^3\tilde{c}_0^3}$. But, even with neglect of the interaction between photons, accounting for vacuum fluctuations leads to appearance of the additional energy U_V in the total energy of blackbody radiation, and a negative contribution from vacuum fluctuations appears in the expression for the pressure (30). Hence, instead of the usual relation between energy and pressure $pV = E$,[10] with account of fluctuations we obtain $3pV = E - 4U_V$. At temperatures $T < T_0$ the total pressure proves to be negative and changes sign, becoming positive, at $T > T_0$. Accounting for the interaction between photons leads to a little shift of the temperature at which the pressure changes sign. This temperature T_1 can be found from Eqs. (26) and (30), that gives $T_1 \approx T_0(1+3\chi)$. Note that vacuum fluctuations do not give a contribution into the enthalpy

$$W = E + pV = 4U_V\sigma_0\frac{\tau^4}{\tilde{\sigma}^3}, \qquad (33)$$

as well as into the entropy (31).

For calculation of the heat capacity of a gas of photons $C_V = T(\partial S/\partial T)_V$, it is already necessary to account for the dependence of the speed of light on temperature, using the formula (26), so that we obtain

$$C_V = \frac{12\sigma_0 U_V}{T_0}\frac{\tilde{\sigma}}{(5\tilde{\sigma}-4)}\frac{\tau^3}{\tilde{\sigma}^3}. \qquad (34)$$

Let us also give the formula for the number of photons

$$N = \frac{90\zeta(3)}{\pi^4}\frac{U_V}{T_0}\sigma_0\frac{\tau^3}{\tilde{\sigma}^3}. \qquad (35)$$

In the low-temperature limit $T \ll T_0$ formulas (33)–(35), of course, turn into the known formulas of the theory of blackbody radiation.[10]

The photon distribution function and thermodynamics of the equilibrium radiation at high temperatures

Now we consider separately the most interesting region of high temperatures $\tau \gg b^{-1/4}$. The distribution functions for the number of photons and the energy with respect to wave numbers have the Planck form at all temperatures

$$n_k = \frac{k^2}{\pi^2(e^{Lk}-1)}, \qquad \varepsilon_k = \frac{\hbar\tilde{c}k^3}{\pi^2(e^{Lk}-1)}, \qquad (36)$$

where $L \equiv \hbar\tilde{c}/T$, so that the total densities of the number of photons and energy are respectively $n = \int_0^\infty n_k dk$ and $\varepsilon = \int_0^\infty \varepsilon_k dk$. However, the parameter L entering into (36) depends on temperature differently in the low-temperature and high-temperature limits. At low temperatures $L \equiv \hbar\tilde{c}_0/T = [2\pi/90^{1/4}](_0/\tau)$, and at high temperatures $L = B_0/\tau^{1/5}$, where $B = 2\pi b^{1/5}/90^{1/4} = 0.11$. In particular, in the case of low temperatures, as is known, maximums of the distributions (36) shift to higher energies proportional to temperature, respectively as $k_{\max 0} = 0.782\tau$ and $k_{\max 0} = 1.383\tau$ (Wien's displacement law).[10] In the limit of high temperatures $\tau \gg b^{-1/4}$ maximums of the distributions (36) also shift to higher energies with increasing temperature, but much slower, as $k_{\max 0} = 14.51\tau^{1/5}$ and $k_{\max 0} = 25.65\tau^{1/5}$ respectively.

When going over to the distribution functions with respect to frequencies $\tilde{\omega} = \tilde{c}k$ in (36) $Lk \equiv \hbar\tilde{\omega}/T$, and maximums of the

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distributions with respect to frequencies shift with temperature in the same way at all temperatures: as $\hbar\tilde{\omega}_{\max}/T = 1.594$ for the number of photons and $\hbar\tilde{\omega}_{\max}/T = 2.821$ for the energy.

In the high-temperature limit the temperature dependencies of thermodynamic functions of the equilibrium radiation are determined by the formulas:

$$p = \frac{5}{2}\frac{U_V\sigma_0}{V}\frac{\tau^{8/5}}{b^{3/5}}, \qquad E = \frac{3}{2}U_V\sigma_0\frac{\tau^{8/5}}{b^{3/5}},2mm$$
$$S = \frac{4U_V\sigma_0}{T_0}\left(\frac{\tau}{b}\right)^{3/5}, \qquad C_V = \frac{12U_V\sigma_0}{5T_0}\left(\frac{\tau}{b}\right)^{3/5},2mm \qquad (37)$$
$$W = 4U_V\sigma_0\frac{\tau^{8/5}}{b^{3/5}}, \qquad N = \frac{90\zeta(3)}{\pi^4}\frac{U_V\sigma_0}{T_0}\left(\frac{\tau}{b}\right)^{3/5}.$$

Here the pressure and the energy are connected by the relation $pV = \frac{5}{3}E$. As seen, the thermodynamic quantities increase with temperature much slower than at low temperatures.

Conclusion

The considered effect of dependence of the speed of light in vacuum on the radiation temperature is of fundamental importance for understanding the world around us and the early stage of evolution of the Universe. In the theories of special and general relativity the speed of light in vacuum is considered to be a cosmological constant. The equations of Einstein's theory of general relativity are usually written in such form that their left part is expressed through the space-time curvature tensor and has a purely geometric nature, and the right part contains the energy-momentum tensor of matter and fields of different nature. As is known, Einstein himself was dissatisfied with such separation of geometry and matter in the equations. Accounting for dependence of the speed of light on conditions, at which its propagation occurs, results in that now the metric tensor itself, through the speed of light contained in it, proves to be directly dependent on the state of matter, and thus the interdependence of matter and geometry becomes closer.

Einstein was rather interested in evidence for the possible dependence of the speed of light on the external conditions. As PL Kapitsa[13] recalled, when, working in the 30s of past century in the Cavendish Laboratory with Rutherford, he obtained magnetic fields 10 times stronger than those obtained before, a number of scientists advised him to make experiments on studying the influence of strong magnetic field on the speed of light. The one who insisted the most was Einstein. He said to Kapitsa: "I don't believe that God created such the Universe, that the speed of light depends on nothing in it". Yet Kapitsa refused the proposed experiment, on the ground that the experiment promised to be extremely difficult and the effect, if it had been discovered, for sure would have been at the edge of experimental accuracy and there would have been no credit to these results.

The above-stated calculations of dependence of the speed of light on temperature allow to definitely conclude that, as Einstein surmised, the magnetic field, similarly to temperature, will affect the speed of light propagation. We can estimate the order of magnitude of fields, at which the speed of light will change substantially, by equating the energies (2) and (3) $w_0 \sim w_I$. The estimation gives $H \sim 10^{16}$ Gs. So, PL Kapitsa was right when he refused to perform a labor-consuming experiment, because the fields necessary for observation of such effect should be so strong that they could hardly be realized in modern conditions.

So long as, according to above mentioned estimates, in the first instances of existence of the Universe the speed of light exceeded its present value by many orders of magnitude, this should substantially affect the existing scenarios of the evolution of the Universe at its early stage.

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None

Conflicts of interest

Authors declare there is no conflicts of interest.

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