Majority-vote model in one-dimensional on directed small-world networks

Abstract

We investigate the critical properties of the Majority-Vote model (MVM) one-dimensional (1D) on directed small-world networks. The MVM is studied by applying the Monte Carlo method. We calculate the critical points, as well as the critical exponent’s ratio, and . We find that MVM presents identical exponents to the Ising model one-dimensional on directed Small-World networks (DSW). Our results are in agreement with the Grinstein criterion for models with up and down symmetry on regular lattices.

Keywords: Majority-vote, networks, monte Carlo simulations

Introduction

Grinstein et al. argue that non-equilibrium spin systems on square lattices (SL) with up-down symmetry belongs to the same class of universality of the Lenz-Ising model in two dimensions (2D). This hypothesis was endorsed for some non-equilibrium models on other regular lattices.

In 1992 Oliveira proposed the a non-equilibrium model known as MVM which disobeyes the detailed balance. The update of the MVM follows a Markov sequence of stochastic dynamics with local rules and with up-down symmetry. In 2D, on a square lattice, the MVM presents a continuous phase transition with critical exponents identical of the Ising model. Sousa and Brenda studied the Ising model and MVM on DSW random lattices, respectively. The exponents obtained in both models are identical and in agreement with the conjecture suggested by Grinstein et al.

In this paper, we consider the MVM in 1D on DSW networks and perform an extensive computer simulation study of the MVM. To extract the critical exponents, we applied finite-size scaling (FSS) techniques. Monte Carlo simulations of this system were performed using a master equation to update the spins. Here, there is a continuous phase transition for 0<p<1, where p is the rewiring probability. Besides, the calculated critical exponents for p=0.1 do not belong to the same universality class as the 2D Ising model.

We consider the non-equilibrium MVM on DSW networks by a set of spins variables , assuming values values ±1 located on every node i of a DSW networks with L sites, where L is the length of a linear chain. The small-world network in one-dimension is built from a regular network with two closest neighbors, connected to L nodes and J neighbors. In this network, each node is randomly reconnected with n edges with probability . When p = 0 for the network it is regular (received no long-range connection), but for 0 <p <1 the network is small world (short-range links) and p = 1 random network (long-range connections), as shown in Figure 1.

In the MVM on a network, the system dynamics traditionally is as follows: We assign a spin variable with values ±1 at each node of the net. At each step, we try to spin- flip a node. The flip is accepted with probability

\[ w_i = \frac{1}{2} \left( 1 - (1 - 2q) \sigma_i S \left( \sum_j \sigma_j \right) \right) \quad (1) \]

where \( S(x) \) is the sign of \( x \) if \( x \neq 0 \), \( S(x) = 0 \) if \( x = 0 \). To calculate \( w_i \) our sum runs over the \((J=2)\) nearest neighbours of spin \( i \) on the network. In this model, we add a long-range connection connecting to another site \( k \) with \( p=0.1 \). This connection is only one way, that is, the site \( k \) does not send back a connection to the site \( i \). \( w_i \) means that with probability \((1-q)\) the spin will adopt the same state as its neighbours. The control noise parameter \( q \in [0,1] \) works like the temperature in the Ising model: the smaller the value of \( q \), the greater the likelihood of parallel alignment with the local majority. The simulations have been performed on different DSW networks sizes comprising a number \( L=5000, 10000, 20000, 40000, 60000, 80000, 120000, 160000, 200000, \) and \( 2600000 \) of sites. For each \( L \) size quenched averages over the connectivity disorder are approximated by averaging over independent realizations. For each simulation, we have started with a stable configuration of spins. We ran \( 3 \times 10^5 \) Monte Carlo steps (MCS) per spin with \( 2 \times 10^4 \) configurations discarded for thermalization using a random-number generator.

Results and discussions

The molar magnetization, \( m = \sum \sigma_i / L \), were measured. From magnetization, we can obtain other measures such as the average magnetization, susceptibility and the fourth-order Binder cumulant,

Figure 1 Random networks.

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\[ m = \langle m \rangle_v, \]
\[ \chi(q) = \frac{N}{T} \langle m^2 \rangle_{av} - \langle m \rangle_{av}^2 \]
\[ U_4(q) = 1 - \frac{\langle m^4 \rangle_{av}}{3! \langle m \rangle_{av}^4}, \]

in the above equations \(<...>\) stands for thermodynamic averages and \( ... \rangle_{av} \) for averages over different realizations.

In order to calculate the exponents of these models, we apply finite-size scaling (FSS) theory. We then expect, for large system sizes, an asymptotic FSS behavior of the form

\[ m = L^{\beta/\nu} f_m(x)[1+...], \]
\[ \chi = L^{\gamma/\nu} f_\chi(x)[1+...], \]

Where \( \beta \) and \( \gamma \) are the usual critical exponents, and \( f_m(x) \) and \( f_\chi(x) \) are FSS functions with

\[ x = (q - q_c)L^{1/\nu} \]

being the scaling variable. The dots in the brackets \([1+...]\) indicate corrections-to-scaling terms. We calculated the error bars from the fluctuations among the different realizations. Therefore, from the size dependence of \( m \) and \( \chi \) we obtain the exponents ratios \( \beta/\nu \) and \( \gamma/\nu \) respectively. The susceptibility at its maximum also scales as \( L^{\nu} \). Moreover, the value of \( T^a = T_c(L) \) for which \( \chi \) has a maximum scales with the lattice size as

\[ T_c(L) = T_c + bL^{-1/\nu} \]

In this way, Eq. 7 may be used to get \( 1/\nu \).

In the Figure 2, we plot the magnetization, Binder Cumulant, and susceptibility versus the noise parameter \( q \) for sizes \( L=5M, 10M, 20M, 30M, 40M, 60M, 80M, 120M, 160M, 200M, \) and \( 260M \) with rewiring probability \( p=0.1 \). The shape of these figures indicates that this model exhibits a continuous phase transition.

In the Figure 3, we plot logarithm of the magnetization at \( q_c \) versus \( \ln L \) for \( p=0.01 \) and of the eq. (5), we obtain the exponents ratio \( \beta/\nu = 0.218(11) \).

\[ \text{Figure 3 Plot of the logarithm of the magnetization at } q_c \text{ as a function of the logarithm of } L, \text{ for } p=0.01 \text{ and of the eq. (5), we obtain the exponents ratio } \beta/\nu = 0.218(11). \]

In the Figure 4, we plot logarithm of the susceptibility \( \chi \) at \( q_c \) and \( \chi_{\text{max}} \) versus \( \ln L \). Of the eq. (6), we obtain the exponents ratio \( \gamma/\nu_{q_c} = 0.535(7) \), and \( \gamma/\nu_{\chi_{\text{max}}} = 0.533(8) \) for \( p = 0.1 \).

\[ \text{Figure 4 Plot of the logarithm of the susceptiblity } \chi \text{ at } q_c \text{ and } \chi_{\text{max}} \text{ versus } \ln L \text{. Of the eq. (6), we obtain the exponents ratio } \gamma/\nu_{q_c} = 0.535(7) \text{ and } \gamma/\nu_{\chi_{\text{max}}} = 0.533(8) \text{ for } p = 0.1. \]

In the Figure 5, we plot the log-log of \( [q_c(L) - q_e] \) versus \( \ln L \) and the eq. (7), we obtain the exponents ratio \( 1/\nu = 0.51(3) \).

\[ \text{Figure 5 Plot of the log-log of } [q_c(L) - q_e] \text{ versus } \ln L \text{ and the eq. (7), we obtain the exponents ratio } 1/\nu = 0.51(3). \]
Conclusion

In the present work, we have shown that, by considering the ferromagnetic MVM in one-dimension on DSW networks there is a continuous phase transition. The exponents ratio $\beta / \nu = 0.218(11)$, $\gamma / \nu = 0.535(7)$ and $1 / \nu = 0.51(3)$ for $p=0.1$ indicate that they are identical from Ising model in one-dimension on DSW networks. Therefore, our results agree with the Grinstein criterion for DSW networks.

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Conflict of interest

The author states that there is no conflict of interest.

References