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# Majority-vote model in one-dimensional on directed small-world networks


## Abstract

We investigate the critical properties of the Majority-Vote model (MVM) one-dimensional (1D) on directed small-world networks. The MVM is studied by applying the Monte Carlo method. We calculate the critical points, as well as the critical exponent's ratio , and . We find that MVM presents identical exponents to the Ising model one-dimensional on directed Small-World networks (DSW). Our results are in agreement with the Grinstein criterion for models with up and down symmetry on regular lattices


**Keywords:** Majority-vote, networks, monte Carlo simulations

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## Introduction

Grinstein et al.[1] argue that non-equilibrium spin systems on square lattices (SL) with up-down symmetry belongs to the same class of universality of the Lenz-Ising model in two dimensions (2D). This hypothesis was endorsed for some non-equilibrium models on other regular lattices.[2–8]

In 1992 Oliveira[9] proposed the a non-equilibrium model known as MVM which disobeys the detailed balance. The update of the MVM follows a Markov sequence of stochastic dynamics with local rules and with up-down symmetry. In 2D, on a square lattice, the MVM presents a continuous phase transition with critical exponents identical[9] of the Ising model[10]. Sousa[11] and Brenda[12] studied the Ising model and MVM on DSW random lattices, respectively. The exponents obtained in both models are identical and in agreement with the conjecture suggested by Grinstein et al.[1]

In this paper, we consider the MVM in 1D on DSW networks and perform an extensive computer simulation study of the MVM. To extract the critical exponents, we applied finite-size scaling (FSS) techniques. Monte Carlo simulations of this system were performed using a master equation to update the spins. Here, there is a continuous phase transition for $0<p\leqslant 1$, where $p$ is the rewiring probability. Besides, the calculated critical exponents for $p=0.1$ do not belong to the same universality class as the 2D Ising model.

We consider the non-equilibrium MVM on DSW networks by a set of spins variables $\sigma_i$ assuming values values $\pm 1$ located on every node $i$ of a DSW networks with $L$ sites, where $L$ is the length of a linear chain. The small-world network in one-dimension is built from a regular network with two closest neighbors, connected to $L$ nodes and $J$ neighbors. In this network, each node is randomly reconnected with $n$ edges with probability $p$. When $p = 0$ for the network it is regular (received no long-range connection), but for $0 < p < 1$ the network is small world (short-range links) and $p = 1$ random network (long-range connections), as shown in Figure 1.

In the MVM on a network, the system dynamics traditionally is as follows: We assign a spin variable $\sigma_i$ with values $\pm 1$ at each node of the net. At each step, we try to spin- flip a node. The flip is accepted with probability

$$w_i = \frac{1}{2}\left(1-\left(1-2q\right)\sigma_i . S\left(\sum_J \sigma_J\right)\right) \qquad (1)$$

where S(x) is the sign of $x$ if $x \neq 0$, S(x)=0 if x=0. To calculate $w_i$ our sum runs over the ($J$=2) nearest neighbours of spin $i$ on the network. In this model, we add a long-range connection connecting to another site $k$ with $p=0.1$. This connection is only one way, that is, the site $k$ does not send back a connection to the site $i$. $w_i$ means that with probability ($1$-$q$) the spin will adopt the same state as the majority of its neighbours. The control noise parameter $q\left(0\leqslant q\leqslant 1\right)$ works like the temperature in the Ising model: the smaller the value of q, the greater the likelihood of parallel alignment with the local majority. The simulations have been performed on different DSW networks sizes comprising a number $L$=5000, 10000, 20000, 40000, 60000, 80000, 120000, 160000, 200000, and 2600000 of sites. For each L size quenched averages over the connectivity disorder are approximated by averaging over independent realizations. For each simulation, we have started with a stable configuration of spins. We ran $3\times 10^5$ Monte Carlo steps (MCS) per spin with $2\times 10^5$ configurations discarded for thermalization using a` random-number generator.

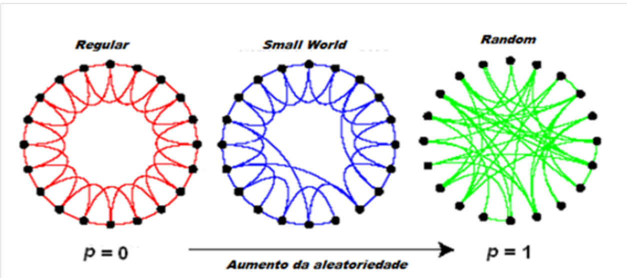


**Figure 1** Random networks.

## Results and discussions

The molar magnetization, $m=\sum_i \sigma_i / L$, were measured. From magnetization, we can obtain other measures such as the average magnetization, susceptibility and the fourth-order Binder cumulant,

$$m=\left[\langle m\rangle\right]_{av}, \quad (2)$$

$$\chi(q)=\frac{N}{T}\left[\left\langle m^2\right\rangle_{av}-\langle m\rangle_{av}^2\right]_{av} \quad (3)$$

$$U_4(q)=1-\frac{\left\langle m^4\right\rangle_{av}}{3\left[\langle m\rangle\right]_{av}^2}, \quad (4)$$

in the above equations <...> stands for thermodynamic averages and $...[...]_{av}$ for averages over different realizations.

In order to calculate the exponents of these models, we apply finite-size scaling (FSS) theory. We then expect, for large system sizes, an asymptotic FSS behavior of the form

$$m=L^{-\beta/\nu}f_m(x)[1+...], \quad (5)$$

$$\chi=L^{\gamma/\nu}f_\chi(x)[1+...], \quad (6)$$

Where $\beta$ and $\gamma$ are the usual critical exponents, and $f_i(x)$ are FSS functions with

$$x=(q-q_c)L^{1/\nu} \quad (7)$$

being the scaling variable. The dots in the brackets [1+...] indicate corrections-to-scaling terms. We calculated the error bars from the fluctuations among the different realizations. Therefore, from the size dependence of $m$ and $\chi$ we obtain the exponents ratios $\beta/\nu$ and $\gamma/\nu$ respectively. The susceptibility at its maximum also scales as $L^{\gamma/\nu}$. Moreover, the value of $T^0=T_c(L)$ for which $\chi$ has a maximum scales with the lattice size as

$$T_c(L)=T_c+bL^{-1/\nu}$$

In this way, Eq. 7 may be used to get $1/\nu$.[7]

In the Figure 2, we plot the magnetization , Binder Cumulante, and susceptibility versus the noise parameter $q$ for sizes L=5M, 10M, 20M, 30M, 40M, 60M, 80M, 120M, 160M, 200M, and 260M and rewiring probability $p=0.1$. The shape of these figures indicates that this model exhibits a continuous phase transition.

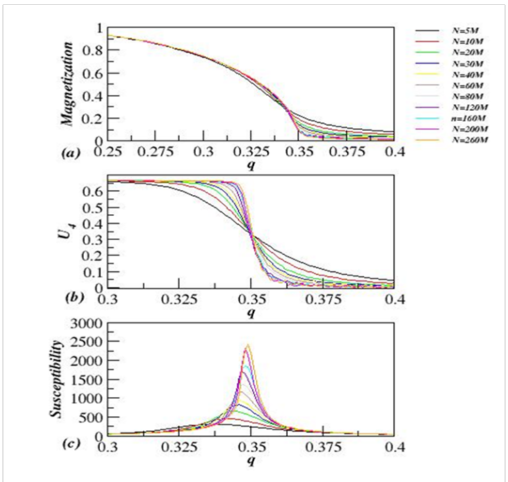

**Figure 2** Plot of the magnetization (A) Binder Cumulant ($U_4$) (B) and susceptibility $\chi$ (C) as a function of the noise parameter $q$ and for sizes L=5M, 10M, 20M, 30M, 40M, 60M, 80M, 120M, 160M, 200M, and 260M with rewiring probability p=0.1. Here 1M=1000.

In the Figure 3, we plot logarithm of the magnetization at $q_c$ versus $\ln L$ for p=0.01 and of the eq. (5), we obtain the exponents ratio $\beta/\nu=0.218(11)$.

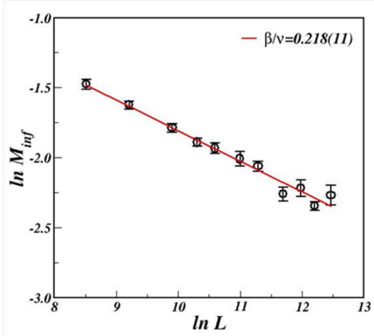

**Figure 3** Plot of the logarithm of the magnetization at $q_c$ as a function of the logarithm of *L*. noise parameter *p=0.1*.

In the Figure 4, we plot logarithm of the susceptibility $\chi$ at $q_c$ and $\chi_{max}$ versus $\ln L$. Of the eq. (6), we obtain the exponents ratio $\gamma/\nu_{q_c}=0.535(7)$, and $\gamma/\nu_{q_{max}}=0.533(8)$ for $p=0.1$.

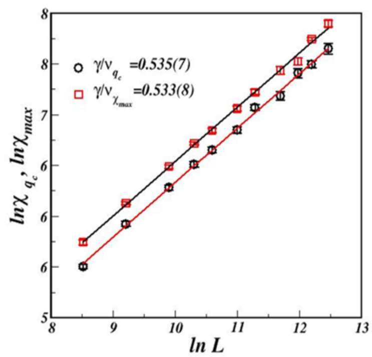

**Figure 4** Plot of the logarithm of the susceptibility $\chi_{qc}$ and $\chi_{max}$ versus *ln L* and noise parameter *p=0.1*.

In the Figure 5, we plot the log-log of $\left[q_c(L)-q_c\right]$ versus $L$ and the eq. (7), we obtain the exponents ratio $1/\nu=0.51(3)$.

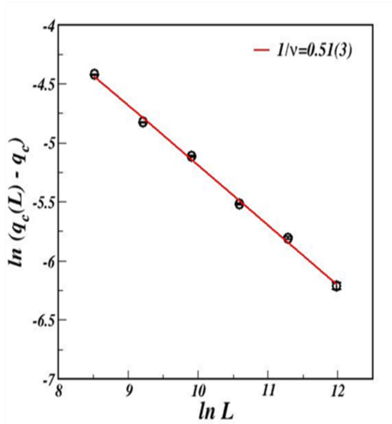

**Figure 5** Plot of the $ln\left[q_c(L)-q_c\right]$ versus ***ln L*** and noise parameter *p=0.1*.

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## Conclusion

In the present work, we have shown that, by considering the ferromagnetic MVM in one-dimension on DSW networks there is a continuous phase transition. The exponents ratio $\beta / \nu = 0.218(11)$, $\gamma / \nu_{qc} = 0.535(7)$ and $1/\nu = 0.51(3)$ for p=0.1 indicate that they are identical from Ising model in one-dimension on DSW networks.[13] Therefore, our results agree with the Grinstein criterion for DSW networks.


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## Conflict of interest

The author states that there is no conflict of interest.

## References


1. G Grinstein, C Jayaprakash, Yu He. Statistical Mechanics of Probabilistic Cellular Automata. *Phys Rev Lett*. 1985;55,:2527.
2. CH Bennett. Role of Irreversibility in Stabilizing Complex and Nonergodic Behavior in Locally Interacting Discrete Systems. *Phys Rev Lett*. 1985;55:657.
3. JS Wang, JL Lebowitz J. Phase transitions and universality in nonequilibrium steady states of stochastic Ising models. *Stat Phys*. 1988;51(5−6):893−906.
4. MC Marques. Nonequilibrium Ising model with competing dynamics: A MFRG approach. *Phys Lett A*. 1990;145(6−7):379−382.
5. HWJ Blote, JR Heringa, A Hoogland, et al. Stability of Ising systems against non-hamiltonian, symmetry breaking dynamics. *Int J Mod Phys B*. 1991;5(4):685−695.
6. JM Gonzalez-Miranda, PL Garrido, J Marro, et al. Nonequilibrium phase diagram of Ising model with competing dynamics. *Phys Rev Lett*. 1987;59:1934.
7. FWS Lima. Majority-vote model on (3, 4, 6, 4) And ($3^4$, 6) Archimedean Lattices. *Int J Mod Phys C*. 2006;17(9):1273−1283.
8. JC Santos, FWS Lima, K Malarz. Majority-vote model on triangular, honeycomb and Kagomé lattices. *Physica A*. 2011;390(2):359−364.
9. MJ de Oliveira. Isotropic majority-vote model on a square lattice. *Journal of Statistical Physics*. 1992;66(1-2):273−281.
10. RJ Baxter. *Exactly solved models in statistical mechanics*. London: Academic Press; 1982. p. 502.
11. EM Sousa, FWS Lima. Ising model on directed small-world Voronoi Delaunay random lattices. *European Physical Journal Plus*. 2013;128:150.
12. Brenda RSC, FWS Lima. Majority-vote model on directed Small-World-Voronoi–Delaunay random lattices. *Int J Mod Phys C*. 2018;29(7):291850061.
13. FWS Lima. Ising Model in One-Dimensionalon Directed Small- World Networks. *RMES*. 2017;3(1):553.