Theorems

Jiang prime $k$-tuple theorem with true singular series.

We define the prime $k$-tuple equation

$$p, p+n_1,$$

where $2 | n_i = 1, \cdots k - 1$.

we have Jiang function

$$J_2(a) = \prod (P-1-\chi(P)),$$

where $\chi = \prod P, \chi(P)$

$$\prod_{i=1}^{k-1}(q+n_i) \equiv 0 \pmod P, q = 1, \cdots, p-1$$

which is true.

If $\chi(P) < P - 1$ then $J_2(a) \neq 0$. There exist infinitely many primes $P$ such that each of $P+n_i$ is prime. If $\chi(P) = P - 1$ then $J_2(a) = 0$. There exist finitely many primes $P$ such that each of $P+n_i$ is prime. $J_2(a)$ is a subset of Euler function $\mathcal{O}(co)$.

If $J_2(a) = 0$, then we have the best asymptotic formula of the number of prime $P$.

$$\pi_k(N, 2) = \left\lfloor \frac{N}{2} \right\rfloor C(k) \frac{N}{\log N} + \frac{N}{2} \sum_{l=1}^{k} \left\lfloor \frac{N}{l} \right\rfloor \chi(l) $$

is Jiang true singular series.

Example 1

Let $k = 2, P, P+2$, twin primes theorem.

From (3) we have

$\chi(2) = 0, \chi(P) = 1$ if $P > 2$

Substituting (6) into (2) we have

$$J_2(a) = \prod_{P+2}(P - 2) \neq 0$$

There exist infinitely many primes $P$ such that $P + 2$ is prime. Substituting (7) into (4) we have the best asymptotic formula

$$\pi_k(N, 2) = \left\lfloor \frac{N}{2} \right\rfloor C(k) \frac{N}{\log N} + \frac{N}{2} \sum_{l=1}^{k} \left\lfloor \frac{N}{l} \right\rfloor \chi(l) $$

Example 2

Let $k = 3, P, P+2, P+4$.

From (3) we have

$\chi(2) = 0, \chi(3) = 2$

From (2) we have

$$J_2(a) = 0$$

It has only a solution $P = 3, P+2 = 5, P+4 = 7$. One of $P, P+2, P+4$ is always divisible by 3. Example 2 is not admissible.

Example 3

Let $k = 4, P, P+n$, where $n = 2, 6, 8$.

From (3) we have

$\chi(2) = 0, \chi(3) = 1, \chi(P) = 3$ if $P > 3$

Substituting (11) into (2) we have

$$J_2(a) = \prod_{P+5}(P - 4) \neq 0$$

There exist infinitely many primes $P$ such that each of $P+n$ is prime. Example 3 is admissible.

Substituting (12) into (4) we have the best asymptotic formula

$$\pi_k(N, 2) = \left\lfloor \frac{N}{2} \right\rfloor C(k) \frac{N}{\log N} + \frac{N}{2} \sum_{l=1}^{k} \left\lfloor \frac{N}{l} \right\rfloor \chi(l) $$

Example 4

Let $k = 5, P, P+n$, where $n = 2, 6, 8, 12$.

From (3) we have

$\chi(2) = 0, \chi(3) = 1, \chi(5) = 3, \chi(P) = 4$ if $P > 5$

Substituting (14) into (2) we have
There exist infinitely many primes \( P \) such that each of \( P + n \) is prime. Example 4 is admissible. Substituting (15) into (4) we have the best asymptotic formula

\[
\pi_\scriptsize{n}(N, 2) = |\{P \leq N : P + n = \text{prime}\}| - \frac{15^4}{2^7 \prod_{p \geq 5} (P - 5)^4 N \log^7 N} \tag{16}
\]

Example 5

Let \( k = 6, P, P + n \), where \( n = 2, 6, 8, 12, 14 \).

From (3) and (2) we have

\[
\chi(2) = 0, \chi(3) = 1, \chi(5) = 4, J_2(5) = 0 \tag{17}
\]

It has only a solution \( a \) such that \( P = 5, P + 2 = 7, P + 6 = 11, P + 8 = 13, P + 12 = 17, P + 14 = 19 \). One of \( P + n \) is always divisible by 5. Example 5 is not admissible.

The Hardy-Littlewood prime \( k \)-tuple conjecture with wrong singular series.

This conjecture is generally believed to be true, but has not been proved. 18

We define the prime \( k \)-tuple equation

\[
P, P + n, \ldots, P + (k-1) \text{ where } 2 | n, \ldots, k - 1. \tag{18}
\]

In 1923 Hardy et al. 1 conjectured the asymptotic formula

\[
\pi_k(N, 2) = |\{P \leq N : P + n = \text{prime}\}| - \frac{15^4}{2^7 \prod_{p \geq 5} (P - 5)^4 N \log^7 N} \tag{19}
\]

where

\[
H(k) = \prod_{P \geq 5} \left[ 1 - \frac{V(P)}{P} \right] \left[ 1 - \frac{1}{P} \right] \tag{20}
\]

is Hardy-Littlewood wrong singula series,

\[
\nu(P) \text{ is the number of solutions of congruence } \prod_{p | n} (q + n) \equiv 0 \text{ (mod } P) , \quad q = 1, \ldots, P. \tag{21}
\]

which is wrong.

From (21) we have \( \nu(P) < P \) and \( H(k) \neq 0 \). For any prime \( k \)-tuple equation there exist infinitely many primes \( P \) such that each of \( P + n \) is prime, which is false.

Conjecture 1

Let \( k = 2, P, P + 2 \), twin primes theorem

From (21) we have

\[
\nu(P) = 1 \tag{22}
\]

Substituting (22) into (20) we have

\[
H(2) = \prod_{p | P + 2} \frac{P}{P - 1} \tag{23}
\]

Substituting (23) into (19) we have the asymptotic formula

\[
\pi_2(N, 2) = |\{P \leq N : P + 2 = \text{prime}\}| - \prod_{p \geq 5} \frac{P N}{P - 4 \log^2 N} \tag{24}
\]

which is wrong see example 1. They do not get twin primes formula (8):

Conjecture 2

Let \( k = 3, P, P + 2, P + 4 \).

From (21) we have

\[
\nu(2) = 1, \nu(P) = 2 \text{ if } P > 2 \tag{25}
\]

Substituting (25) into (20) we have

\[
H(3) = 4 \prod_{p | P + 2} \frac{P^3 (P - 2)}{(P - 1)^3} \tag{26}
\]

Substituting (26) into (19) we have asymptotic formula

\[
\pi_3(N, 2) = |\{P \leq N : P + 2 + 4 = \text{prime}\}| - \frac{15^4}{2^7 \prod_{p \geq 5} (P - 2)^4 N \log^7 N} \tag{27}
\]

which is wrong see example 2.

Conjecture 3

Let \( k = 4, P, P + 2, P + 4 \), where \( n = 2, 6, 8, 12 \).

From (21) we have

\[
\nu(2) = 1, \nu(3) = 2, \nu(P) = 3 \text{ if } P > 3 \tag{28}
\]

Substituting (28) into (20) we have

\[
H(4) = \frac{27}{2} \prod_{P \geq 7} \frac{P^3 (P - 3)}{(P - 1)^3} \tag{29}
\]

Substituting (29) into (19) we have asymptotic formula

\[
\pi_4(N, 2) = |\{P \leq N : P + 2 + 4 = \text{prime}\}| - \frac{27}{2} \prod_{P \geq 7} \frac{P^3 (P - 3)^4 N \log^4 N} \tag{30}
\]

Which is wrong see example 3.

Conjecture 4

Let \( k = 5, P, P + 2, P + 4 \), where \( n = 2, 6, 8, 12 \).

From (21) we have

\[
\nu(2) = 1, \nu(3) = 2, \nu(5) = 3, \nu(P) = 4 \text{ if } P > 5 \tag{31}
\]

Substituting (31) into (20) we have

\[
H(5) = \frac{15^4}{4^2} \prod_{P \geq 5} \frac{P^3 (P - 4)}{(P - 1)^3} \tag{32}
\]

Substituting (32) into (19) we have asymptotic formula

\[
\pi_5(N, 2) = |\{P \leq N : P + 2 + 4 = \text{prime}\}| - \frac{15^4}{4^2} \prod_{P \geq 5} \frac{P^3 (P - 4)^4 N \log^4 N} \tag{33}
\]

Which is wrong see example 4.

Conjecture 5

Let \( k = 6, P, P + 2, P + 4 \), where \( n = 2, 6, 8, 12, 14 \).

From (21) we have

\[
\nu(2) = 1, \nu(3) = 2, \nu(5) = 4, \nu(P) = 5 \text{ if } P > 5 \tag{34}
\]

Substituting (34) into (20) we have

\[
H(6) = \frac{15^5}{2^5} \prod_{P \geq 5} \frac{(P - 5)^2}{(P - 1)^6} \tag{35}
\]

Substituting (35) into (19) we have asymptotic formula

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\[ \pi_{\#}(N,2) = \left\{ P \leq N : P + n = \text{prime} \right\} - \frac{155}{213} \prod_{P} \frac{(P - 5)P^N}{(P - 1)^5\log^2 N} \]

which is wrong see example 5.

**Conclusion**

The Jiang prime \( k \)-tuple theorem has true singular series. The Hardy-Littlewood prime \( k \)-tuple conjecture has wrong singular series. The tool of additive prime number theory is basically the Hardy-Littlewood wrong \( k \)-tuple conjecture which are wrong. Using Jiang true singular series we prove almost all prime theorems. Jiang prime \( k \)-tuple theorem will replace Hardy-Littlewood prime \( k \)-tuple Conjecture. There cannot be really modern prime theory without Jiang function.

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**Conflict of interest**

Author declares that there is no conflict of interest.

**References**


