Review Article

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Secular influence of time variation of the gravitational constant on the periods of pulsars

Abstract

The theoretical formulae for the influence of the change of the theoretical formulae for the influence of the change of moment of inertia due to a time variation of the gravitational constant on the period of a pulsar are given by the method for solving the first order linear differential equations. The analytical and numerical solutions of the period of a pulsar slow down due to time variation of the gravitational constant are derived and calculated for five pulsars (PSR1749–28, PSR2045–16. JP1933+16, HP1508+55 and CP0834+06). Numerical results are given in Table 1 the results are discussed and conclusions are given

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Introduction

Some authors studied the variation of pulse period arising from the change of moment of inertia and they always use the method of the angular momentum conservation ($L = I\Omega = const$) or energy conservation ($E_{rot} = \frac{1}{2}I\Omega^2 = const$). However, the angular momentum is not conservative due to energy loss arising from the radiating power. Hence the change of the period of pulsar cannot be researched by using the angular momentum and the rotating energy conservation. In the formula of the magnetic dipole model of pulsars the moment of inertia is not variable (constant). But when we consider the energy–loss or time variation of the gravitational constant the moment of inertia may be changed. Hence it is necessary to give the formula for the magnetic dipole model which suits the change of moment of inertia. This is an important work in this paper.

It is well known that the gravitational constant is variable with time since the large number hypothesis suggested by Dirac. However, the variation of gravitational constant influences the change of moment of inertia according to the formula $\dot{I}/I = -\varepsilon\dot{G}/G$. Therefore the variation of G with time also influences the change of the rotational angular velocity or period of pulsar through the change of moment of inertia. Heintzman and Hillebrant[1] studied the relation between pulsar slow down and the temporal change of the gravitational constant. They estimated the variable value of gravitational constant $\dot{G}/G$ per year from the above relation. However, they did not research the influence of time variation of G on the change of period of a pulsar. Li[2] studied the retardation of rotation of the Earth due to the variation of the gravitational constant, but he did not research spin down of pulsar due to the gravitational constant. In the present paper the author researched pulsar slow down due to time variation of the gravitational constant through the change of moment of inertia

The equation determining the influence of change of moment of inertia on the period of a pulsar

The pulsar radiating power W is transformed from the rotational energy at a rate $\frac{dE}{dt}$, i e

$$-W = \frac{dE}{dt} \text{ or } W + \frac{dE}{dt} = 0 \tag{1}$$

According to the theory of magnetic model

$$\frac{dE}{dt} = -\frac{2}{3c^3}(M\sin\alpha)^2\Omega^4 = -\frac{32}{3c^3}\frac{\pi^4\mu^2}{P^4}, = -W. \quad \Omega = \frac{2\pi}{P}. \tag{2}$$

Here P is the period of pulsar, and μ is the projection of the magnetic dipole moment on the direction perpendicular to the rotational axis. We assume that when we consider time variation of the gravitational constant, $\mu = \mu_0$ (const), which does not influence the magnetic dipole moment.

The energy carried away by radiation from the rotational energy of pulsar can be written

$$E = \frac{1}{2}I\Omega^2 = 2\pi^2 I / P^2. \tag{3}$$

Here I denotes moment of inertia. If we consider the variation of moment of inertia with time, then

$$\dot{E} = 2\pi^2\left[\frac{1}{P^2}\frac{dI}{dt} - \frac{2I}{P^3}\frac{dP}{dt}\right]. \tag{4}$$

Substituting the formula (2) and (4) into the formula (1), we obtain the Bernoulli equation for n=1

$$\frac{dP}{dt} - \frac{1}{2}\left(\frac{1}{I}\frac{dI}{dt}\right)P = \frac{8\pi^2\mu_0{}^2}{3c^3 I(t)}(P^{-1}) \tag{5}$$

When both sides of the equation (5) are multiplied by 2P,

$$2P\frac{dP}{dt} - \frac{\dot{I}}{I}P^2 = \frac{16\pi^2\mu^2{}_0}{3c^3 I(t)}$$

Or

$$\frac{dP^2}{dt} - \frac{\dot{I}}{I}P^2 = \frac{16\pi^2\mu_0{}^2}{3c^3 I(t)}.$$

We can transform Bernoulli equation into the first order linear differential equation. i. e. the equation (5) may be written as the form of the first order linear differential equation

$$\frac{dP^2}{dt}+NP^2=Q(t), \tag{6}$$

Comparing the equation (6) with the above equation, we define that

$$N=-\dot{I}/I\ ,\ Q(t)=\frac{16\pi^2\mu_0{}^2}{3c^3I(t)} \tag{7}$$

The solution of the equation determining influence of time variation of g on the period of pulsars

Some authors give the relation between the variation of moment of inertia I and time variation of the gravitational constant G as follows (Blake 1978, Will 1981)[3,4]

$$\dot{I}/I=-\kappa\dot{G}/G. \tag{8}$$

Blake gives that the coefficient κ lies the range 0.1 to 0.2, and Will[4] gives $\varepsilon=0.17$. However, the formula (8) is derived from the equation of hydrostatic equilibrium, which is suitable to an Earth model and does not suite to the neutron stellar model.

Heintzmann & Hillebrant[1] studied pulsar slow down and the temporal change of G. They gave the formulas for the connection of the change of moment of inertia with time variation of the gravitational constant for the white dwarf star and neutron star. For the neutron stars

$$\frac{d\ln G}{d\ln I}=\frac{4-3\gamma-A\sigma}{2} \tag{9}$$

According to (7) this can be written as

$$\frac{\dot{I}}{I}=(\frac{2}{4-3\gamma-A\sigma})\frac{\dot{G}}{G}=-N \tag{10}$$

Here $\sigma=\frac{2GM}{c^2R}$, M and R denote mass and radius of the neutron star. The parameter A is determined by $\gamma=n+1/n$. n is the index of polytropic model. For $\gamma=5/3,\ 2\ ,3$, A=10, 4, 1. $\frac{\dot{G}}{G}=-10^{-13}/yr\sim-10^{-12}/yr$

$$N=-\frac{\dot{I}}{I}=-(\frac{2}{4-3\gamma-A\sigma})\frac{\dot{G}}{G}=const. \tag{11}$$

According to the first linear differential equation (6), N is a function of time t or it is a constant value. In this paper N is a constant value as shown in the expressions (11) and (20). Integrating (7), yields $I=I_0e^{-Nt}$, (12) Substitution of (12) into the second expression of (7), we get

$$Q(t)=\frac{16\pi^2\mu_0{}^2}{3c^2I_0}e^{Nt} \tag{13}$$

Integrating the equation (6), we get

$P(t)^2=e^{-\int Ndt}[\int Q(t)e^{\int Ndt}dt+C]$. C is an integrating constant, Substituting (13) into the above integral expression, we obtain

$$P(t)^2=e^{-Nt}[C+(\frac{16\pi^2\mu_0{}^2}{3c^3I_0})\int e^{2Nt}dt].$$

When we take t=0, $P^2(t)=P^2(0)$, $\therefore C=P(0)^2$, i.e.

$$P(t)^2=e^{-Ntt}[P(0)^2+(\frac{16\pi^2\mu_0{}^2}{3c^3I_0})\int_0^t e^{2Nt}\,dt.$$

Integrating the above expression, one yields

$$P(t)^2=e^{-Nt}[P(0)^2+\frac{8\pi^2}{3c^3N}(\frac{\mu_0{}^2}{I_0{}^2})(e^{2Nt}-1)]. \tag{14}$$

In the formula (5) when $t=0$, $I=I_0$, $\frac{dI_0}{dt}=0$, $P=P(0)$, $\dot{P}=\dot{P}(0)$, one yields

$$\frac{8\pi^2\mu_0{}^2}{3c^3I_0}=P(0)\dot{P}(0) \tag{15}$$

Substituting the expression (15) into the formula (14), then, the formula (14) become as

$$P(t)^2=e^{-kt}[P(0)^2+(\frac{P(0)\dot{P}(0)}{N})(e^{2Nt}-1)] \tag{16}$$

Hence, we can estimate the variable rate of the pulse period per century as follows

$$\delta P=[P(t)-P(0)](s/century). \tag{17}$$

Where P(0) is the initial value as $t=0$.

Numerical results

We use the formulas (16)—(17) to estimate the periodic variation of five pulsars PSR0843+06, PSR1508+55, PSR1933+16, PSR1749–28 and PSR2045–16 due to time variation of the gravitational constant per century. The P and $\dot{P}$ of these pulsars are adopted from data in a Table given by Allen[5], We assume these pulsars have M=1.4 (solar mass) and R=1.2km,[6] the polytropic index n=1 (Alan, Riper,,1975). $\gamma=(n+1)/n=2,\ A=4$ (Heintzmann & Hillerant, 1975),[1] $\sigma=2GM/c^2R=0.3439$, $4-3\gamma-A\sigma=-3.3756$. Substituting these data into the expression (10) which can be written as

$$N=-\frac{\dot I}{I}=-\{\frac{2}{4-3\gamma-A\sigma}]\frac{\dot G}{G}=-[\frac{2}{3.3756}]\frac{\dot G}{G}=-0.592487\frac{\dot G}{G.} \quad (18)$$

We cited the time variation of $\frac{\dot G}{G}$ given by AI–Rawaf[7]

$$-2.8\times10^{-13}/yr\prec\frac{\dot G}{G}\prec-6.0\times10^{-13}/yr. \quad (19)$$

In this paper we calculate the lower limit for $\frac{\dot G}{G}=-2.8\times10^{-13}/yr.$

Substituting this value for $\frac{\dot G}{G}$ into the expression (18), we obtain

$$N=1.66\times10^{-13}/yr=constant \quad (20)$$

Substituting the values of N, $P(0)$, and $\dot P(0)$ into the formula (16) and takes $t=100\,year$ (1st century), we obtain the numerical results listed in Table 1 for the slow down of periods of five pulsars due to a time variation of the gravitational constant.

Table 1 shows that pulse periods of five pulsars are prolonged in the range $0.0000149s\sim0.0000363s$ per century due to time variation of the gravitational constant

Table 1 Numerical results for spin down of periods of five pulsars due to time variation of gravitational constant in the lower limit

Pulsars	$P(t_0)(s)$	$\dot P(t_0)\times10^{-15}(s/s)$	$P(t)(s)$	ät(s / cent)
PSR1749–28	0.5625532	8.15	0.5625789	0.0000257
PSR2045–16	1.9615669	10.96	1.9616032	0.0000363
JP 1933+16	0.3587354	6.00	0.3587543	0.0000189
HP 1508+55	0.7396779	5.04	0.7396928	0.0000149
CP 0834+06	1.2737635	6.80	1.2737849	0.0000214

Discussions and conclusion

In the quadrupole elastic energy model of neutron stars the total energy and moment of inertia connect with oblateness ε,[8] i e,

$$E=E_0+\frac{1}{2}I\Omega^2+A\varepsilon^2+B(\varepsilon-\varepsilon_0)^2,\ I=I_0(1+\varepsilon).$$

But in the magnetic dipole model we may not consider oblateness $\varepsilon=0$, the ε_0 is the primary value. We consider pulsars as spherical stars.

1) Some pulsar, such as Crab and Vela speed up suddenly due to stellar quakes at some time.[8] They do not effect all pulsars. It is a temporary happning and is not a secular happing. It cannot influence the secular variation of a slow down due to time variation of gravitational constant.

Because the value for $\frac{\mu_0^2}{I_0}$ cannot be obtained from the observation, it may be written as the formula (14) in terms of $P(0)$ and $\dot P(0)$

$$\frac{\mu_0^2}{I_0}=\frac{3c^3}{8\pi^2}P(0)\dot P(0)$$

Hence, the formula (14) can be expressed by using the formula (16)

2) In this paper the results are obtained under the condition of no variation effect for the magnetic dipole moment and magnetic inclination without variation.

We also obtained the conclusions:

1) The variation of the gravitational constant with time may be determined known from the observation and theories. It connects with the large number hypothesis in cosmology

Time variation of gravitational constant can influences the change of moment of inertia through $\dot I/I=-\varepsilon\dot G/G.$.

The change of moment of inertia can influences the spin down of the period of pulsar due to time variation of the gravitational constant, and the variation of the moment of inertia is an exponential formulation under the condition for time variation of the gravitational constant.

The variable rate of the spin down of the period of five pulsars are on the order 10^{-5} seconds per century. This effect can be observed by the current astronomical instruments over a long time.[9]

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Conflict of interest

The author declares no conflict of interest.

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