1. Appendix

A Equal–time commutation relation

Nakanishi has shown the equal–time commutation relation of the vierbein as follows[1]:

$$\left[\hat{\mathcal{E}}^{a}_{\mu}(r),\frac{d\mathcal{E}^{b}_{\nu}}{d\tau'}(r')\right]=-\frac{i}{2}\kappa\left(g^{00}\mathcal{E}\right)^{-1}\delta^{3}\left(\mathbf{r}-\mathbf{r}'\right)[\mathcal{E}^{a}_{\mu}\mathcal{E}^{b}_{\nu}-\mathcal{E}^{b}_{\mu}\mathcal{E}^{a}_{\nu}-g_{\mu\nu}\eta^{ab}+$$

$$\left(g^{00}\right)^{-1}\left(\delta^{0}_{\mu}\delta^{0}_{\nu}\eta^{ab}+\delta^{0}_{\mu}\mathcal{E}^{b0}\mathcal{E}^{a}_{\nu}+\delta^{0}_{\nu}\mathcal{E}^{a0}\mathcal{E}^{b}_{\nu}+\mathcal{E}^{a0}\mathcal{E}^{b0}g_{\mu\nu}\right)], \qquad (49)$$

where $\tau=\tau'$, $\kappa=1$ is the Einstein's gravitational constant, and $\mathcal{E}$ is a determinant of the vierbein. Here "*time*" does not mean the physical time, but merely the zeroth component of the space–time coordinate. By substituting the FLRW solution of $\mathcal{E}^{a}_{\mu}=diag(\Omega,\Omega/f,\Omega r,\Omega r\sin\theta)$ into (49) at the same space–time point, the commutation relation for the scale function between Ω and $\dot{\Omega}(=d\Omega/d\tau)$ is given as

$$\left[\hat{\mathcal{E}}^{0}_{0}(r),\frac{d\mathcal{E}^{0}_{0}}{d\tau'}(r')\right]=-i\frac{3}{2}\delta^{3}\left(\mathbf{r}-\mathbf{r}'\right)\left(r'^{2}\sin\theta/f(r')\right)^{-1}, \qquad (50)$$

by direct calculations. On the other hand, $\hat{\mathcal{E}}^{0}_{0}(r)=\Omega(\tau)$. Thus, the comutation relation can be ontained as

$$\left[\Omega,\dot{\Omega}\right]d^{3}\mathbf{r}'=-i\frac{3}{2}\delta^{3}\left(\mathbf{r}-\mathbf{r}'\right) \qquad (51)$$

This result is consistent with ours (21) except an overall factor.

[1]The equation (3.42) in Nakanishi[28]