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Physics & Astronomy International Journal

Letter to Editor

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Theorem on spaces and long–ranged interaction forces forming these spaces

Abstract

The aim of this paper is to find connection between the spaces of different dimensions $\mathbf{i}$ (from zero up to '$\mathbf{n}$') and the long–range attractive forces that create these spaces and have (forces) its dimension $\mathbf{j}$ (from zero up to '$\mathbf{m}$'). A theorem is formulated and strictly proved showing in which cases the long–ranged attractive forces can form real spaces of different dimensions (from zero up to $\mathbf{i}=1,2,\ldots\mathbf{n}$). The existence of the attraction between masses is defined by divergence the vector of interaction between masses.

Keywords: attractive forces, spaces of different dimensions, real spaces, attraction between masses, divergence

Volume 2 Issue 4 - 2018

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Received: August 02, 2018 | **Published:** August 28, 2018

Pacs: 04.50.+h ; 02.40.–k

Letter to editor

As is well–known from affine geometry,[1] there are spaces with the allowable systems of orthogonal coordinates having the common origin, an identical unit volume and the same orientation. Such is our real 3d–space. Why? Because our real space could be created only owing to long ranged attractive forces, e.g., by the forces of gravitation. An empty space, i.e., the space without any matter, can have any dimension–from zero up to '$\mathbf{n}$'. The mathematical space is the empty.

The main goal of this article is to find connection between the spaces of different dimensions $\mathbf{i}$ (from zero up to '$\mathbf{n}$') and the long–range attractive forces that create these spaces and have (forces) its dimension $\mathbf{j}$ (from zero up to '$\mathbf{m}$'). By the dimension of long–range attraction forces F is meant the value of the exponent j in the denominator of the formula $F = km_1m_2/r^j$,

where m_1 and m_2 are interacting masses (kg); k is a coefficient; r the distance between these masses (m); $\mathbf{j}(j)=1,2,3,\ldots,\mathbf{m}(m)$.

Our problem should not be confused with problem that P. Ehrenfest was solving 100 years ago.[2–5] He made attempt to link the dimension of space with fundamental laws of physics but he did not concern the problems connected with the creature of spaces under influence of long–ranged interaction forces.

Call any space containing matter real space. Any real space has to contain the sources of long–range interactions, in our case of the attraction between masses. The existence of these sources is defined by divergence the vector of interaction $\boldsymbol{a}$ between masses. For example, for our 3–D real space the divergence of $\boldsymbol{a}$ will have the following form if $\boldsymbol{a}$ is only the function of the coordinate ρ (spherical coordinate system):

$$div_3\mathbf{a} = \lim_{V\to 0}\frac{\oint_3 a_\rho n_\rho dS}{V}, V=\frac{4}{3}\pi r^3, S=4\pi r^2, \quad (1)$$

$$\boldsymbol{F} = km_1m_2\boldsymbol{r}/r^3;$$

$$\boldsymbol{a}=\boldsymbol{E}=(\boldsymbol{F}/m_2)=\boldsymbol{r}\frac{km_1}{r^3}; \quad (2)$$

where index "3" in (1) indicates that the above formulae refer to 3D–space; dS is a surface element (in our case of the spherical surface); n_ρ the unit vector perpendicular to dS; V volume $\boldsymbol{F}$ the interaction force between masses m_1 and m_2; k the constant of gravitation ($kg^{-1}{\bullet}m^3{\bullet}s^{-2}$); $\boldsymbol{E}$ the vector of gravitation field intensity ($m{\bullet}s^{-2}$), as $m_2=1$.

If $V\to 0$, then we can write down (1) as

$$div_3\mathbf{a} = \oint_3 n_\rho \frac{k}{r^2}\frac{\delta m_1}{\delta V}dS. \quad (3)$$

Since $ds=\rho^2\sin\theta d\theta d\phi$ in spherical system of coordinate (ρ,θ,ϕ), we have after integration (3):

$$div_3\mathbf{a} = 4\pi k n_\rho \frac{\delta m_1}{\delta V}. \quad (4)$$

If we would use another formula instead $F = km_1m_2\mathbf{r}/r^3$, e.g.,

$$\boldsymbol{F} = km_1m_2\mathbf{r}/r^2, \quad (5)$$

then we have

$$div_3\mathbf{a} = 4\pi krn_\rho \frac{\delta m_1}{\delta V}. \quad (6)$$

It means that $div_3\mathbf{a}$ depends on "r", it means, in turn, that the law of energy conservation is broken. Indeed, if $V\to 0$ (see (1)), then $r\to 0$ and $div_3\mathbf{a}=0$. It means that the gravitation source at this point of space is not observed. The analogous picture takes place at other points of our space. In fact, it means as well that there is no any real space, since the interaction (5) cannot maintain its existence.

Now take such a law instead (5):

$$\boldsymbol{F} = km_1m_2\mathbf{r}/r^4, \quad (7)$$

then instead (6) we have

$$div_3\mathbf{a} = 4\pi kr^{-1}n_\rho \frac{\delta m_1}{\delta V}, \quad (8)$$

and $div_3\mathbf{a}\to\infty$ if V and, consequently, r tends to zero. It means that our space collapses into a point, i.e., we obtain a black hole.

Here we should make an important remark. As seen, studying the above case, we have used the sphere of dimension 3. It means that we have been studying an isotropic space. If the space investigated had a fractional dimension, e.g., 2.9, then we had to take for our investigations not a sphere but an ellipsoid. Consequently, we cannot take the relation for the element of the ellipsoid surface as $ds = \rho^2 \sin\theta d\theta d\phi$, since in this case ρ should be a function of the angles φ and θ . In this work we have been studying only isotropic spaces.

Now we can put a question: how will things be going for spaces of other dimensions–from zero to $\mathbf{n}$? To answer this question, first of all write down expressions for volumes and surfaces of different ranks. We begin to study spaces whose dimensions $\mathbf{i} \le 3$, i.e., 0,1,2.

If $\mathbf{i} = 2$, then we consider a circumference and a space inside it (we we call this space a flat sphere). Therefore we have:

$$div_2\mathbf{a} = Lim\frac{\oint_2 a_\rho n_\rho dL}{S}, S = \pi r^2, L = 2\pi r, \quad (9)$$

$$S \to 0$$

$$\boldsymbol{F} = km_1m_2\boldsymbol{r} / r^2; \boldsymbol{a} = \boldsymbol{E} = (\boldsymbol{F} / m_2)\boldsymbol{r}\frac{km_1}{r^2}; \quad (10)$$

here the coefficient k in $kg^{-1} \bullet m^2 \bullet s^{-2}$ units.

If $S \to 0$, then we can write down (9) as

$$div_2\mathbf{a} = \oint_2 n_\rho \frac{k}{r}\frac{\delta m_1}{\delta S}dL. \quad (11)$$

Since $dL = \rho d\phi$ in polar system of coordinate (ρ, ϕ) , we have after integration in (11):

$$div_2\mathbf{a} = 2\pi k n_\rho \frac{\delta m_1}{\delta S}. \quad (12)$$

If we would have another formula instead $F = km_1m_2\mathbf{r} / r^2$, e.g.,

$$\boldsymbol{F} = km_1m_2\mathbf{n}_\rho, \quad (13)$$

Then we would have

$$div_2\mathbf{a} = 2\pi k r n_\rho \frac{\delta m_1}{\delta S}. \quad (14)$$

It means that $div_2\mathbf{a}$ depends on " r ', it means, in turn, that the law of energy conservation is broken. Indeed, if $S \to 0$ (see (9)), then $r \to 0$ and $div_2\mathbf{a} = 0$. It means that the gravitation source at this point of space is not observed. The analogous picture takes place at other points of this space. In fact, it means as well that there is no any real 2D–space. Since the interaction (13) cannot maintain its existence.

Now take such a law instead (10):

$$\boldsymbol{F} = km_1m_2\mathbf{r} / r^3, \quad (15)$$

Then instead (14) we have

$$div_2\mathbf{a} = 2\pi k r^{-1} n_\rho \frac{\delta m_1}{\delta S}, , \quad (16)$$

and $div_3\mathbf{a} \to \infty$, if $r \to 0$, i.e., the flat sphere collapses into point and we have black hole but in 2D–space. Below the Greek letters φ and θ will be replaced by Greek letter ζ_m with index $m = 1,2$, since we shall study spaces with $\mathbf{i} \ge 4$.

If $\mathbf{i} = 1$, a straight–line segment will be as if an analogue of the above flat sphere and a pair of points will be as if analogue of the above circumference bounding the above flat one. At this case we have:

$$div_1\mathbf{a} = 2a, \quad (17)$$

$$F = km_1m_2, a = E \quad (18)$$

Here the coefficient k is in $kg^{-1} \bullet m \bullet s^{-2}$ units. At last, if $\mathbf{i} = 0$, then the space is a point here and its $div_0 a = \frac{0}{0}$, i.e. we have uncertainty. Now we shall study the spaces having the dimensions from $\mathbf{i} = 4$ up to $\mathbf{i} = \mathbf{n}$.

If $\mathbf{i} = 4$, then we have:[6]

$$div_4 A_{(4)} = Lim\frac{\oint_4 a_\rho n_\rho d\Lambda_{(4)}}{\Omega_{(4)}}, \Omega_{(4)} = \frac{1}{2}\pi^2 R_{(4)}^4; \Lambda_{(4)} = 2\pi^2 R_{(4)}^3 \quad (19)$$

$$\Omega_{(4)} \to 0$$

$$\boldsymbol{F}_{(4)} = km_1m_2\boldsymbol{R}_{(4)} / R_{(4)}^4; \boldsymbol{A}_{(4)} = \boldsymbol{E}_{(4)} = (\boldsymbol{F}_{(4)} / m_2) = \boldsymbol{R}_{(4)}\frac{km_1}{R_{(4)}^4}; \quad (20)$$

Where index "4" in (19–20) indicates that these formulae refer to 4D–space; $d\Lambda_{(4)}$ is the element of 3D–surface; n_ρ component of unit vector perpendicular to each point of this 3D–surface; Ω_4 is 4D–volume; $F_{(4)}$ interaction force between masses m_1 and m_2 in 4D–space; k constant of gravitation in 4D–space ($kg^{-1} \bullet m^4 \bullet s^{-2}$); $\boldsymbol{E}$ vector of gravitation field intensity ($m \bullet s^{-2}$), as $m_2 = 1$.

If $\Omega_{(4)} \to 0$, then we can write down (19) as

$$div_4\mathbf{A}_{(4)} = \oint_4 n_\rho \frac{k}{R^3}\frac{\delta m_1}{\delta\Omega_{(4)}}d\Lambda_{(4)}. \quad (21)$$

Since $d\Lambda_{(4)} = \rho^3 F(\zeta_1, \zeta_2, \zeta_3)d\zeta_1 d\zeta_2 d\zeta_3$ in a spherical system of coordinate $(\rho, \zeta_1, \zeta_2, \zeta_3)$, then we have after integration (21):

$$div_4\mathbf{A}_{(4)} = 2\pi^2 k n_\rho \frac{\delta m_1}{\delta\Omega_{(4)}}. \quad (22)$$

If we would have another formula instead $\boldsymbol{F} = km_1m_2\boldsymbol{R}_{(4)} / R_{(4)}^4$, e.g.,

$$\boldsymbol{F} = km_1m_2\mathbf{R}_{(4)} / R_{(4)}^3, \quad (23)$$

Then we would have

$$div_4\mathbf{A}_{(4)} = 2\pi^2 k R_{(4)} n_\rho \frac{\delta m_1}{\delta\Omega_{(4)}}. \quad (24)$$

It means that $div_4\mathbf{A}_{(4)}$ depends on " R ", it means, in turn, that the law of energy conservation is broken. Indeed, if $\Omega_{(4)} \to 0$ (see (19), then $R \to 0$ and $div_4\mathbf{A}_{(4)} = 0$. The analogous picture takes place at other points of our space. In fact, it means as well that there is no any real space, since the interaction (23) cannot maintain its existence.

Now take such a law instead (23):

$$\boldsymbol{F} = km_1m_2\mathbf{R}_{(4)} / R_{(4)}^5, \quad (25)$$

Then instead (24) we have

$$div_4\mathbf{A}_{(4)} = 2\pi^2 k R_{(4)}^{-1} n_\rho \frac{\delta m_1}{\delta\Omega_{(4)}}, \quad (26)$$

And $div_4\mathbf{A}_{(4)} \to \infty$ if $\Omega_{(4)}$ and, consequently, $R_{(4)}$ tend to zero. It means that our space collapses into a point, i.e., we obtain a black hole.

In principle, we get a similar picture for the cases i=5,..,n. Show it for the case $\mathbf{i} = \mathbf{n}$.

$$div_n\mathbf{A}_{(\mathbf{n})} = \lim_{\Omega_{(n)} \to 0}\frac{\oint_n a_\rho n_\rho d\Lambda_{(n-1)}}{\Omega_{(n)}}; \Omega_{(n)} = C_{(n)}\rho^n . \Lambda_{(n-1)} = nC_{(n)}\rho^{n-1}, C_{(n)} = \frac{\pi^{n/2}}{\Gamma\left(\frac{n}{2}+1\right)}, \quad (27)$$

Citation: Veitsman EV. Theorem on spaces and long–ranged interaction forces forming these spaces. *Phys Astron Int J.* 2018;2(1):399–401. DOI: 10.15406/paij.2018.02.00116

$$F_{(n)} = km_1m_2\mathbf{R}_{(n)} / R^n_{(n)}; \boldsymbol{A}_{(n)} = \boldsymbol{E}_{(n)} = \left(\boldsymbol{F}_{(n)} / m_2\right) = \boldsymbol{R}_{(n)}\frac{km_1}{R^n_{(n)}}; \quad (28)$$

where index "n" in (27–28) indicates that these formulae refer to **n**D–space; $d\Lambda_{(n-1)}$ is the element of nD–surface; n_ρ a component of unit vector perpendicular to each point of this **(n–1)**D–surface; $\Omega_{(n)}$ **n**D–volume; $F_{(n)}$ the interaction force between of masses m_1 and m_2 in **n**D–space; k the constant of gravitation in **n**D–space ($kg^{-1} \cdot m^n \cdot s^{-2}$); $\boldsymbol{E}$ the vector of gravitation field intensity ($m \cdot s^{-2}$), as $m_2 = 1$; $\Gamma\left(\frac{n}{2}+1\right)$ gamma function.

If $\Omega_{(n)} \to 0$, then we can write down (27) as

$$div_n\mathbf{A}_{(n)} = \oint_n n_\rho \frac{k}{R^{n-1}}\frac{\delta m_1}{\delta\Omega_{(n)}}d\Lambda_{(n)}. \quad (29)$$

Since $d\Lambda_{(n)} = \rho^{n-1}F(\zeta_1,\zeta_2,\zeta_3,...,\zeta_{n-1})d\zeta_1 d\zeta_2 d\zeta_3,...,d\zeta_{n-1}$ in a spherical system of coordinate $(\rho,\zeta_1,\zeta_2,\zeta_3,...,\zeta_{n-1})$, we have after integration (29):

$$div_n\mathbf{A}_{(n)} = kn_\rho \frac{n\pi^{n/2}}{\Gamma\left(\frac{n}{2}+1\right)}\frac{\delta m_1}{\delta\Omega_{(n)}}. \quad (30)$$

If we would have another formula instead $F_{(n)} = km_1m_2 / R^{n-1}$, e.g.,

$$\boldsymbol{F}_{(n)} = km_1m_2\mathbf{R}_{(n)} / R^{n-1}_{(n)}, \quad (31)$$

Then we would have

$$div_n\mathbf{A}_{(4)} = kn_\rho \frac{nR_{(n)}\pi^{n/2}}{\Gamma\left(\frac{n}{2}+1\right)}\frac{\delta m_1}{\delta\Omega_{(n)}}. \quad (32)$$

It means that $div_n\mathbf{A}_{(n)}$ depends on "$R_{(n)}$", it means, in turn, that the law of energy conservation is broken. Indeed, if $\Omega_{(n)} \to 0$ (see (27), then $R_{(n)} \to 0$ and $divn\mathbf{A}_{(n)} = 0$. It means that the gravitation source at this point of space is not observed. The analogous picture takes place at other points of this space. In fact, it means as well that there is no any real space since the interaction (31) cannot maintain its existence.

Now take such a law instead (31):

$$F_{(n)} = km_1m_2\mathbf{R}_{(n)} / R^{n+1}_{(n)}, \quad (33)$$

Then instead (24) we have

$$div_n\mathbf{A}_{(n)} = kn_\rho \frac{nR^{-1}_{(4)}\pi^{n/2}}{\Gamma\left(\frac{n}{2}+1\right)}\frac{\delta m_1}{\delta\Omega_{(n)}}, \quad (34)$$

And $div_n\mathbf{A}_{(n)} \to \infty$ if $\Omega_{(n)}$ and, consequently, $R_{(n)}$ tend to zero. It means that our space collapses to point, i.e., we obtain a black hole.

Now we can assume that vacuum is a $\mathbf{nD}$–space where the interaction law between masses has the rank $\mathbf{n}+1$. There are fluctuations of the number $\mathbf{n}+1$ in the interaction one and the rank of the interaction may become less than the dimension of the space. As a result, there occurs the Big Bang. Thus we have shown that long–raged interaction forces of the dimensions $\mathbf{i} = 0, 1, 2, \ldots, \mathbf{n}$ can form real isotropic Euclidean spaces if and only if, when the dimensions $\mathbf{j}$ of these spaces equals $\mathbf{j} = \mathbf{i}+1$. Then we can affirm, using the method of mathematical induction, that long–raged interaction forces of the dimensions $\mathbf{i} = \mathbf{n}+1$ can form a real isotropic Euclidean space of the rank $\mathbf{j} = \mathbf{i}+1 = \mathbf{n}+2$.

This is a theorem which we name "Theorem on spaces and long–ranged interaction forces forming these spaces" or, more shortly, "Theorem on spaces and forces forming them".

Corrigendum

We have omitted the sign "–" (minus) in the right sides formulae of the type (2), (3), (5) and so on after the sign "=" (equality), since we are only interested in an absolute volume of this divergence.

Acknowledgements

None.

Conflict of interest

Author declares there is no conflict of interest.

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