MedCrave Step into the World of Research
Physics & Astronomy International Journal

Review Article

Open Access CrossMark

Finite periodic orbits around L_4 in photogravitational restricted three–body problem

Abstract

The motion of the infinitesimal mass in the restricted three–body problem is considered in the vicinity of the triangular point L_4, when the more massive primary is considered as a source of radiation. General coordinates are taken as polar coordinates (r, θ) centered at the triangular point L_4. A time–independent nonlinear second–order ordinary differential equation for r as a function of θ is derived. Approximations to periodic solutions of finite size are obtained following the geometrical dynamics approach of Rand and Podgorski.[1]

Keywords: restricted three–body problem, triangular liberation point, polar coordinates, geometrical dynamics approach, solar radiation pressure, periodic solutions of finite size

Volume 2 Issue 4 - 2018

Hariprasad P, Prashant Kumar, Ram Krishnan Sharma

Department of Aerospace Engineering, Karunya Institute of Technology and Sciences, India

Correspondence: Ram Krishnan Sharma, Department of Aerospace Engineering, Karunya Institute of Technology and Sciences, Coimbatore–641114, Tamil Nadu, India, Tel +9194 8784 6632, Email ramkrishan@karunya.edu

Received: July 29, 2018 | **Published:** August 24, 2018

Introduction

The simplicity and elusiveness of the three–body problem have attracted a number of mathematicians for centuries. There are names of many great mathematicians (Euler, Lagrange, Jacobi, Hill, Hamilton, Poincaré, Birkhoff etc.), who have worked on this problem and made important contributions. The book of Szebehely[2] provides systematic coverage of the literature on the subject as well as derivations of some of the important results. Even today the problem of three–body is as enigmatic as ever. If two of the finite bodies move in circular coplanar orbits about their common center of mass and the third body is too small to affect the motion of the two bodies, then the problem is called circular restricted three body problem (RTBP). In the circular problem, two finite masses are fixed in a co–ordinate system rotating with the orbital angular velocity and origin is at the center of mass of the two bodies. It resembles an important dynamical system for the study of new investigations regarding motions not only in the solar system but also in other planetary systems. Motion of small space objects (asteroid, comet, ring, spacecraft, satellite etc.) in the solar system as well as Sun–planet systems (Sun–Earth system, Sun–Jupiter system etc.) are the best examples of RTBP. In 1772, the famous mathematician Lagrange discovered that in a rotating frame, there are five stationary or equilibrium points at which the restricted mass would remain fixed if placed there. Three of them lie on the line connecting the two finite masses, called collinear equilibrium points and remaining two are located at equidistant from the two finite masses, called triangular equilibrium points. That is, the two masses and the triangular points are thus located at the vertices of the equilateral triangle in the plane of the circular orbits. The problem becomes more interesting when it also includes the other type of space structures such as belt, disk, ring etc., which are present in the solar system.[3] Different aspects of this problem such as conditions for existence of equilibrium points, stability property (linear and nonlinear), periodicity of the orbits etc., with perturbation factors in the form of radiation pressure, oblateness etc. have been studied by many authors, Some of the studies.[4–9] Periodic orbits of finite size around the Lagrangian point L_4 had been the subject of investigations.[10,11] Geometrical dynamics is the study of the geometry of the orbits in configuration space of a dynamical system without reference to the system's motion in time. It is an alternative approach to study the motion around the Lagrangian points. Rand & Podogorski[1] were the first to introduce this approach to planar RTBP it terms of the polar coordinates (r, θ) centered at L_4 to study the motion around it in the RTBP. Sharma and Subba Rao[12] employed their method to study the motion around L_4 in the planar RTBP when the more massive primary is an oblate spheroid with its equatorial plane coincident with the plane of motion. In this paper we have utilized the same approach in the planar RTBP when the more massive primary is a source of radiation. We have used the polar coordinates (r, θ) centered at the triangular liberation point L_4. A time–independent nonlinear second–order ordinary differential equation for r as a function of θ is derived. Approximations to periodic solutions are obtained by perturbations and Fourier series. These solutions represent periodic orbits around L_4.

Equation of motion

The equations of motion for the circular photogravitational planar RTBP in the dimensionless barycentric synodic coordinates x, y are:[2,13]

$$\ddot{x}-2\dot{y}-x=-V_x \tag{1}$$

$$\ddot{y}+2\dot{x}-y=-V_y \tag{2}$$

where

$$V=-q(1-\mu)/r_1-\mu/r_2$$

$$r_1^2=(x-\mu)^2+y^2; r_2^2=(x+1-\mu)^2+y^2. \tag{3}$$

q is the mass reduction factor for a given particle. In order to obtain the equations of motion in terms of polar coordinates (r, θ);

$$x=a+r\cos(\theta+\alpha),$$

$$y=b+r\sin(\theta+\alpha), \tag{4}$$

α is the angle which the major axis of the ellipse makes with x–axis. (a,b) are the coordinates of L_4. Differentiating, we get

$$\dot{x}=\dot{r}\cos(\theta+\alpha)-r\dot{\theta}\sin(\theta+\alpha),$$

$$\dot{y}=\dot{r}\sin(\theta+\alpha)+r\dot{\theta}\cos(\theta+\alpha).$$

Differentiating again, we get

$$\ddot{x}=\ddot{r}\cos(\theta+\alpha)-2\dot{\theta}\dot{r}\sin(\theta+\alpha)-r\dot{\theta}^2\cos(\theta+\alpha)-r\ddot{\theta}\sin(\theta+\alpha),$$

$$\ddot{y} = \ddot{r}sin(\theta+\alpha) + 2\dot{\theta}\dot{r}sin(\theta+\alpha) - r\dot{\theta}^2 sin(\theta+\alpha) + r\ddot{\theta}cos(\theta+\alpha).$$

Substituting these expressions in the equations of motion (1 and 2) and simplifying, we get

$$\ddot{r} - r\dot{\theta}^2 - 2r\dot{\theta} - V_X\cos(\theta+\alpha) - V_y\sin(\theta+\alpha) + \alpha\cos(\theta+\alpha) + \beta\sin(\theta+\alpha) + r, \tag{5}$$

$$r^2\ddot{\theta} + 2r\dot{r}(\theta+1) = V_X\sin(\theta+\alpha) - V_y\cos(\theta+\alpha) - a\sin(\theta+\alpha) + b\cos(\theta+\alpha), \tag{6}$$

where

$$V_X = q(1-\mu)(X-\mu)/r_1^3 + \mu(X+1-\mu)/r_2^3,$$

$$V_y = y\left[(1-\mu)/r_1^3 + \mu/r_2^3\right],$$

$$V_r = q(1-\mu)/r_1^3\left[(x-\mu)\cos(\theta+\alpha) + y\sin(\theta+\alpha)\right] + \mu/r_2^3\left[(x+1-\mu)\cos(\theta+\alpha) + y\sin(\theta+\alpha)\right],$$

$$V_r = q(1-\mu)/r_1^3\left[(x-\mu)\cos(\theta+\alpha) + y\sin(\theta+\alpha)\right] + \mu/r_2^3\left[(x+1-\mu)\cos(\theta+\alpha) + y\sin(\theta+\alpha)\right],$$

$$V_\theta = q(1-\mu)/r_1^3\left[-r(x-\mu)\sin(\theta+\alpha) + yr\cos(\theta+\alpha)\right] + \mu/r_2^3\left[-r(x+1-\mu)\sin(\theta+\alpha) + yr\cos(\theta+\alpha)\right],$$

$$V_x\cos(\theta+\alpha) + V_y\sin(\theta+\alpha) = V_r,$$

$$V_x\sin(\theta+\alpha) - V_y\cos(\theta+\alpha) = -V_\theta/r.$$

The equations of motion then become

$$r^2\ddot{\theta} + 2r\dot{r}(\theta+1) = -V_\theta + \left[b\cos(\theta+\alpha) - a\sin(\theta+\alpha)\right]r = -U_\theta, \tag{7}$$

$$r^2\ddot{\theta} + 2r\dot{r}(\theta+1) = -V_\theta + \left[b\cos(\theta+\alpha) - a\sin(\theta+\alpha)\right]r = -U_\theta, \tag{8}$$

where

$$U = V - r^2/2 - r\left[a\cos(\theta+\alpha) + b\sin(\theta+\alpha)\right]. \tag{9}$$

The Jacobian Integral is

$$\dot{r}^2/2 + \dot{r}^2\dot{\theta}^2/2 + U = h = constant \tag{10}$$

Taking r as a function of $\theta(\boldsymbol{r}')$

$$\dot{r} = r'\dot{\theta},$$

$$\ddot{r} = r''\dot{\theta}^2 + r'\ddot{\theta}.$$

Also

$$\ddot{r} = r\dot{\theta}^2 + 2r\dot{\theta} - U_r,$$

$$\ddot{\theta} = -U_\theta/r^2 - 2\dot{r}(\theta+1)/\mathrm{r}$$

$$\dot{\theta}^2 = 2(h-U)/\left(r^2 + r'^2\right)$$

After substitution, we get

$$r\dot{\theta}^2 + 2r\dot{\theta} - U_r = r''\dot{\theta}^2 + r'\left[-U_\theta/r^2 - 2\dot{r}(\theta+1)/\mathrm{r}\right]$$

Substituting to above equation, we get a time–independent second–order ordinary differential equation

$$2(\mathrm{h\text{-}U})(\mathrm{r}^2 - \mathrm{rr}'' + 2\mathrm{r}'^2) - 2[2(\mathrm{h}-u)]^{\frac{1}{2}}[\mathrm{r}^2 + \mathrm{r}'^2]^{\frac{3}{2}} + (\mathrm{r}^2 + \mathrm{r}'^2)((\mathrm{r}'/\mathrm{r})\mathrm{U}_e - \mathrm{rU}_r) = 0.$$

Here it has been assumed that $\dot{\theta} < 0$ for periodic orbits around L_4.[2]

An approximate solution

Location of the triangular liberation point L_4,

$$a = -\frac{1}{2} + \frac{\grave{o}}{3} + \mu \tag{13}$$

$$b = \sqrt{3}\left(\frac{1}{2} - \frac{\grave{o}}{9}\right) \tag{14}$$

$$tan2\alpha = \sqrt{3}\left(1 - \frac{8\grave{o}}{9} - 2\mu\right) \tag{15}$$

$$q = 1 - \varepsilon, \tag{16}$$

where ε is a small quantity. Transforming of potential equation and substituting it in the equation below, we get

$$U = V - r^2/2 - r\left[a\cos(\theta+\alpha) + b\sin(\theta+\alpha)\right] \tag{17}$$

$$U = U_0 + r^2 g(\theta) + r^3 f(\theta) + 0\left(r^4\right) \tag{18}$$

where

$$U_0 = -1 + \grave{o}$$

$$g(\theta) = \frac{3}{4}(-1 + \grave{o} + \left(-\frac{5\grave{o}}{3\lambda} + \lambda\right)\mathrm{Cos}[2\theta])$$

$$f(\theta) = \left(-\frac{3}{16} + \frac{15\grave{o}}{16} + \frac{3\mu}{8}\right)\mathrm{Cos}[\alpha+\theta] + \left(\frac{5}{8} - \frac{5\grave{o}}{4} - \frac{5\mu}{4}\right)\mathrm{Cos}[3\alpha+3\theta] + \left(\frac{3\sqrt{3}}{16} - \frac{41\grave{o}}{16\sqrt{3}}\right)\mathrm{Sin}[\alpha+\theta] - \frac{5\grave{o}\mathrm{Sin}[3\alpha+3\theta]}{8\sqrt{3}}$$

Solution of the differential equation will be of the form

$$r^2 = \frac{M}{N + \grave{e}os2}.$$

Substituting this equation into (12) and

at $\theta = 0°$

$$-2\sqrt{2}\left(\frac{M}{1+N}\right)^{3/2}\sqrt{1 + h - \epsilon - \frac{3M\left(-1 + \epsilon - \frac{5\epsilon}{3\lambda} + \lambda\right)}{4(1+N)}} + \frac{2M(-1+N)\left(1 + h - \epsilon - \frac{3M\left(-1 + \epsilon - \frac{5\epsilon}{3\lambda} + \lambda\right)}{4(1+N)}\right)}{(1+N)^2} - \frac{M^2\left(3(-1+\lambda)\lambda + \epsilon(-5+3\lambda)\right)}{2(1+N)^2\lambda} = 0 \tag{19}$$

$\theta = 90°$

$$-2\sqrt{2}\left(\frac{M}{-1+N}\right)^{3/2}\sqrt{1 + h - \epsilon - \frac{3M\left(-1 + \epsilon + \frac{5\epsilon}{3\lambda} - \lambda\right)}{4(-1+N)}} + \frac{2M(1+N)\left(1 + h - \epsilon - \frac{3M\left(-1 + \epsilon + \frac{5\epsilon}{3\lambda} - \lambda\right)}{4(-1+N)}\right)}{(-1+N)^2} + \frac{M^2\left(3\lambda(1+\lambda) - \epsilon(5+3\lambda)\right)}{2(-1+N)^2\lambda} = 0 \tag{20}$$

After equating both the equations and simplifying, we get

$$M = -\frac{(-1 + h + \epsilon)\left(4\left(1 + 3N^2\right) + 6N\left(-1 + N^2\right)\left(-\frac{5\epsilon}{3\lambda} + \lambda\right)\right)}{-6N(-1+\epsilon) + \frac{5\epsilon}{\lambda} - 3\lambda + N^2\left(-\frac{10\epsilon}{\lambda} + 6\lambda\right)}$$

Where

Citation: Hariprasad P, Kumar P, Ram KS. Finite periodic orbits around L_4 in photogravitational restricted three–body problem. *Phys Astron Int J.* 2018;2(4):382–387. DOI: 10.15406/paij.2018.02.00114

$$N = -\frac{4 + \sqrt{-8 + 9\lambda^2}\,\mathrm{Sign}\left[-1 + h + \in\right]}{3\lambda}$$

Values of $h > U_0$ correspond to the short–period orbits and $h < U_0$ corresponds to the long–period orbits.

For M and N to be real, it is necessary that

$$-8 + 9\lambda^2 > 0,$$

which is equivalent to the usual stability criterion,

$$\mu(1-\mu) < 1/27.$$

To obtain the periodic solution when r^3 term is considered for inclusion in U, we adopt the perturbation scheme:

$$r = X_1\epsilon + X_2\epsilon^2 + \ldots$$

where

$$X_1 = M\,\cancel{\epsilon}\left(N + \cos 2\ \right)$$

$$\epsilon = \left|h + U_0\right|^{1/2}.$$

Substituting it in (12) and (18) and equating to zero the coefficients of like powers of ϵ, we obtain a set of linearized differential equations $X_n(\theta)$. After some algebra, the equation $X_2(\theta)$ becomes

$$F_1 X_2'' + F_2 X_2' + F_3 X_2 + F_4 = 0, \tag{21}$$

where

$$F_1 = -4k_1^2 k_2$$

$$F_2 = \frac{2k_1\left(-3k + k_2\left(4k_1 + 2k_3 + \dfrac{G'k_2X_1}{X_1'}\right)\right)X_1'}{X_1}$$

$$F_3 = 2Gkk_2 + 2k_1\left(-3k + k_2\left(2k_3 + k_2\left(-4G + \frac{G'X_1'}{X_1}\right) + 2k_1\left(2 - \frac{X_1''}{X_1}\right) - 4G\left(k_2 + X_1'^2 - X_1X_1''\right)\right)\right)$$

$$F_4 = 2Fk_1k_2^2X_1X_1' - FX_1^2k_2(k_2\left(6 - k_3 - 6GX_1^2\right) + 2k_1\left(k_2 + X_1'^2 - X_1X_1''\right))$$

with

$$k_1 = 1 - GX_1^2$$

$$k_2 = X_1^2 + X_1'^2$$

$$k_3 = -2GX_1^2 + G'X_1X_1'$$

$$k = k_2k_3 + 2k_1(k_2 + X_1'^2 - X_1X_1'')$$

We know that

$$F = \beta_1\mathrm{Cos}\theta + \beta_2\mathrm{Sin}\theta + \beta_3\mathrm{Cos}3\theta + \beta_4\mathrm{Sin}3\theta,$$

Where

$$\beta_1 = \beta_1^{(1)}\mathrm{Cos}\alpha + \beta_1^{(2)}\mathrm{Sin}\alpha$$

$$\beta_2 = -\beta_1^{(1)}\mathrm{Sin}\alpha + \beta_1^{(2)}\mathrm{Cos}\alpha$$

$$\beta_3 = \beta_3^{(1)}\mathrm{Cos}3\alpha + \beta_3^{(2)}\mathrm{Sin}3\alpha$$

$$\beta_4 = -\beta_3^{(1)}\mathrm{Sin}3\alpha + \beta_3^{(2)}\mathrm{Cos}3\alpha$$

with

$$\beta_1^{(1)} = \left(-\frac{3}{16} + \frac{15\in}{16} + \frac{3\mu}{8}\right)$$

$$\beta_1^{(2)} = \left(\frac{3\sqrt{3}}{16} - \frac{41\in}{16\sqrt{3}}\right)$$

$$\beta_3^{(1)} = \left(\frac{5}{8} - \frac{5\in}{4} - \frac{5\mu}{4}\right)$$

$$\beta_3^{(2)} = \frac{-5\in}{8\sqrt{3}}$$

After some algebra, we obtain

$$F_1(\theta) = \sum_{j=0}^{7}\alpha_{2j}Cos[(2j)\theta]$$

$$\mathrm{F}_2(\theta) = \sum_{j=1}^{7} z_{2j}\mathrm{Sin}\ [(2\mathrm{j})\theta]$$

$$\mathrm{F}_3(\theta) = \sum_{j=0}^{7}\tau_{2j}\mathrm{Cos}\ [(2\mathrm{j})\ \theta]$$

$$\mathrm{F}_4(\theta) = \sum_{j=0}^{4} o_{2j+1}\mathrm{Cos}\ [(2\mathrm{j}+1)\ \theta] + \sum_{j=0}^{4}\phi_{2j+1}\mathrm{Sin}\ [(2\mathrm{j}+1)\ \theta]$$

where α's are known constants, which depend on λ and N only. The coefficients of α's occurring in the expressions for $F_i\left(i = 1, 2, 3, 4\right)$ are provided hereunder. Taking

$$\sigma = 3m + 4n,\ \rho = 4 - 3m\lambda,\ \gamma = -n + \lambda,\ \delta = -1 + n\lambda,$$

the coefficients of $\mathbf{F_1}$ are

$$\alpha_0 = 2a_0b_0 + a_2b_2 + a_4b_4 + a_6b_6$$

$$\alpha_2 = a_6 + 2a_2b_0 + 2a_0b_2 + a_4b_2 + a_2b_4 + a_6b_4 + a_4b_6$$

$$\alpha_4 = a_4 + 2a_4b_0 + a_2b_2 + a_6b_2 + 2a_0b_4 + a_2b_6$$

$$\alpha_6 = a_2 + 2a_6b_0 + a_4b_2 + a_2b_4 + 2a_0b_6$$

$$\alpha_8 = 2a_0 + a_6b_2 + a_4b_4 + a_2b_6$$

$$\alpha_{10} = a_2 + a_6b_4 + a_4b_6$$

$$\alpha_{12} = a_4 + a_6b_6$$

$$\alpha_{14} = a_6$$

where

$$a_0 = 4n\rho\sigma + \left(1 + n^2\right)(\rho^2 + 2\sigma^2)$$

$$a_2 = n\rho^2 + 4\left(1 + n^2\right)\rho\sigma + 2n(\rho^2 + 2\sigma^2)$$

$$a_4 = \left(1 + n^2\right)\rho^2 + 4n\rho\sigma$$

$$a_6 = n\rho^2$$

$$b_0 = 3 + 24n^2 + 8n^4$$

$$b_2 = 8n(3 + 4n^2)$$

$$b_6 = 8n.$$

If

$$\zeta_0 = 4(n^2\left(-4 - 15m\lambda + 12mn + 8n^2\right) + 6mn - \left(4 + 3m\lambda\right))$$

$$\zeta_2 = 16n(-3mn^2\lambda - 3m\lambda + 6mn + 4n^2 - 4)$$
$$\zeta_4 = n(-36mn\lambda + 24m + 16n - 9\lambda^2 m)$$

and

$$\eta_0 = -48m\left(\left(1+n^2\right)\gamma + n\delta\right) - 24m\left(1+4n^2+n^4\right)\lambda + 16\left(2n\left(n\rho+\sigma\right) + \left(1+n^2\right)\left(\rho+2n\sigma\right)\right) - 3\zeta_0$$
$$\eta_2 = -48m\left(2n\gamma + \left(1+n^2\right)\delta\right) - 96mn\left(1+n^2\right)\lambda + 16\left(n\rho + 2\left(1+n^2\right)\left(n\rho+\sigma\right) + 2n\left(\rho+2n\sigma\right)\right) - 3\zeta_2$$
$$\eta_4 = -48mn\delta - 48mn^2\lambda + 16\left(\left(1+n^2\right)\rho + 2n\left(n\rho+\sigma\right)\right) - 3\zeta_4$$
$$\eta_6 = 16n\rho.$$

The coefficients of $\mathbf{F_2}$ are

$$z_2 = \left(1+4n^2\right)\Omega_2 + 4n\Omega_4 + \Omega_6$$
$$z_4 = 4n\Omega_2 + 2\left(1+2n^2\right)\Omega_4 + 4n\Omega_6 + \Omega_8$$
$$z_6 = \Omega_2 + 4n\Omega_4 + 2\left(1+2n^2\right)\Omega_6 + 4n\Omega_8 + \Omega_{10}$$
$$z_8 = \Omega_4 + 4n\Omega_6 + 2\left(1+2n^2\right)\Omega_8 + 4n\Omega_{10}$$
$$z_{10} = \Omega_6 + 4n\Omega_8 + 2\left(1+2n^2\right)\Omega_{10}$$
$$z_{12} = \Omega_8 + 4n\Omega_{10}$$
$$z_{14} = \Omega_{10}$$

where

$$\Omega_2 = 4\sigma\eta_0 + \rho\eta_2 - 2\sigma\eta_4 - \rho\eta_6$$
$$\Omega_4 = 2\rho\eta_0 + 2\sigma\eta_2 - 2\sigma\eta_6$$
$$\Omega_6 = \rho\eta_2 + 2\sigma\eta_4$$
$$\Omega_8 = \rho\eta_4 + 2\sigma\eta_6$$
$$\Omega_{10} = \rho\eta_6.$$

The coefficients of $\mathbf{F_3}$ are given by

$$\tau_0 = 2n\omega_0 + \omega_2$$
$$\tau_2 = 2\omega_0 + 2n\omega_2 + \omega_4$$
$$\tau_4 = \omega_2 + 2n\omega_4 + \omega_6$$
$$\tau_6 = \omega_4 + 2n\omega_6 + \omega_8$$
$$\tau_8 = \omega_6 + 2n\omega_8 + \omega_{10}$$
$$\tau_{10} = \omega_8 + 2n\omega_{10} + \omega_{12}$$
$$\tau_{12} = \omega_{10} + 2n\omega_{12}$$
$$\tau_{14} = \omega_{12}$$

where

$$\omega_0 = 2B_0P_0 + B_2P_2 + B_4P_4 - 2\pi_0\zeta_0 - \pi_2\zeta_2 - \pi_4\zeta_4,$$
$$\omega_2 = 2B_2P_0 + \left(2B_0 + B_4\right)P_2 + B_2P_4 + B_4P_6 - 2\pi_0\zeta_2 - \pi_4\zeta_2 - \pi_6\zeta_4 - \pi_2\left(2\zeta_0 + \zeta_4\right),$$
$$\omega_4 = 2B_0P_4 + B_2\left(P_2 + P_6\right) + B_4\left(2P_0 + P_8\right) - 2\pi_4\zeta_0 - \left(\pi_2 + \pi_6\right)\zeta_2 - 2\pi_0\zeta_4$$
$$\omega_6 = B_4P_2 + 2B_0P_6 + B_2\left(P_4 + P_8\right) - 2\pi_6\zeta_0 - \pi_4\zeta_2 - \pi_2\zeta_4$$
$$\omega_8 = B_4P_4 + B_2P_6 + 2B_0P_8 - \pi_6\zeta_2 - \pi_4\zeta_4$$
$$\omega_{10} = B_4P_6 + B_2P_8 - \pi_6\zeta_4,$$
$$\omega_{12} = B_4P_8$$

with

$$B_0 = n\rho + (1+n^2)\sigma,$$
$$B_2 = \left(1+n^2\right)\rho + 2n\sigma,$$
$$B_4 = n\rho.$$

and

$$P_0 = -24m\left(-1+n^2\right)\left(-n+\gamma\right) - 12m\left(2\left(1+n^2\right)\gamma + 4n\delta\right) - 4m(12n\delta +$$
$$\left(1+n^2\right)\left(-12n+3\lambda\right)) + 3\rho + 8n\left(n\rho+\sigma\right) + 2\left(-3+4n^2\right)\left(\rho+2n\sigma\right),$$
$$P_2 = -48m\left(-1+n^2\right)\delta - 12m\left(8n\gamma + 3\delta + 4n^2\delta\right) - 4m(6n\left(-3n+\gamma\right) +$$
$$12\left(1+n^2\right)\delta + 9n\lambda) + 10n\rho + 6\sigma + 4\left(-3+4n^2\right)\left(n\rho+\sigma\right) + 8n\left(\rho+2n\sigma\right),$$
$$P_4 = -24m\left(\gamma+2n\delta\right) - 24m\left(-1+n^2\right)\lambda - 4m\left(-12n+9\lambda+21n^2\lambda\right)$$
$$+2\left(-3+4n^2\right)\rho + 8n\left(n\rho+\sigma\right) + 6\left(\rho+2n\sigma\right),$$
$$P_6 = -12m\delta - 36mn\lambda + 10n\rho + 6\sigma,$$
$$P_8 = 3\rho.$$
$$\pi_0 = 6(2\left(m\left(1+n^2\right) - mn\lambda + n\rho\right) + \left(1+2n^2\right)\left(\rho+\sigma\right)),$$
$$\pi_2 = 3\left(-4m\left(1+n^2\right)\lambda + \rho + 8n\left(m+\sigma\right)\right),$$
$$\pi_4 = -12mn\lambda + 12n\rho + 6\sigma,$$
$$\pi_6 = 3\rho.$$

Introducing

$$q_2 = 4n\rho + 8\left(1+n^2\right)\sigma,$$
$$q_4 = 4\left(1+n^2\right)\rho + 8n\sigma,$$
$$q_6 = 4n\rho.$$
$$s_1 = -4\left(\left(1+5n^2\right)\rho + 6n\sigma\right) - 8n\left(3n\rho + \left(1+5n^2\right)\sigma\right) + \zeta_0,$$
$$s_2 = -12n\rho - 8n\left(\left(1+5n^2\right)\rho + 6n\sigma\right) - 8\left(3n\rho + \left(1+5n^2\right)\sigma\right) + \zeta_2,$$
$$s_4 = -24n^2\rho - 4\left(\left(1+5n^2\right)\rho + 6n\sigma\right) + \zeta_4,$$
$$s_6 = -12n\rho.$$

The coefficients of $\mathbf{F_4}$ are

$$o_1 = \left(1+n^2\right)m_1 + n(m_{-1} + m_3)$$
$$o_3 = \left(1+n^2\right)m_3 + n(m_1 + o_3 = \left(1+n^2\right)m_3 + n(m_1 + m_5)$$
$$o_5 = \left(1+n^2\right)m_5 + n(m_3 + m_7)$$
$$o_7 = \left(1+n^2\right)m_7 + n(m_5 + m_9)$$
$$o_9 = \left(1+n^2\right)m_9 + n(m_7 + m_{11})$$
$$\phi_1 = \left(1+n^2\right)n_1 + n(n_{-1} + n_3)$$

$$\phi_3 = \left(1+n^2\right)n_3 + n(n_1+n_5)$$
$$\phi_5 = \left(1+n^2\right)n_5 + n(n_3+n_7)$$
$$\phi_7 = \left(1+n^2\right)n_7 + n(n_5+n_9)$$
$$\phi_9 = \left(1+n^2\right)n_9 + n(n_7+n_{11})$$

where

$$m_1 = m_{-1} = \left(-q_2+2s_1+s_2\right)\beta_1 + \left(-3q_2-3q_4+s_2+s_4\right)\beta_3$$
$$n_1 = \left(q_2+2s_1-s_2\right)\beta_2 + (-3q_2+3q_4+s_2-s_4)\beta_4$$
$$n_{-1} = -\left(q_2+2s_1-s_2\right)\beta_2 - (-3q_2+3q_4+s_2-s_4)\beta_4$$
$$m_3 = \left(q_2-q_4+s_2+s_4\right)\beta_1 + \left(-3q_6+2s_1+s_6\right)\beta_3$$
$$n_3 = \left(q_2+q_4+s_2-s_4\right)\beta_2 + \left(3q_6+2s_1-s_6\right)\beta_4$$
$$m_5 = \left(q_4-q_6+s_4+s_6\right)\beta_1 + \left(3q_2+s_2\right)\beta_3$$
$$n_5 = \left(q_4+q_6+s_4-s_6\right)\beta_2 + \left(3q_2+s_2\right)\beta_4$$
$$m_7 = \left(q_6+s_6\right)\beta_1 + \left(3q_4+s_4\right)\beta_3$$
$$n_7 = \left(q_6+s_6\right)\beta_2 + \left(3q_4+s_4\right)\beta_4$$
$$m_9 = m_{11} = n_9 = n_{11} = 0$$

For a periodic solution to (21), it is sufficient to set.[12]

$$x_2(\theta) = \sum_{\substack{n=1 \\ n\,odd}}^{\infty} (a_n \mathrm{Cos}\ n\theta + b_n \mathrm{Sin}\ n\theta)$$

Substituting this Fourier series in (21) and equating to zero coefficients of $\cos n\,\theta$ and $\sin n\,\theta$, we obtain a set of linear algebraic equations on a_n and b_n, respectively. Periodic expression for $x_3(\theta)$ and higher order terms can be obtained in a similar fashion.

There will be only one periodic orbit for given values of h, ϵ and μ. The value $h > -1$ corresponds to the equilibrium solution at L_4. Value of $h > -1$ corresponds to short–periodic orbits while $h < -1$ correspond to long–periodic orbits.

Numerical results

Geometrical illustration of the foregoing analysis is provided in Figures 1–5 for some typical values of μ, ϵ and h. The curves with dotted lines therein correspond to the linear analysis with X_1 term for r while the others correspond to the non–linear analysis with the inclusion of X_2 terms. Fourier series solution for X_2 has been attempted retaining terms up to 29θ. Figure 1 & 2 illustrate the impact of the higher order terms in the analysis for the perturbed and unperturbed problems. However, it may be noted that the relative location of the origin with reference to the primaries in the two cases is not the same. Figure 3 refers to the short–period eigen frequency and is meant to illustrate the effects of radiation pressure. The curves corresponding to the long–period eigen frequency are indistinguishable and hence are omitted. It may thus be highlighted that the effect of higher order terms becomes significant for higher values of μ and surpasses the considerations of assumed radiation pressure effect.

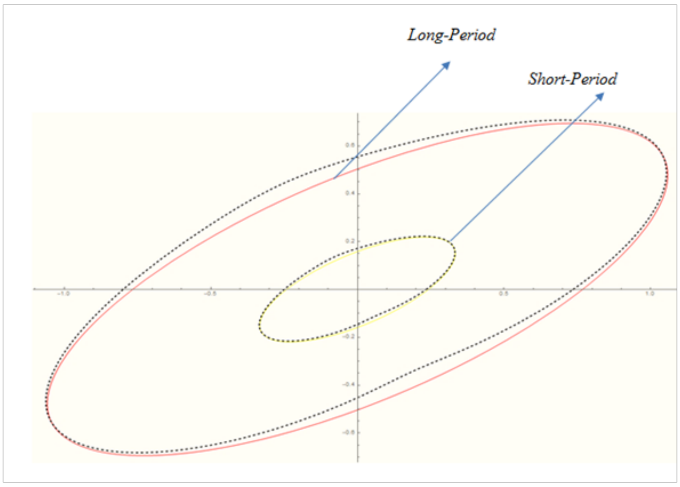

Figure 1 $\mu = 0.0369$, $\xi^2 = .0009$ and $\epsilon = 0.01$.

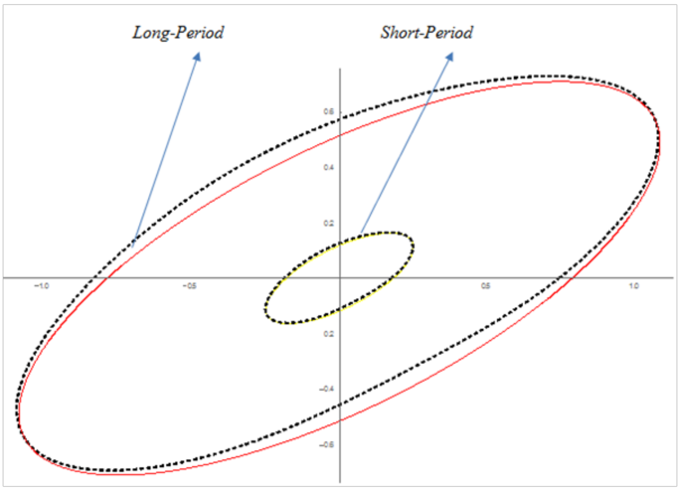

Figure 2 $\mu = 0.0369$, $\xi^2 = .0005$ & $\epsilon = 0.01$.

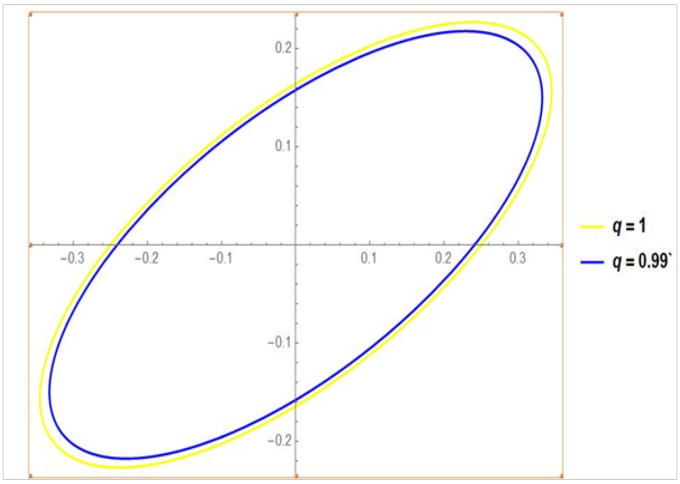

Figure 3 $\mu = 0.0369$, $\xi^2 = .0005$.

Citation: Hariprasad P, Kumar P, Ram KS. Finite periodic orbits around L_4 in photogravitational restricted three–body problem. *Phys Astron Int J.* 2018;2(4):382–387. DOI: 10.15406/paij.2018.02.00114

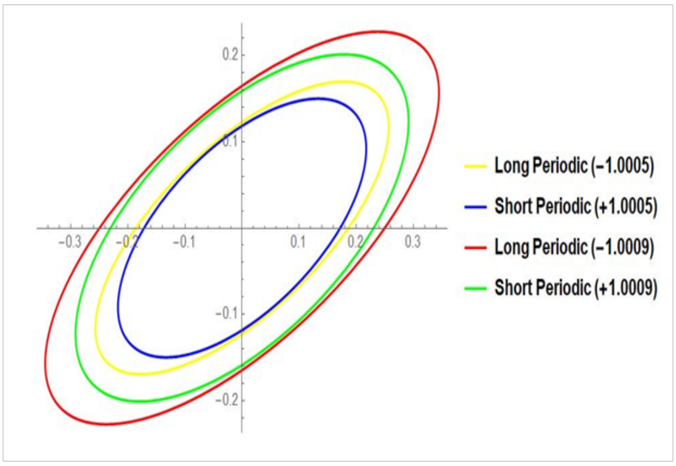

Figure 4 $\mu = 0.0369$, $\xi^2 = .0005$ & 0.0009 & $\epsilon = 0.01$.

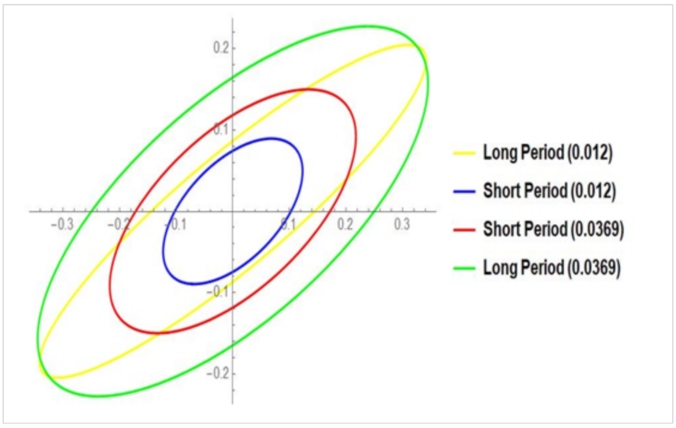

Figure 5 $\mu = 0.012$ & 0.0369, $\xi^2 = .0005$ & $\epsilon = 0.001$.

Conclusion

Finite periodic orbits have been generated at the triangular point L_4 in the photogravitational restricted three–body problem by considering the more massive primary as a source of radiation. The geometrical illustration of the periodic orbit shows the effects of μ, ϵ and h . The effect of higher order terms becomes significant for higher values of μ .

Acknowledgements

None.

Conflicts of interest

Authors declare there is no conflict of interest.

References

1. Rand R, Podgorski W. Geometrical Dynamics: A new approach to periodic orbits around L_4. *Celestial mechanics.* 1972l;6(4):416–420.
2. Szebehely V. *Theory of Orbits*. USA: Academic Press; 1967.
3. Jiang IG, Yeh LC. On the chaotic orbits of disk–star–planet systems. *Astronomical Journal.* 2004;128(2):923–932.
4. Radzievskii VV. The restricted problem of three bodies taking account of light pressure. *Astronomicheskii Zhurnal.* 1950;27:250.
5. Sharma RK, Subba Rao PV. Stationary solutions and their characteristic exponents in the restricted three–body problem when the more massive primary is an oblate spheroid. *Celestial Mechanics.* 1976;13(2):137–149.
6. Bhatnagar KB, Chawla JM. A study of the lagrangian points in the photogravitational restricted three body problem. Indian *Journal of Pure and Applied Mathematics.* 1979;10(11):1443–1451.
7. Sharma RK. *On linear stability of triangular libration points of the photogravitational three–body problem when the more massive primary is an oblate spheroid, Sun and Planetary System*. In: Fricke W, Teleki G, Editors. Netherlands: D. Reidel Publishing Company; 1982. p. 435–436.
8. Subba Rao PV, Sharma RK. Effect of oblateness on the non–linearity of L_4 in the restricted three–body problem. *Celestial Mechanics and dynamical Astronomy.* 1997;65(3):291–312.
9. Elipe A, Lara M. Periodic orbits in the restricted three body problem with radiation pressure. *Celestial Mechanics and Dynamical Astronomy.* 1997;68(1):1–11.
10. Pedersen P. On the periodic orbits in the neighborhood of the triangular equilibrium points in the restricted problem of three bodies. *Monthly Notices of the Royal Astronomical Society.* 1933;94:167–185.
11. Deprit A, Delie A. Trojan orbits: I. d'Alembert Series at L_4. *Icarus.* 1965;4(3):242–262.
12. Sharma RK, Subba Rao PV. On finite periodic orbits around the equilateral solutions of the planar restricted three–body problem. *International workshop on Space Dynamics and Celestial mechanics.* 1986. p. 71–85.
13. Sharma RK. The linear stability of libration points of the photogravitational restricted three–body problem when the smaller primary is an oblate spheroid. *Astrophysics and Space Science.* 1987;135(2):271–281.