

Bulk viscous Bianchi–I cosmological model in $f(R, T)$ gravity theory

Abstract

This paper deals with a new spatially homogeneous Bianchi–I cosmological model of the universe filled with a bulk viscous fluid in $f(R, T)$ gravity theory. We obtain exact solutions of the field equations by using a special law of variation of Hubble's parameter for the average scale factor that yields a negative constant value of the deceleration parameter, which correspond to the model of the universe with a big-bang singularity at the initial time. The universe subsequently expands with power law expansion and gives essentially an empty universe for large time. The physical and dynamical properties of the model are discussed which are consistent with cosmological scenario of the present–day accelerated universe.

Keywords: bulk viscous, bianchi –I cosmology, $f(R, T)$ gravity

Volume 2 Issue 4 - 2018

Shri Ram,¹ Surendra K Singh,² Verma MK³

^{1,3}Department of Mathematical Sciences, Banaras Hindu University, India

²Pt Ramadhar J Tiwari College of Polytechnic, India

Correspondence: Shri Ram, Department of Mathematical Sciences, Banaras Hindu University, Varanasi–2210 05, India, Tel 0542 2316 318, Email srmathitbhu@rediffmail.com

Received: December 09, 2017 | **Published:** July 25, 2018

Introduction

A wide range of recent cosmological observations^{1,2} has suggested that the universe in its present state is in the phase of accelerated expansion. The cause of this acceleration is supposed to be some kind of anti gravitational force. A large class of cosmological models has explained the acceleration of the universe in terms of a component with negative pressure, the so called dark energy (DE). The limitations of general relativity in providing satisfactory explanation of this phase of evolution have led cosmologists to adopt hypotheses and study their implications in this context. The hypotheses include those assigning (I) the time–dependence of the gravitational constant and cosmological term (II) some other geometries or physical fields associated with the universe and (III) modified or alternative theories of gravity. Modified gravity theories certainly provide a way of understanding the problem of DE and the possibility to reconstruct the gravitational field theories that would be capable to reproduce the late–time acceleration of the universe. Among several modified theories, $f(R)$ gravity theory formulated by Nojiri et al.,³ is indeed a realistic alternative to general relativity consistent with DE. In $f(R)$ gravity theory, the cosmic acceleration could be achieved by replacing Einstein–Hilbert action of general relativity with a general function of Ricci scalar R . Various attributes of cosmological models in $f(R)$ gravity theory depicting time inflation and late–time cosmic acceleration have been investigated by a number of authors.⁴ A further extension of $f(R)$ gravity theory is the $f(R, T)$ gravity theory.⁵ In this theory, an arbitrary function of the Ricci scalar R and trace T of the energy momentum tensor represents the gravitation Lagrangian. Since the inception of this theory, its characteristics have been studied both with isotropic and anisotropic Bianchi models by several cosmologists which certainly promote understanding of many associated issues in respect of the accelerated expansion of the present–day universe. Houndjo⁶ has developed the cosmological reconstruction of $f(R, T)$ gravity theory and discussed transaction of matter dominated phase to an acceleration phase. Further, Houndjo et al.,⁷ considered cosmological scenario based on $f(R, T)$ reconstructed numerically from Holographic DE. The cosmic acceleration in this theory results not only from the geometrical contribution but also from the matter content. The role played by viscosity and the consequent dissipative mechanism in cosmology have been studied by several cosmologists.^{8–12} These

dissipative processes may indeed responsible for smoothing out of initial anisotropy. Bulk viscosity is the only dissipative phenomena occurring in FRW models and is significant in causing the accelerated expansion of the universe known as inflationary phase as discussed by Setare et al.¹³ Several cosmologists have discussed the role of bulk viscosity in the early evolution of the universe in different physical contexts. The cosmological and astrophysical implications of $f(R, T)$ gravity theory in the presence of perfect fluid and bulk viscous fluids have been studied by several cosmologists. Adhav et al.,¹⁴ derived an LRS Bianchi type–I model in $f(R, T)$ gravity theory. Subsequently Reddy et al.,^{15,16} Shamir et al.,¹⁷ Chaubey et al.,¹⁸ Shri Ram et al.,^{19,20} Chandel et al.,²¹ etc. have discussed Bianchi models in the presence of perfect fluids in different physical contexts in the framework of $f(R, T)$ gravity theory. Kiran et al.,²² has shown the non–existence of Bianchi type–III bulk viscous string cosmological model. Shri Ram et al.,²³ investigated Bianchi type–I and V bulk viscous fluid cosmological models. Sahu et al.,²⁴ discussed cosmic transits and anisotropic models of Bianchi type–III. Further, Sahoo et al.,²⁵ studied cosmological models in $f(R, T)$ theory with variable deceleration parameter. This motivates the theorists to construct various models in different Bianchi space–time in different contexts.^{26–29} Sahoo et al.,³⁰ has investigated a locally rotationally symmetric Bianchi type–I cosmological model in the presence of one–dimensional cosmic strings within the framework of $f(R, T)$ gravity theory. Recently, Sahoo et al.,³¹ have studied homogeneous and anisotropic locally rotationally symmetric (LRS) Bianchi type–I model with magnetized strange quark matter distribution and cosmological constant in $f(R, T)$ gravity. It deserves mention that Momeni et al.,³² have discussed cylindrically symmetric solutions of cosmic strings in gravity rainbow scenario and calculated the gravitational fields equations corresponding to energy dependent background.

In this paper, we investigate a new exact spatially homogeneous Bianchi type–I space–time in the presence of bulk viscous fluid in $f(R, T)$ gravity theory. The paper is organized as follows: In Section 2, we present the metric and field equations of $f(R, T)$ gravity theory. In Section 3, we obtain exact solution of the field equations by utilizing the special law of variation for the average scale factor that yields a negative constant value of the deceleration parameter. The resulting cosmological model is continuously expanding, shearing

and accelerating universe which would essentially gives an empty space at late time. The other physical and geometrical features of the model are discussed in Section 4. Some concluding remarks are given in Section 5.

Metric and field equations

We consider the Bianchi type-I space-time represented by the metric

$$ds^2 = dt^2 - A^2 dx^2 - B^2 dy^2 - C^2 dz^2 \quad (1)$$

Where, A, B and C are functions of cosmic time t only.

We assume that the cosmic matter is represented by the energy-momentum tensor of a bulk viscous fluid given by

$$T_{ij} = (\rho + \bar{p})u_i u_j - \bar{p}g_{ij} \quad (2)$$

Where the effective pressure \bar{p} is given by

$$\bar{p} = p - \xi u^i_{;i}. \quad (3)$$

Here p is the isotropic pressure, ρ the energy density of matter, ξ is the bulk viscous coefficient and u^i the 4-velocity vector of the fluid satisfying $u^i u_j = 1$. A comma and semicolon denote ordinary and covariant differentiation respectively. On thermodynamically reasons the bulk viscosity coefficient is positive, assuring that the viscosity pushes the dissipative pressure \bar{p} towards negative values. However, the correction applied to the thermodynamic pressure p due to bulk viscosity is very small. Therefore, the dynamics of cosmic evolution is not fundamentally influenced by the inclusion of viscous term in energy-momentum tensor.

The field equations in $f(R, T)$ gravity theory with the special choice of the function

$$f(R, T) = R + 2f(T) \quad (4)$$

Are given by Reddy:³³

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi T_{ij} + 2\dot{f}(T)T_{ij} + [2\bar{p}\dot{f}(T) + f(T)]g_{ij} \quad (5)$$

Where a dot denotes derivative with respect to T . We further choose

$$f(T) = \lambda T, \quad (6)$$

Where λ is a constant. This gravity theory is equivalent to cosmological scenario with an effective cosmological term $\Lambda \sim H^2$, where H is the Hubble parameter.³⁴ In comoving coordinates, the field equation (5) together with (2), (4) and (6) for the metric (1) yield the following system of equation:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = \lambda\rho - (8\pi + 3\lambda)\bar{p}, \quad (7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = \lambda\rho - (8\pi + 3\lambda)\bar{p}, \quad (8)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = \lambda\rho - (8\pi + 3\lambda)\bar{p}, \quad (9)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} = (8\pi + 3\lambda)\rho - \lambda\bar{p}. \quad (10)$$

An over dot denotes derivation with respect to t .

Now we define certain physical and kinematical parameters associated with the metric (1). The spatial volume V and the average scale factor a is defined by

$$V = a^3, a^3 = ABC. \quad (11)$$

The expansion scalar θ and the shear scalar σ are given by

$$\theta = u^i_{;i} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \quad (12)$$

$$\sigma^2 = \frac{1}{2} \left[\left(\frac{\dot{A}}{A} \right)^2 + \left(\frac{\dot{B}}{B} \right)^2 + \left(\frac{\dot{C}}{C} \right)^2 \right] - \frac{1}{6} \theta^2. \quad (13)$$

The generalized mean Hubble's parameter H is defined by

$$H = \frac{\dot{a}}{a} = \frac{1}{3} (H_1 + H_2 + H_3) \quad (14)$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$, $H_3 = \frac{\dot{C}}{C}$ are directional Hubble's parameter in the direction of x , y and z respectively. The anisotropy parameter Δ is given by

$$\Delta = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i - H}{H} \right)^2. \quad (15)$$

The deceleration parameter q has the usual definition

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \quad (16)$$

The sign of q indicates whether the model inflates or not. The positive sign of q corresponds to a standard deceleration model whereas the negative sign indicates inflation.

Exact solution

We obtain solution of the field equations (7)–(10) which are four equations in six unknowns A , B , C , ρ , \bar{p} and ξ . Therefore to find a deterministic solution we shall need two extra conditions on physical ground of the problem or for simply for mathematical convenience.

Subtracting (7) from (8), (8) from (9) and (9) from (7) and integrating the results, we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{a^3}, \quad (17)$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_2}{a^3}, \quad (18)$$

$$\frac{\dot{A}}{A} - \frac{\dot{C}}{C} = \frac{k_3}{a^3} \quad (19)$$

where k_1, k_2 and k_3 are constants of integration. Using (17), (18), (19) in (13) and simplifying, we find that

$$\sigma = \frac{k}{a^3} \quad (20)$$

Where the constant k is given by $k^2 = k_1^2 + k_2^2 + k_3^2$. Equations (17), (18) and (19) suggest that

$$A^n = BC, \quad (21)$$

n is a positive constant. In view of (21), we can write

$$B = A^{n/2} D, C = A^{n/2} D^{-1} \quad (22)$$

Where D is a function of time. Substituting (22) in to (18), we obtain

$$\frac{\dot{D}}{D} = \frac{K}{a^3} \quad (23)$$

Where $K = \frac{1}{2} k_2$. From (23) we can determine $D(t)$ if the average scale $a(t)$ is explicitly known. We assume that the Hubble parameter H to be related to the average scale factor a by the relation

$$H = ma^{-1/m} \quad (24)$$

Where $m > 0$ is a constant.³⁵ Combining (14) and (24) and solving the resulting equation, we get

$$a(t) = (t + c)^m, c = \text{constant}. \quad (25)$$

This gives a constant value of the deceleration parameter as

$$q = -1 + \frac{1}{m}. \quad (26)$$

Since recent observational data indicates that the universe is accelerating and the value of deceleration parameter lies somewhere in the range $-1 < q < 0$, so we have $m > 1$ for the accelerating universe.

From (23) and (25), we obtain

$$D = M \exp \left\{ \frac{K}{(1-3m)} (t+c)^{\frac{1}{(1-3m)}} \right\} \quad (27)$$

Where M is the constant of integration. Also, equation (11) and (25) lead to

$$A = (t+c)^{\frac{3m}{n+1}}. \quad (28)$$

Substituting (27) in (22), we find that the solutions for the scale function B and C as

$$B = M (t+c)^{\frac{3mn}{2(n+1)}} \exp \left[\frac{K}{(1-3m)} (t+c)^{1-3m} \right], \quad (29)$$

$$C = M (t+c)^{\frac{3mn}{2(n+1)}} \exp \left[-\frac{K}{(1-3m)} (t+c)^{1-3m} \right]. \quad (30)$$

provided $m \neq \frac{1}{3}$. Without loss of any generality we can take $m = 1$. Hence, the metric of our solution can be written in the form

$$ds^2 = dT^2 - T^{\frac{6m}{n+1}} dx^2 - T^{\frac{3mn}{n+1}} \exp \left\{ \frac{2K}{(1-3m)} T^{(1-3m)} \right\} dy^2 - T^{\frac{3mn}{n+1}} \exp \left\{ \frac{-2K}{(1-3m)} T^{(1-3m)} \right\} dz^2 \quad (31)$$

Where $T = t + c$.

Physical features of the model

We discuss the physical and kinematical features of the cosmological model represented by the line-element (31). The expression for energy density and effective pressure as calculated from (7)–(10) are

$$\rho = \frac{1}{(8\pi + 3\lambda)^2 - \lambda^2} [(8\pi + 3\lambda) \left\{ \frac{9m^2 n(n+2)}{4(n+1)^2 T^2} + \frac{3mk}{(n+1)T^{(3m+1)}} - \frac{K^2}{T^{6m}} \right\} - \lambda \left\{ \frac{9m^2 (n+2)^2}{4(n+1)^2 T^2} + \frac{3m(3n+2)}{2(n+1)^2 T^2} + \frac{2K^2}{T^{6m}} - \frac{3m}{T^2} \right\}], \quad (32)$$

$$p = \frac{1}{(8\pi + 3\lambda)^2 - \lambda^2} [\lambda \left\{ \frac{9m^2 n}{(n+1)^2 T^2} + \frac{9m^2 n^2}{4(n+1)^2 T^2} - \frac{K^2}{T^{6m}} \right\} - (8\pi + 3\lambda) \left\{ \frac{9m^2}{(n+1)^2 T^2} + \frac{9m^2 n^2}{4(n+1)^2 T^2} + \frac{2K^2}{T^{6m}} - \frac{3m}{T^2} \right\}]. \quad (33)$$

For the specification of $\xi(t)$, we assume that the fluid obeys the equation state

$$p = \gamma p \quad (34)$$

Where $0 \leq \gamma \leq 1$ is a constant. Then from (3), we obtain

$$\xi(t) = \frac{1}{3m[(8\pi + 3\lambda)^2 - \lambda^2]} [\gamma(8\pi + 3\lambda) \left\{ \frac{9m^2 n}{(n+1)^2 T} + \frac{9m^2 n^2}{4(n+1)T} - \frac{K^2}{T^{6m-1}} \right\} - \gamma\lambda \left\{ \frac{9m^2}{(n+1)^2 T} + \frac{9m^2 n^2}{4(n+1)^2 T} + \frac{2K^2}{T^{6m-1}} - \frac{3m}{T} \right\} - \lambda(8\pi + 3\lambda) \left\{ \frac{9m^2}{(n+1)^2 T} + \frac{9m^2 n^2}{4(n+1)^2 T} + \frac{2K^2}{T^{6m-1}} - \frac{3m}{T} \right\}]. \quad (35)$$

The physical and kinematical parameter has the values given by the following expression:

$$V = T^{3m}, \quad (36)$$

$$\theta = \frac{3m}{T}, \quad (37)$$

$$\sigma = \frac{k}{T^{3m}}. \quad (38)$$

The directional Hubble's parameters and the average Hubble parameter are given by

$$H_2 = \frac{3mn}{2(n+1)T} + \frac{K}{T^{3m}}, \quad (40)$$

$$H_2 = \frac{3mn}{2(n+1)T} - \frac{K}{T^{3m}} \quad (41)$$

And

$$H = \frac{m}{T}. \quad (42)$$

The anisotropic parameter Δ has the value

$$\Delta = \frac{1}{2} \left(\frac{n-2}{n+1} \right)^2 + \frac{2K^2}{3m^2 T^{6m-2}}. \quad (43)$$

We observe that the spatial volume and the three scale factors are zero at $T = 0$. At this epoch the energy density and thermodynamic pressure assume infinite values. The expansion scalar and shear scalar are infinite at $T = 0$

Therefore; this model describes a continuously expanding shearing and accelerating universe with a big-bang start at $T = 0$. The energy density and pressure are infinite at $T = 0$ and are decreasing functions with the passage of time, and ultimately tend to zero for large time. The bulk viscosity coefficient decreases as time increases and tend to zero. Thus, the model would essentially give an empty universe for large time. The fluid is highly anisotropic near the initial singularity. The anisotropy in the model is maintained throughout the passage of time. The ratio θ tends to zero as $T \rightarrow \infty$, which indicates that the shear scalar tends to zero faster than the expansion scalar. The directional

Hubble's parameters are different in three spatial directions for finite time and tend to zero for large time.

Conclusion

Evolution of spatially homogeneous Bianchi type-I cosmological model is studied in the presence of a bulk viscous fluid within the framework of f(R, T) gravity theory. We have obtained exact solutions of the field equations by using the special law of variation for Hubble's parameter that yields a negative constant value of the deceleration parameter. The resulting cosmological model corresponds to continuously expanding, shearing and accelerating universe which would essentially gives an empty space for large time. The nature of the physical and kinematical parameters is discussed at the initial singularity and at infinity. The effect of the bulk viscosity is to produce a change in perfect fluid and hence exhibits the essential influence on the character of the solution. This effect is clearly visible in the expression of the isotropic pressure. Initially the fluid is highly viscous and viscosity decreases monotonically as time increases and becomes negligible for large time.

Acknowledgements

The authors would like to place on record their sincere thanks for honorable referees for their valuable suggestions which have helped in improving the quality of the paper.

Conflict of interest

The author declares there is no conflict of interest.

References

1. Riess AG, Alexei V Filippenko, Peter Challis, et al. Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. *Astrophysics J.* 1998;116;1009.
2. Purlmulter S, Aldering G, Della Valle M. Discovery of a Supernova Explosion at Half the Age of the Universe and its Cosmological Implications. *Nature.* 1998;391:51-54.
3. Nojiri S, Odinstov SD. Modified f(R) gravity consistent with realistic cosmology: From a matter dominated epoch to a dark energy universe. *Phys Rev D.* 2006;74.
4. Harko T, Francisco SN Lobo, Shinichi Nojiri, et al. f(R,T) gravity. *Phys Rev D.* 2011;84.
5. Copeland EJ. Dynamics of dark energy. *J Mod Phys D.* 2006;15:93.
6. Houndjo MJS. Reconstruction of f(R, T) gravity describing matter dominated and accelerated phases. *Int J Mod Phys D.* 2012;21(1).
7. Houndjo MJS, Piattella OF. Reconstructing f(R, T) gravity from holographic dark energy. *Int J Mod Phys D.* 2012;21(3).
8. Weinberg S. A Model of Leptons. *Phys Rev Lett.* 1967;19:1264.
9. Misner CW. Transport Processes in the Primordial Fireball. *Nature.* 1967;214:40-41.
10. Misner CW. The Isotropy of the Universe. *Astrophys J.* 1968;151:431.
11. Belinski VA, Khalatnikov M. *Sov Phys.* 1976;42:205.
12. Murphy GL. *Phys Rev D.* 1978;8:4231.
13. Setaren MR, Sheykh A. Statefinder diagnostic and stability of modified gravity consistent with holographic and new agegraphic dark energy. *Int J Mod Phys. D.* 2010;19:171.

14. Adhav KS. *Bulg. J Phys.* 2007;34:260.
15. Reddy DRK, Santikumar R, Naidu R. L. Bianchi type-III cosmological model in $f(R, T)$ theory of gravity. *Astrophys Space Sci.* 2012;342(1):249–252.
16. Reddy DRK, Naidu RL, Satyanarayana B, et al. Cosmological Model in $f(R, T)$ Gravity. *Int J Theor Phys.* 2012;51(1):3222–3227.
17. Shamir MF, Adil Jhangeer, Akhlaq Ahmad Bhatti. Exact Solutions of Bianchi Types I and V Models in $f(R, T)$ Gravity. 2012;16.
18. Chaubey R, Shukla AK. A new class of Bianchi cosmological models in $f(R, T)$ gravity. *Astrophys Space Sci.* 2012;343(1):415–422.
19. Shri Ram, Priyanka. Some Kaluza–Klein cosmological models in $f(R, T)$ gravity theory. *Astrophys Space Sci.* 2013;347(2):389–397.
20. Shri Ram et al. *Pramana. J Phys.* 2013;81:67.
21. Chandel S, Shri Ram. *Indian J Phys.* 2013;87:1283.
22. Kiran M, Reddy DRK. Non-existence of Bianchi type-III bulk viscous string cosmological model in $f(R, T)$ gravity. *Astrophys Space Sci.* 2013;346:521.
23. Shri Ram, Priyanka. Bianchi types I and V bulk viscous fluid cosmological models in $f(R, T)$ gravity theory. *Cent Eur J Phys.* 2014(10);12:744–754.
24. Sahu SK. *Chin J Phys.* 2017;55:862.
25. Sahoo PK, LRS Bianchi type-I cosmological model with constant deceleration parameter in $f(R, T)$ gravity. *Int J Geometric Methods in Modern Phys.* 2017;4:1750097.
26. Sahoo PK, Sivakumar M. *Astrophys Space Sci.* 2015;357:60.
27. Sahoo PK, Parbati Sahoo, Binaya K Bishi, S Aygünet al: Magnetized strange quark model with Big Rip singularity in $f(R, T)$ gravity. *Mod Phys Letter.* 2017;32.
28. Sahoo PK. *Can J Phys.* 2014;2:1062.
29. Sahoo PK, Mishra B, Parbati Sahoo, et al. Bianchi type string cosmological models in $f(R, T)$ gravity et al: *Eur Phys J Puls.* 2016;131:333.
30. Sahoo PK, *Fortschr Phys.* 2018;64: 414.
31. Sahoo PK, Parbati Sahoo, Binaya K. Bishi, et al. Magnetized strange quark matter in $f(R, T)$ gravity with bilinear and special form of time varying deceleration parameter. *New Astronomy.* 2018;60:80.
32. Memeni D. *Astrophys Space Sci.* 2017;362:148.
33. Reddy DRK. *Astrophys Space Sci.* 2012;342:249.
34. Poplowski NJ. *A Lagrangian description of interacting dark energy.* USA: Cornell University Library; 2006.
35. Berman MS. *Nuovo Cimento B.* 1983;74:182.