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Remote state preparation with quantum remote control

Abstract

We put forward two novel and perfect protocols for preparing an arbitrary single–particle state and simultaneously implementing a rotation operator or an arbitrary unitary operator on that particle. There are schemes for either only teleporting particles or only remotely controlling quantun particles. If one has to remotely control a prepared quantum, then he/she has to first do remote state preparation and then executes the remote control on the prepared quantum. Both operations ware done separately on two sets of entanglements. However, this intuitive solution is inefficient because many resources are wasted. Thus, we attempt to complete both operations by sharing one three–particle Greenberger–Horne–Zeilinger (GHZ) state. Comparing our two protocols with the corresponding intuitive solutions, respectively, the results show that our protocols are more efficient than the corresponding intuitive solutions. Only Pauli operators, controlled–NOT gate, Bell–state measurement, single–qubit measurement, Einstein–Podolsky–Rosen (EPR) state and GHZ state are used in our schemes, so these schemes are easily realized in physical experiment.

Keywords: remote state preparation, quantum remote control, rotation operator, arbitrary unitary operator, GHZ state, EPR state

Volume 2 Issue 4 - 2018

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Received: June 27, 2018 | **Published:** July 09, 2018

PACS: 03.67.Hk, 03.65.BZ

Introduction

In 1993, Bennett et al.[1] first presented the quantum teleportation, which is one of the ingenious applications in the quantum information field.[2,3] As extensions of the teleportation protocol, two recognized versions are the remote state prepartion (RSP) scheme[4–5] and the joint remote state preparation (JRSP) scheme.[6] Up to now, related works appeared widely and achieved great advancements in both theoretical[7–24] and experimental[25–26] aspects. In the above works, the quantum nonlocality of entangled particles contained in quantum states plays a very important role, and information is hidden in these quantum states, so it is called the nonlocality of quantum states.

Another kind of nonlocality comes from the quantum operator described by another kind of information, and it is also very important and useful. For example, in order to realize distributed quantum computation, Grover et al.[27] & Cirac et al.[28] have efficiently transferred not only quantum data but also quantum operators between the nodes of the network. That is, except for teleporting an arbitrary particle, the entanglement can also be employ to transmit the information of a unitary operator. In 2001, Huelga et al.[29] first proposed the concept of quantum operation teleportation, in which a unitary operator is transferred on a qubit, would function in a manner similar to that of a remote control apparatus, and so we shall also refer to it as quantum remote control. Since then, various quantum remote control protocols[30–35] have been presented.

To draw forth research questions, let's consider a scenario: suppose that Alice, Bob, and Charlie are three legitimate participants in spatial separation. Bob intends to help Charlie prepare an arbitrary particle $|\xi\rangle = x|0\rangle + ye^{i\theta}|1\rangle$ ($x, y \ge 0, \theta \in [0, 2\pi]$ with $x^2 + y^2 = 1$), and Alice wants to do a unitary operator on the particle $|\xi\rangle$. How to accomplish this task? The question looks very simple, but it may be useful in quantum cryptography, such as quantum private comparison, controlled remote state preparation, quantum secret sharing and so on. Specially, in the future quantum network, such an operator can be taken as a control (encryption or decyption) on the quantum information inhabiting the particle, which can be used as a key to activate some important actions such as missile emissions, quantum collective seal or unseam, remote joint destruction of quantum money, etc.

In order to achieve the above task by utilizing the existing protocols, an intuitive solution can be adopted. That is, Bob first makes use of remote state preparation method to prepare the particle $|\xi\rangle$ at Charlie's site by employing an EPR state as quantum channel, and then Alice employs the quantum remote control with the other EPR state to perform her operator on the particle $|\xi\rangle$ held by Charlie. In this paper, we attempt to provide two more efficient protocols for this question by using two different unitary operators. We aim to carry out the remote state preparation and quantum remote control simultaneously by sharing one three–qubit GHZ state.

The proposed protocols

Let us consider the following situation: there are three legitimate participants Alice, Bob and Charlie in our scheme. Assume that Bob wants to help his coworker, Charlie, prepare an arbitrary single–qubit state taking the general form

$$|\xi\rangle_b = (x|0\rangle + ye^{i\theta}|1\rangle)_b, \tag{1}$$

where $|0\rangle$ and $|1\rangle$ are two eigenstates of a single–qubit, the real coefficients $x, y \ge 0$ and $\theta \in [0, 2\pi]$ with the normalization condition $x^2 + y^2 = 1$. Bob knows the information about the qubit $|\xi\rangle_b$, whereas it is unknown to his coworkers. At the same time, Alice intends to remotely perform a unitary operator U on $|\xi\rangle_b$.

In order to complete the above tasks, let Alice, Bob and Charlie pre–share a three–qubit GHZ state of the form

$$|\vartheta\rangle_{ABC} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{ABC}, \tag{2}$$

where Alice, Bob and Charlie have the first, the second and the third qubits, respectively. The proposed protocol is described in the following steps:

Step 1: Bos first takes an ancillary qubit B' in initial state $|0\rangle_{B'}$, then performs a $CNOT_{BB'}$ on qubits B and B', where $CNOT_{XY}$ s the Controlled–NOT gate acting on two qubits Y and Y as $CNOT_{XY}|i\rangle_X|j\rangle_Y = |i\rangle_X|i\oplus j\rangle_Y$ with $\oplus$ an addition mod 2. After those actions, qubi $|B'\rangle$ became entangled with those in state (3), i.e., $|\mathcal{G}\rangle_{ABC}|0\rangle_{B'} \to |\mathcal{G}'\rangle$:

$$|\mathcal{G}'\rangle = \frac{1}{\sqrt{2}}(|0000\rangle_{ABB'C} + |1111\rangle_{ABB'C}). \tag{3}$$

Note that, though $|\mathcal{G}'\rangle$ is a 4–qubit entangled state, the actual non–local resource cots just 3 qubits because the entanglement of qubit B' with those in state (3) is made locally.

Step 2: Bob performs a projective measurement on her qubit $B(PM_B)$ with the basis $\{|\xi_1\rangle, |\xi_2\rangle\}$:

$$|\xi_1\rangle = x|0\rangle + y|1\rangle, |\xi_2\rangle = x|1\rangle - y|0\rangle. \tag{4}$$

According to the measurement postulate of quantum mechanics, the $|\mathcal{G}'\rangle$ can be rewritten in terms of Bob's measurement basis as

$$|\mathcal{G}'\rangle = \frac{1}{\sqrt{2}}[|\xi_1\rangle_B(x|000\rangle + y|111\rangle)_{AB'C} + |\xi_2\rangle_B(-y|000\rangle + x|111\rangle)_{AB'C}]. \tag{5}$$

From Equation (5) it is easily verified that Bob's PM_B will project the joint state of qubits A, B' and C onto one of the two possible states

$$x|000\rangle_{AB'C} + y|111\rangle_{AB'C}, y|000\rangle_{AB'C} - x|111\rangle_{AB'C} \tag{6}$$

with equal probability of $1/2$. The next stage work is tricky in the sense that the feed–forward measurement strategy will be exploited. Concretely, in this stage Bob's correct action depends essentially on the measurement outcomes of the preceding stage. Namely, if the result is $|\xi_1\rangle_B$, his needs to apply to his qubit B' the following unitary operator

$$U_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & e^{-i\theta} \\ 1 & -e^{-i\theta} \end{pmatrix}, \tag{7}$$

which is written in the computational basis $\{|0_{B'}\rangle, |1_{B'}\rangle\}$, whereas the result is $|\xi_2\rangle_B$, then the correct unitary operator should be

$$U_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\theta} & 1 \\ e^{-i\theta} & -1 \end{pmatrix}. \tag{8}$$

That is, using the basis

$$|\eta_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{-i\theta}|1\rangle), |\eta_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle - e^{-i\theta}|1\rangle)$$

and the basis

$$|\eta_1'\rangle = \frac{1}{\sqrt{2}}(e^{-i\theta}|0\rangle + |1\rangle), |\eta_2'\rangle = \frac{1}{\sqrt{2}}(e^{-i\theta}|0\rangle - |1\rangle),$$

expression (6) can be rewritten as

$$\begin{aligned} x|000\rangle|_{AB'C} + y|111\rangle_{AB'C} &= \frac{1}{\sqrt{2}}\{|\eta_1\rangle_{B'}(x|00\rangle + ye^{i\theta}|11\rangle)_{AC} \\ &\quad + |\eta_2\rangle_{B'}(x|00\rangle - ye^{i\theta}|11\rangle)_{AC}\}, \\ y|000\rangle_{AB'C} - x|111\rangle_{AB'C} &= \frac{1}{\sqrt{2}}\{|\eta_1'\rangle_{B'}(ye^{i\theta}|00\rangle - x|11\rangle)_{AC} \\ &\quad + |\eta_2'\rangle_{B'}(ye^{i\theta}|00\rangle + x|11\rangle)_{AC}\}. \end{aligned} \tag{9}$$

After measuring on the ancillary qubit $B'(PM_{B'})$, he broadcasts two bits classical information about his measurement results to Alice and Charlie. What is interesting is that it is not necessary for his to send secret massages. Instead, he just needs to broadcast his outcomes via any public media since these outcomes in fact mean nothing to any outside parties. It is worth noting that in the second step Bob used the forward feedback measurement techniques. Namely, the choice of basis for measuring qubit B depends essentially on the outcomes of the prior measurements on qubit B, respectively.

From Equation (9) we can see that after the PM_B and $PM_{B'}$, the qubits held by the Alice and Charlie collapse correspondingly into the following entangled states:

$$\begin{aligned} |\mu^{\pm}\rangle_{AC} &:= x|00\rangle_{AC} \pm ye^{i\theta}|11\rangle_{AC}, \\ |\nu^{\pm}\rangle_{AC} &:= x|11\rangle_{AC} \pm ye^{i\theta}|00\rangle_{AC}. \end{aligned} \tag{10}$$

To be precise, the complete relation between the outcomes of $PM_{B'}$ and $PM_{B'}$ of Bob and the combined state of Alice and Charlie is shown in Table 1.

Table 1 Relation among bob's measurement results (PM_B & $PM_{B'}$), the combined states of alice and charlie after the measurement of bob.

PM_B	$PM_{B'}$	Combined state of alice and charlie
$\|\xi_1\rangle_B$	$\|\eta_1\rangle_{B'}$	$\|\mu^+\rangle_{AC} = x\|00\rangle_{AC} + ye^{i\theta}\|11\rangle_{AC}$
$\|\xi_1\rangle_B$	$\|\eta_2\rangle_{B'}$	$\|\mu^-\rangle_{AC} = x\|00\rangle_{AC} - ye^{i\theta}\|11\rangle_{AC}$
$\|\xi_2\rangle_B$	$\|\eta_1'\rangle_{B'}$	$\|\nu^-\rangle_{AC} = x\|11\rangle_{AC} - ye^{i\theta}\|00\rangle_{AC}$
$\|\xi_2\rangle_B$	$\|\eta_2'\rangle_{B'}$	$\|\nu^+\rangle_{AC} = x\|11\rangle_{AC} + ye^{i\theta}\|00\rangle_{AC}$

The remaining steps are described separately according to different operators U, see the subsection 2.1 and 2.2.

The proposed protocol with remote rotation

In this subsection, consider that Alice wants to remotely perform a rotation operator $U = R_z(\omega)$ (about the Z axis) on $|\xi\rangle_b$, where $\omega \in [0, 2\pi]$,

$$R_z(\omega) \equiv e^{-i\omega z/2} = \cos\frac{\omega}{2}I - i\sin\frac{\omega}{2}Z = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\omega/2} & 0 \\ 0 & e^{i\omega/2} \end{pmatrix}. \tag{11}$$

This will produce a new quantum state:

$$|\varphi\rangle_b = R_z(\omega)|\xi\rangle_b = xe^{-i\omega/2}|0\rangle_b + ye^{i(\theta+\omega/2)}|1\rangle_b. \tag{12}$$

Step 3: According to Bob's measurement outcomes PM_B $PM_{B'}$, Alice performs the rotation $R_z(\omega)$ of an angle ω on the first qubit A of GHZ state. If PM_B $PM_{B'}$ is $|\xi_1\rangle_B|\eta_j\rangle_{B'}$ ($j = 1, 2$), Alice performs $R_z(\omega)$ on qubit A. Otherwise, Alice performs $R_z(-\omega)$ on A. Then, Alice measures A with X basis $\{|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\}$ to obtain the measurement outcome, PM_A, which will be sent to Charlie via a classical channel.

Step 4: After hearing the messages from Alice and Bob, Charlie can perform a corresponding unitary operation on qubit C to adjust the state to the one given in Equation (12). The one–to–one correspondence between Charlie's operations and the measurement outcomes of Alice and Bob is shown in Table 2.

Table 2 The one–to–one correspondence between charlie's unitary operations and the measurement outcomes of alice and bob.

PM_A	PM_B	$PM_{B'}$	Operations of charlie
$\|+\rangle_A$	$\|\xi_1\rangle_B$	$\|\eta_1\rangle_{B'}$	I
$\|-\rangle_A$	$\|\xi_1\rangle_B$	$\|\eta_1\rangle_{B'}$	σ_z
$\|\xi_1\rangle_B$	$\|\xi_1\rangle_B$	$\|\eta_2\rangle_{B'}$	σ_z
$\|\xi_1\rangle_B$	$\|\xi_1\rangle_B$	$\|\eta_2\rangle_{B'}$	I
$\|+\rangle_A$	$\|\xi_2\rangle_B$	$\|\eta_1'\rangle_{B'}$	$i\sigma_y$
$\|-\rangle_A$	$\|\xi_2\rangle_B$	$\|\eta_1'\rangle_{B'}$	σ_x
$\|+\rangle_A$	$\|\xi_2\rangle_B$	$\|\eta_2'\rangle_{B'}$	σ_x
$\|\xi_2\rangle_B$	$\|\xi_2\rangle_B$	$\|\eta_2'\rangle_{B'}$	$i\sigma_y$

For example, let $PM_{B'}$ $PM_{B'}$ is $|\xi_2\rangle_B\,|\eta_1'\rangle_{B'}$, the the state of the composite quantum system of qubits A and C is $|\nu^-\rangle_{AC} = x|11\rangle_{AC} - ye^{i\theta}|00\rangle_{AC}$. Alice needs to perform $R_z(-\omega)$ on A, then the state $|\nu^-\rangle_{AC}$ becomes the following form

$$[R_z(-\omega)\otimes I]|\nu^-\rangle_{AC} = xe^{-i\omega/2}|11\rangle_{AC} - ye^{i\theta}e^{i\omega/2}|00\rangle_{AC}.$$

Suppose Alice's measurement outcome is $|+\rangle_A$, then the state of qubit C held by Charlie will be $xe^{-i\omega/2}|1\rangle_C - ye^{i\theta}e^{i\omega/2}|0\rangle_C$. Once Charlie performs a unitary operation $i\sigma_y$ on qubit C, he will obtain the state in Equation (12), and the quantum task has been completed.

Remark 1: In the intuitive solution corresponding to the 2.1 subsection, Bob first has to help Charlie prepare the state $|\xi\rangle_b$ which requires an EPR state, a Controlled–NOT gate, two sing–qubit projective measurements, a corresponding operator and 2 bits of classical message. After that, Alice employs the quantum remote control to perform her operation on the qubit $|\xi\rangle_b$ held by Charlie.

In the remote control protocol, Alice and Charlie also have to pre–share a Bell state

$$|\phi^+\rangle_{A'C'} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A'C'}, \tag{13}$$

where the subscripts A' and C' represent the first and the second qubit of the Bell state belonged to Alice and Charlie, respectively.

Charlie first performs a $CNOT_{C'b}$ on qubits C' and b, and measures b with Z basis. The composite state becomes

$$|\chi\rangle_{A'C'b} = \frac{1}{2}[(x|00\rangle + ye^{i\theta}|11\rangle)_{A'C'}|0\rangle_b + (x|11\rangle + ye^{i\theta}|00\rangle)_{A'C'}|1\rangle_b]. \tag{14}$$

Then, Charlie sends a one–bit classical message to Alice about his measurement. If the measurement outcome is $|0\rangle$, Alice and Charlie will do nothing. Otherwise, they both perform σ_x on their qubits. Thus the state they shared becomes

$$|\chi'\rangle_{A'C'} = (x|00\rangle + ye^{i\theta}|11\rangle)_{A'C'}. \tag{15}$$

Now Alice performs $R_z(\omega)$ on her qubit A' and measures it with X basis. Alice sends a one–bit classical message to Charlie about her measurement outcome. According to Alice's message, Charlie performs a corresponding unitary operation to complete the remote control process. Hence, the remote control requires an EPR state, a Controlled–NOT gate, two single–qubit projective measurements, 4 unitary operators and 2 bits of classical message in 2 rounds of transmission.

On the other hand, in the proposed protocol, a GHZ state is used to prepare the qubit as well as remotely control the qubit. In total, the proposed protocol requires a three–qubit GHZ state, a Controlled NOT gate, three sing–qubit projective measurement, 2 unitary operators and 3 bits of classical message in 2 rounds of transmission. The contrast between the intuitive solution and the proposed protocol is given in Table 3. From Table 3, we can see that our protocol is more efficient than the intuitive solution.

Table 3 The comparison of the intuitive solution to the proposed protocol for the remote rotation.

	Intuitive solution	Our protocol
Entanglement state	2 EPR states	1 GHZ state
Unitary operator	5	2
Number of rounds in classical message	3	2
Classical message	4 bits	3 bits
Number of Controlled NOT gate	2	1
Number of single–qubit measurement	4	3

The proposed protocol with remote arbitrary unitary operator

In the subsection, assume that Alice intends to remotely perform an arbitrary unitary operator U on $|\xi\rangle_b$, where

$$U = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}, (|a|^2 + |b|^2 = 1), \tag{16}$$

which is unknown to three distant parties Alice, Bob and Charlie. Alice possesses a device able to perform the operator U on her single qubit that is referred to as the control. It will produce a new quantum state:

$$|\varphi'\rangle_b = U|\xi\rangle_b = xU|0\rangle_b + ye^{i\theta}U|1\rangle_b. \tag{17}$$

Step 3': After hearing Bob's message, Charlie implement a projective measurement on his qubit C with X basis $\{|+\rangle, |-\rangle\}$, and sends the measurement outcome to Alice with 1 bit of classical message. According to Charlie's message, Alice performs one of the unitary operations $\{I, \sigma_z, \sigma_x, i\sigma_y\}$ on their qubit A so that the state of qubit A becomes

DOI: 10.15406/paij.2018.02.00101

$$|\tau\rangle_A = x|0\rangle_A + ye^{i\theta}|1\rangle_A. \tag{18}$$

Step 4': To complete quantum tasks, an auxiliary maximally entangled state is introduced

$$|\psi\rangle_{A'C'} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A'C'}, \tag{19}$$

where qubit A' is located in Alice's side, while qubit C' is located in Charlie's side. Alice use the device to implement the operator U such as in Equation (16) on her qubit A. With this, Equation (19) can be evolved as

$$U|\tau\rangle_A = xU|0\rangle_A + ye^{i\theta}U|1\rangle_A. \tag{20}$$

Using the Bell basis $\{|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), |\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)\}$ for Alice's qubits A and A', the composite quantum system of qubits A, A' and C' can be rewritten as

$$\begin{aligned} U|\psi\rangle_A \otimes |\psi\rangle_{A'C'} &= \frac{1}{2}\{|\Phi^+\rangle_{AA'}(xU|0\rangle_{C'}) + ye^{i\theta}U|1\rangle_{C'}) \\ &+ |\Phi^-\rangle_{AA'}(xU|0\rangle_{C'} - ye^{i\theta}U|1\rangle_{C'}) \\ &+ |\Psi^+\rangle_{AA'}(xU|1\rangle_{C'} + ye^{i\theta}U|0\rangle_{C'}) \\ &+ |\Psi^-\rangle_{AA'}(xU|1\rangle_{C'} - ye^{i\theta}U|0\rangle_{C'})\}. \end{aligned} \tag{21}$$

After that, Alice carries out a Bell measurement on her two qubits A and A', and informs the result to Charlie with 2 bits of classical message. After receiving information from Alice, Charlie always end up holding the following correct transformed state

$$xU|0\rangle_{C'}) + ye^{i\theta}U|1\rangle_{C'}$$

by performing one of the unitary operations $\{I, \sigma_z, \sigma_x, i\sigma_y\}$ on his qubit U. That is, the perfect protocol of remote quantum state preparation with remote arbitrary unitary operator U on a GHZ state is successfully executed via entanglement swapping.

Remark 2: In the intuitive solution corresponding to the 2.2 subsection, after Bob helps Charlie prepare the quantum state $|\xi\rangle_b$, Alice needs to employ the quantum remote control to perform her operation U on qubit $|\xi\rangle_b$ held Charlie.

In this remote control scheme, Alice and Charlie also have to pre–shared two EPR states

$$|\psi'\rangle_{12} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{12}, |\psi'\rangle_{34} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{34}, \tag{22}$$

where the qubits 1 and 3 belong to Alice, the qubits 2 and 4 to Charlie, respectively. Charlie implements a Bell measurement on his qubits b and 2 with basis $\{|\Phi^{\pm}\rangle_{b2}, |\Psi^{\pm}\rangle_{b2}\}$, and sends a two–bit classical message to Alice about his measurement. After that, Alice makes one of the unitary operations $\{I, \sigma_z, \sigma_x, i\sigma_y\}$, so that the state of qubit 1 becomes $|\tau'\rangle_1 = x|0\rangle_1 + ye^{i\theta}|1\rangle_1$. Then she use the device to perform the operation U such as in Equation (17) on her qubit 1, the state $|\tau'\rangle_1$ transforms into

$$U|\tau'\rangle_1 = xU|0\rangle_1 + ye^{i\theta}U|1\rangle_1. \tag{23}$$

Subsequently, Alice executes a Bell measurement on her qubits 1 and 3 with basis $\{|\Phi^{\pm}\rangle_{13}, |\Psi^{\pm}\rangle_{13}\}$, and informs the result to Charlie using a classical channel. According to Alice's two classical bits, Charlie carries out one of operations $\{I, \sigma_z, \sigma_x, i\sigma_y\}$ on his qubit 4. As a result of this procedure, Charlie always end up holding the following correct transformed state

$$xU|0\rangle_4 + ye^{i\theta}U|1\rangle_4 = U(x|0\rangle_4 + ye^{i\theta}|1\rangle_4). \tag{24}$$

From the above, the remote control requires two EPR states, two Bell measurements, 3 unitary operators and 4 bits of classical message in 2 rounds of transmission. On the other hand, our protocol in subsection 2.2 requires an EPR state, a three–qubit GHZ state, a Controlled–NOT gate, 3 single–qubit projective measurements, a Bell measurement, 3 unitary operators and 5 bits of classical message in 3 rounds of transmission. Comparing the intuitive solution with my scheme, the results are shown in Table 4. As you can see from table 4, the proposed scheme here is more effective than the intuitive solution.

Table 4 The comparison of the intuitive solution to our protocol for an arbitrary unitary operator.

	Intuitive solution	Our protocol
Entanglement state	3 EPR states	1 EPR state and 1 GHZ state
Unitary operator	4	3
Number of rounds in classical message	3	3
Classical message	6 bits	5 bits
Number of Controlled NOT gate	1	1
Number of single–qubit measurement	2 single and 2 Bell	3 single and 1 Bell

Discussion and conclusion

Quantum nonlocality plays a central role in quantum information. Many novel results have been obtained by using quantum nonlocality of entangled particles. Using quantum nonlocality of GHZ state and EPR state, we have proposed two protocols of remote state preparation with remote control in this paper. From the depiction of our schemes, one can readily see that our schemes are deterministic, that is, the remotely controlling a prepared quantum by using two different operators respectively are conclusively fulfilled with unit probability. In addition, the use of forward feedback measurement techniques in the process of Bob's measurements ensures that the probability of success of our protocols is 1, that is, our protocols are Perfect.

Each of our protocols can be modified to remotely control a joint remote state preparation. In fact, using the four–particle GHZ–type state to replace the three–particle GHZ state in our schemes, we can obtain

$$|\mathcal{G}\rangle = \frac{1}{\sqrt{2}}(|0000\rangle_{ABB'C} + |1111\rangle_{ABB'C}),$$

where, two senders Bob[1] and Bob[2] own particles B and B', respectively. And their colleagues, Alice and Charlie, hold particles A and C respectively. Obviously, the state $|\mathcal{G}\rangle$ is exactly the state shown in equation (3). Therefore, in our schemes, we can use Bob[1] and Bob[2] instead of Bob to do the work of corresponding particles B and B' respectively, then the schemes of joint remote state preparation with remote control on a four–particle GHZ–type state are obtained.

Now let us consider the security issue of the schemes via simple analyses. It depends thoroughly on whether the three legitimate users have securely shared the entanglement/entanglements beforehand. By virtue of the same matured check strategies in treating other similar quantum tasks,[36,37] then any potential inside cheating or outside evil attack can be easily detected. For simplicity, here we do not repeat it. This means that our protocols are conclusively secure, too. In addition, as mentioned in our schemes, except the owner of the operator U or state $|\xi\rangle$. Consequently, the rest of the participants are unknown the information of U and B in advance. neither participant can solely determine in priori to finally accomplish the construction, which can be viewed as a scheme security protection in another manner.

Essentially, our schemes are an ordering hybrid of remote quantum state preparation and the remotely control with demanding operators, in which the remote control is loaded in the remote quantum state preparation process. Their execution procedures seem to be similar to those of operator sharing, but essentially different. For example, their conditions, goals, behavior pattern, characteristics and etc. are different.

In summary, we use quantum nonlocality of a three–particle GHZ state to implement both the remote preparation of an arbitrary single–particle state and the remote rotation on that quantum state simultaneously. Moreover, using quantum nonloclity of EPR state and GHZ state, we can also perform both the preparation of an arbitrary single–particle state and an arbitrary unitary operator on that state at once. These schemes are safe and perfect, and may be useful in quantum cryptography, such as controlled remote state preparation, quantum private comparison, and so on. our protocols can be modified to the protocols of remotely control a joint remote state preparation, respectively, and so it improves the security of the protocols. By comparing our schemes with the intuitive solutions, we find that the proposed protocols are more efficient than the intuitive solutions. One of the reasons for this result is that intuitive solution requires two sets of quantum entanglement to do two operations (remote state preparation and remote control) respectively, resulting in waste of quantum resources, whereas a GHZ state is shared in our schemes (when necessary, an auxiliary EPR state is introduced). In our protocols, it is necessary for single–qubit basis measurement, Bell measurement, controlled–NOT gate, EPR state, GHZ state and simple unitary operations as well as classical communication, therefore our schemes are easy to implement physically.

Acknowledgments

This work is supported by Natural Science Foundation of China (Grant No. 11071178, 11671284) and Sichuan Province Education Department Scientific Research Innovation Team Foundation (NO. 15TD0027).

Conflict of interest

Authors declare there is no conflict of interest.

References

1. Bennett CH, Brassard G, Crepeau C, et al. Teleporting an unknown quantum state via dual classical and Einstein–Podolsky–Rosen channels. *Physical Review Letters.*1993;70(13):1–5.
2. Nielsen MA, Chuang IL. *Quantum computation and quantum information*. UK: Cambridge University Press; 2000.
3. Imre S, Gyongyosi L. *Advanced quantum computing and communications–An engineering apporach*. USA: John Wiley & Sons; 2012.
4. Pati AK. Minimum classical bit for remote preparation and measurement of a qubit. *Physical Review A*. 2000;63(63):94–98.
5. Bennett CH, DiVincenzo DP, Shor PW, et al. Erratum: Remote State Preparation. *Physical Review Letters*. 2002;88(9):1.
6. Xia Y, Song J, Song HS. Multiparty remote state preparation. *Journal of Physics B Atomic Molecular & Optical Physics*. 2007;40(18).
7. Peng JY, Mo ZW. Several teleportation schemes of an arbitrary unknown multi–particle state via different quantum channels. *Chinese Physics B*. 2013;22(5).
8. Wang XW, Zhang DY, Tang SQ, et al. Multipary hierarchical quantum information splitting. *Journal of Physics B Atomic Molecular & Optical Physics*. 2011;44(3):35505–35508.
9. Peng JY, Mo ZW. Quantum sharing an unknown multiparticle state via POVM. *International Journal of Theoretical Physics*. 2013;52(2):620–633.
10. Bennett CH, Wiesner SJ. Communication via one– and two–particle operators on Einstein–Podolsky–Rosen states. *Physical Review Letters*. 1992;69(20).
11. Peng JY, Luo MX, Mo ZW. Quantum taks with non–maximally quantum channels via positive operator–valued measurement. *International Journal of Theoretical Physics*. 2013. 52(1):253–263.
12. Nguyen BA. Quantum dialogue. *Physics Letters A*. 2004;328(1):6–10.
13. Peng JY, Bei MQ, Mo ZW. Hierarchical and probabilistic quantum state sharing via non–maximally entangled$|\chi\rangle$state. *Chinese Physics B*. 2014;23(1).
14. Yand YG, Wen QY. Circular threshold quantum secret sharing. *Chinese Physics B*. 2008;17(2):419–423.
15. Peng JY, Mo ZW. Hierarchical and probabilistic quantum state sharing via non–maximally entangled four–qubit cluster state. *International Journal of Quantum Information*. 2013;11(01).
16. Chen XB, Wen QY, Sun ZX, et al. Deterministic and exact entanglement teleportation via the W state∗. *Chinese Physics B*. 2010;19:1–5.
17. Peng JY, Bei MQ, Mo ZW. Bidirectional quantum states sharing. *International Journal of Theoretical Physics*. 2016;55(5):2481–2489.
18. Chen XB, Ma SY, Su Y, et al. Controlled remote state prepartation of arbitrary two and three–qubit states via the Brown state. *Quantum Information Processing*. 2012;11(6):1653–1667.
19. Peng JY, Luo MX, Mo ZW, et al. Flexible deterministic joint remote state preparation of some states. *International Journal of Quantum Information*. 2013;11(4).
20. Peng JY, Luo MX, Mo ZW. Joint remote state preparation of arbitrary two–particle states via GHZ–type states. *Quantum Information Precessing*. 2013;12(7):2325–2342.
21. Peng JY, Luo MX, Mo ZW, et al. Joint remote prepartion of arbitrary three–qubit states. *Modern Physics Letters B*. 2013;27(22).
22. An NB. Joint remote state preparation via W and W–type states. *Optics Communications*. 2010;283(20):4113–4117.
23. Peng JY, Bei MQ, Mo ZW. Bidirectional Controlled joint remote state prepartion. *Quantum Information Processing*. 2015;14(11):4263–4278.
24. Peng JY, Bei MQ, Mo ZW. Joint remote state preparation of a four–demensional quantum state. *Chinese Physics Letters*. 2014;31(1).
25. Bouwmeester D, Pan JW, Mattle K, et al. Experimental quantum teleportation. *Nature*. 1997;390(390):575–579.

26. Ma XS, Herbst T, Scheidl T, et al. Quantum teleportation over 143 kilometres using active feed–forward. *Nature.* 2012;489(7415):269–273.

27. Grover LK. *Quantum Telecomputation*. USA: Ads Abstract Service; 1997.

28. Cirac JI, Ekert A, Huelga SF, et al. Distributed quantum computation over noisy channels. *Physical Review A*. 1999;59(6):4249–4254.

29. Huelga SF, Vaccaro A, Chefles A, et al. Quantum remote control: Teleportation of unitary operations. *Physical Review A*. 2001;63(4).

30. Chen LB, Lu H, Liu YH. Implementing remotely a single–qubit rotation operation by three–qubit entanglement. *Chinese Physics.* 2005;14(7).

31. Yao CM. Quantum remote control of unitary operations on a qubit of pure entangled states. *Chinese Physics Letters.* 2006;23(3):545–547.

32. Zheng YZ, Ye P, Guo GC. Probabilistic implementation of non–local CNOT operation and Entanglement purification. *Chinese Physics Letters.* 2004;21(1):9–11.

33. Fan QB, Liu DD. Controlled remote implementation of partially unknown quantum operation. *Science in China Series G: Physics, Mechanics & Astronomy.* 2008;51(11):1661–1667.

34. Ye BL, Liu YM, Liu XS, et al. Remotely sharing a single–qubit operation with a five–qubit genuine state. *Chinese Physics Letters.* 2013;30(2).

35. Peng J. Tripartite operation sharing with five–qubit Brown state. *Quantum Information Processing.* 2016;15(6):2465–2473.

36. Yang CP, Guo GC. Disentanglement–free state of two pairs of two–level atoms. *Physical Review A.* 1999;59(6):4217–4222.

37. Deng FG, Long GL, Liu XS. Two–step quantum direct communication protocol using the Einstein–Podolsky–Rosen pair block. *Physical Review A*. 2003;68(4).

Citation: Jia–Yin P, Ming–Qiang B, Zhi–Wen M. Remote state preparation with quantum remote control. *Phys Astron Int J.* 2018;2(4):290–295.
DOI: 10.15406/paij.2018.02.00101