Conceptual Paper

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Standard quantum algorithms and the property of a certain function

Abstract

We discuss a new mathematical structure for standard quantum algorithms. It says a certain property in case of a special function f that the relation $f(x) = f(-x)$ holds. That is, the particular mathematical structure of the special function is that $f(x)$ should be even if we assume $|-x\rangle = -|x\rangle$.

Keywords: quantum algorithms, quantum computation

Volume 2 Issue 2 - 2018

Koji Nagata,[1] Tadao Nakamura[2]

[1]Department of Physics, Korea Advanced Institute of Science and Technology, Korea
[2]Department of Information and Computer Science, Keio University, Japan

Correspondence: Koji Nagata, Department of Physics, Korea Advanced Institute of Science and Technology, Daejeon 34141, Korea, Email ko_mi_na@yahoo.co.jp

Received: March 12, 2017 | **Published:** April 18, 2018

Introduction

In 1985, the Deutsch–Jozsa algorithm was discussed.[1–3] In 1993, the Bernstein–Vazirani algorithm was published.[4,5] This work can be considered an extension of the Deutsch–Jozsa algorithm. In 1994, Simon's algorithm[6] and Shor's algorithm[7] were discussed. In 1996, Grover et al.[8] provided the highest motivation for exploring the computational possibilities offered by quantum mechanics.

In this short contribution, we discuss a new mathematical structure for standard quantum algorithms. They say a certain property in case of a special function f that the relation $f(x) = f(-x)$ holds.

Mathematical structure for standard quantum algorithms

We discuss a new mathematical structure for standard quantum algorithms in case of a special function f. Let us suppose that we are given the following function

$$f:\{-(2^N-1),-(2^N-2),\ldots,2^N-2,2^N-1\}\to\{0,1,\ldots,2^N-2,2^N-1\}. \tag{1}$$

We shall assume that $f(y)\geq 0$. Let us introduce a function $g(x)$ that transforms binary strings into positive integers. We also define $g^{-1}(f(g(x))) = F(x)$. We shall assume, for the time being, that the given function is even. Thus, we have

$$\begin{aligned} F(x) &= F(-x)\in\{0,1\}^N \\ x &\in \{0,1\}^N. \end{aligned} \tag{2}$$

We see that the condition (2) holds in standard quantum algorithms.

What the function $f(x)$ does in (1) is to map a set of discrete values onto another one. In (2), we assume that x is the binary representation of one element. x will be given by a binary string belonging to the Cartesian product $\overbrace{\{0,1\}\times\{0,1\}\times\ldots\times\{0,1\}}^{N}$, for instance, $x = (0,1,1,0,0,1,\ldots,1)$. We then define $-x$ as $-(0,1,1,0,0,1,\ldots,1)$.

Throughout the discussion, we omit any normalization factor. Let us suppose $|-x\rangle = -|x\rangle$. The input state is

$$|\psi_1\rangle = |\overbrace{0,0,\ldots,0,1}^{N}\rangle|\overbrace{1,1,\ldots,1}^{N}\rangle. \tag{3}$$

The function F is evaluated by using the following unitary $2N$ qubits gate

$$U_F: |x,z\rangle \to |x, z+F(x)\rangle \tag{4}$$

with

$$\begin{aligned} &U_F: |x,z\rangle \to |x,z+F(x)\rangle \\ &\Leftrightarrow -|x,z\rangle \to -|x,z+F(x)\rangle \\ &\Leftrightarrow |-x,z\rangle \to |-x,z+F(x)\rangle \\ &\Leftrightarrow |-x,z\rangle \to |-x,z+F(-x)\rangle \end{aligned} \tag{5}$$

And employing the fact that $F(x) = F(-x)$. Here, $z+F(x) = (z_1\oplus F_1(x), z_2\oplus F_2(x),\ldots,z_N\oplus F_N(x))$ (the symbol $\oplus$ indicates addition modulo 2).

We have the following fact

$$U_F\,|\overbrace{0,0,\ldots,0,1}^{N}\rangle|\overbrace{1,1,\ldots,1}^{N}\rangle = |\overbrace{0,0,\ldots,0,1}^{N}\rangle|\overline{F(0,0,\ldots,0,1)}\rangle. \tag{6}$$

Here, for example, if we have $F(0,0,\ldots,0,1) = (0,1,1,0,0,1,\ldots,1)$, then $\overline{F(0,0,\ldots,0,1)} = (1,0,0,1,1,0,\ldots,0)$. Surprisingly the relation $F(x) = F(-x)$ is necessary for the fundamental relation (6) as shown below. From the definition in (5), we have

$$U_F\,|x\rangle|\overbrace{1,1,\ldots,1}^{N}\rangle = |x\rangle|\overline{F(x)}\rangle. \tag{7}$$

This implies for $x\to -x$, wit $x\neq 0$

$$U_F \mid -x\rangle \mid \overbrace{1,1,...,1}^{N}\rangle = \mid -x\rangle \mid \overline{F(-x)}\rangle. \quad (8)$$

We state that $\mid -x\rangle = - \mid x\rangle$. Then it follows that the minus sign on left and right hand side of (8) drop off. This implies

$$U_F \mid x\rangle \mid \overbrace{1,1,...,1}^{N}\rangle = \mid x\rangle \mid \overline{F(-x)}\rangle. \quad (9)$$

We furthermore assume such that

$$\mid P\rangle = \mid Q\rangle \Leftrightarrow P = Q. \quad (10)$$

Comparing (7) with (9) we see $\mid \overline{F(x)}\rangle = \mid \overline{F(-x)}\rangle$. Hence, we cannot avoid the following property of the function in order to maintain consistency for the fundamental relation (6)

$$\overline{F(x)} = \overline{F(-x)}. \quad (11)$$

That is, the function under study is even

$$F(x) = F(-x). \quad (12)$$

Conclusion

In conclusion, we have discussed a new mathematical structure for standard quantum algorithms. They have said a certain property in case of a special function f that the relation $f(x) = f(-x)$ holds.

Acknowledgements

We thank Professor Han Geurdes for valuable discussions.

Conflict of interest

Authors declare there is no conflict of interest.

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Citation: Nagata K, Nakamura T. Standard quantum algorithms and the property of a certain function. *Phys Astron Int J*. 2018;2(2):146–147. DOI: 10.15406/paij.2018.02.00076