Earth similarity index with two free parameters

Abstract

We have derived Earth Similarity Index (ESI) with two free parameters m and T, where m denotes the weight exponent and T denotes the threshold value. These free parameters are optimized with the consideration that the planet Mars is almost similar to the Earth. For the optimized values of free parameters, the interior-ESI, surface-ESI and ESI for some planets are calculated. The results for m= 0.8 and T= 0.8 are compared with the values obtained. We have found that the exoplanet 55 Cnc f is within 10% away from the threshold value T. The exoplanets HD 69830 c, 55 Cnc c, 55 Cnc f, 61 Vir d and HIP 57050 b are found to have ESI within 10% from the threshold value.

Keywords: earth similarity index, parameters, threshold value, planets, binocular telescope, circumstellar habitable zone

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Introduction

In 1584, Catholic Monk Giordano Bruno asserted that there are countless suns and count-less earths, and all these earths are revolving around their respective suns.[1] when he said this in public, he was removed from the Church by charging that he was against the religion. But, this fact was found true when the confirmation of the first extra-solar planet was announced by Mayor et al.[2] revolving around a sun-type 51 Pegasi. Detection of new planets is progressing continuously and as of 9 July 2015, there are 1858 exoplanets, including 468 multiple planetary systems (http://exoplanetarchive.ipac.caltech.edu/).

The number of known exoplanets is increasing each day as the detection methods used for both the ground based and space missions have improved in terms of technology and more scientists are getting interested in this field. There are many ground based and space missions to detect the exoplanets. The Search for Extra Terrestrial Intelligence (SETI) is the name of mission for searching life-forms outside the Earth. Radio telescopes having large antennas are being used to investigate the exoplanets. In Arizona, the Large Binocular Telescope Interferometer (LBTI) is being used for advanced interferometry for detection of exoplanets. These instruments are bound to advance and improve our understanding and detection of a number of exoplanets which may be similar to our Earth or not. The Kepler mission is aimed for detection of earth-like planets and the space missions such as CHEOPS (CHaracterizing ExOPlanet Satellite) and James Webb Space Telescope (JWST) are in progress. With the new generation of telescopes and missions, detection of a large number of exoplanets is expected. The exoplanets need to be classified if one is similar to the Earth or not. We, therefore, need a scheme which can be used for classification on the basis of data available to us. The data contain the information about planets, such as mass, radius, surface temperature, etc. These planetary parameters can be used to know about the probability of existing life-forms somewhere other than the Earth. The probability of existing life-forms outside the Earth is given as a concept of 'circumstellar habitable zone' by Kasting et al.[3] It suggests that the focus should be made on those worlds which can hold atmosphere and liquid water. Some life-forms are most likely to be found on those planetary bodies which have similar Earth-like conditions.

A two-tiered classification scheme of exoplanets is suggested by Schulze-Makuch et al.[1] The first tier consists of Earth Similarity Index (ESI), which decides about the worlds with respect to their similarity to the Earth. The ESI was calculated on the basis of data for exoplanets, such as mass, radius and temperature. The second tier of the scheme is the Planetary Habitability Index (PHI), based on the presence of a stable substrate, available energy, appropriate chemistry and potential for holding liquid solvent.

In the present investigation, we have derived the ESI with two free parameters. These free parameters have been optimized with the consideration that the planet Mars is almost similar to the Earth. For the optimized values of free parameters, the interior-ESI, surface-ESI and ESI for some planets are calculated. The results are compared with the available data.

Earth similarity index

The similarity index is a mathematical tool that can be applied for a set of data. It is used in various fields of science, such as Mathematics (e.g., Set Theory), Ecology (e.g., Sorensen similarity index), Computer imaging (e.g., structural similarity index) and many others.[1] This method is a measure of deviation from a reference system, usually on a scale lying between zero and one. The ESI is a quantitative measure of Earth-likeness.

The ESI, we have proposed, may be understood in the following manner. Consider a physical quantity having value x_0 on the surface of our Earth. Suppose, the value of this quantity varies from x_a to x_b, such that $x_a < x_0 < x_b$, on the Earth. Then, the percentage variations p and q are expressed as

$$p = \frac{x_{0-x_a}}{x_0} \times 100 \quad \text{and} \quad q = \frac{x_{b-x_0}}{x_0} \times 100$$

Now, we define the threshold value T as,

$$T = \left[1 - \left(\frac{x_{0-x_a}}{x_0 + x_a} \right)^m \right]^{w_a} \tag{1}$$

$$T = \left[1 - \left(\frac{x_{b-x_0}}{x_b + x_0} \right)^m \right]^{w_b} \qquad (2)$$

Where w_a and w_b denote the weight exponents and m is a free parameter. The threshold is the limiting value of ESI above which a planet is considered similar to the Earth. Further, the threshold value T can be expressed as

$$T = (1 - \frac{t}{100}) \qquad (3)$$

Where t can assume positive values between zero and 100, so that T is positive, having the value between zero and 1. Thus, we have taken T also as another free parameter. In the work of Schulze-Makuch et al.[1] t is taken as 20, so that T = 0.8, and m as 1. In our investigation, we have considered m to assume the values from 0.6 to 1.2.

From equation (1) and (2), we get

$$w_a = \frac{lnT}{\ln\left[1 - \left(\frac{p}{200 - p} \right)^m \right]} \text{ and } w_b = \frac{lnT}{\ln\left[1 - \left(\frac{q}{200 + q} \right)^m \right]} \qquad (4)$$

We finally take the weight exponent w_x as the geometrical mean of w_a and w_b. That is

$$w_x = \sqrt{w_a \times w_b} \qquad (5)$$

Here, the subscript x denotes a physical quantity. The basic ESI_x for a physical quantity x is expressed as

$$ESI_x = \left(1 - \left| \frac{x - x_0}{x + x_0} \right| \right)^{w_x} \qquad (6)$$

For the planetary property x, the terrestrial reference is x_0. The ESI_x can assume a value between zero (no similarity with the Earth) and one (identical to the Earth). We have calculated ESI_r, ESI_ρ, ESI_e and ESI_T corresponding to radius, density, escape velocity and temperature, respectively. The ESI_r and ESI_ρ for mean radius r and bulk density ρ, respectively, are used to define the interior earth similarity index ESI_I expressed as

Thus, the ESI_I is the geometrical mean of ESI_r and ESI_ρ. The ESI_e and ESI_T for escape velocity v and mean surface temperature T, respectively, are used to define the surface earth similarity index ESI_S, expressed as

$$ESI_I = \left(ESI_r \times ESI_\rho \right)^{1/2} \qquad (7)$$

$$ESI = \left(ESI_I \times ESI_S \right)^{1/2} \qquad (8)$$

Thus, the ESI_S is the geometrical mean of ESI_e and ESI_T. The ESI (sometimes called the global ESI) is expressed as

$$ESI = \left(ESI_I \times ESI_S \right)^{1/2} \qquad (9)$$

Using equations (3) and (4) in (5), we have

$$ESI = \left(ESI_r \times ESI_\rho \times ESI_e \times ESI_T \right)^{1/4}$$

Analysis

In our investigation, we have considered two values of T as 0.8 and

0.9. The value of T can never be greater than 1, as we are considering the Earth to be the most superior planet. The variations of parameters, relative to those of the mean values on the Earth, are taken as the following. The definitional limits for radius are from 0.5 to 1.5 times the Earth's radius Sotin et al.[4] The limit for the mass of an existing planet is from 0.1 to 10 times that of the Earth Gaidos et al.[5] For the density, the definitional limits are 0.7 and 1.5 times the Earth's density. The temperature variation is taken from 273 K to 232 K. The definitional limits for the escape velocity are considered as 0.4 and 1.4 times that of the Earth. For these limits, we have calculated earth similarity indexes corresponding to radius, density, escape velocity and temperature.

For the given values of parameters for the planets in our solar system, we have calculated ESI, where m and T are considered as the free parameters. The values of ESI are found large for the Mars, Mercury and Venus. For these three planets, in Figure 1, we have plotted ESI versus m for T=0.8 and T=0.9. In the figure, solid line is for the Mars, dashed line for the Mercury and dotted line for the Venus. For other solar planets, the ESI is small and therefore is not shown in the figure. We have also drawn a line for ESI=0.8 and ESI=0.9 in the respective part of figure. For the optimization of free parameter m, we have considered that the Mars has the ESI equal to the terminal value T. For both, T=0.8 and T=0.9, we have found m=0.8, where the ESI of Mars is equal to the terminal value T. Hence, in the further calculations of the ESI_S, ESI_I and ESI for various solar planets and satellites, and exoplanets, we have taken two values of T as 0.8 and 0.9, and the value of m as 0.8.
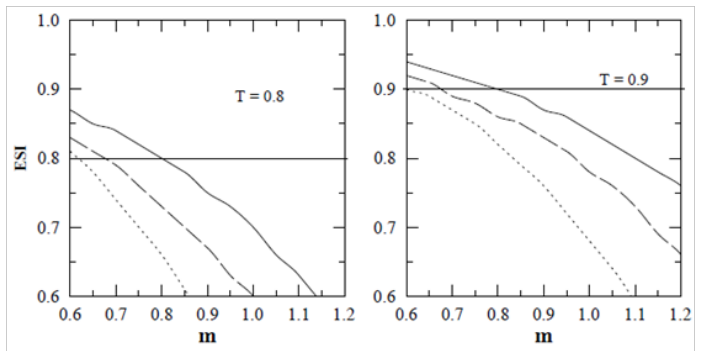

Figure 1 Variation of ESI versus for T= 0.8 and T= 0.9. Solid line is for the Mars, dashedline for the Mercury and dotted line for the Venus. We have also plotted line for ESI=0.8 and ESI=0.9 in the respective figure. For other solar-planets, the value of ESI is very small.

In Table 1, we have given the values of physical parameters, interior-ESI, surface-ESI and ESI for 31 objects. For 21 objects, the values of physical parameters are the same as given by Schulze-Makuch et al.[1] In their calculations, there are m=1 and T=0.8. In the last column of Table 1, we have given the values of Schulze-Makuch et al.[1] for the ESI. Table 2 gives the similar results for m=0.8 and T=0.9.

Most of the exoplanets are detected by transit photometry method and radial velocity method. The transit photometry method cannot measure the mass of exoplanet whereas the radial velocity method cannot measure the radius of exoplanet by Jones et al.[6] So, for the exoplanets for which either mass or radius was not measured, the mass or radius is calculated with the help of mass-radius relation, given by Sotin et al.[4]

Table 1 Values of various ESI for some solar planets and satellites and exoplanets with m=0.8 and T=0.8

S.No.	Body	Radius (EU)	Density (EU)	Esc. Vel. (EU)	Temp K	ESII	ESIS	ESI	ESI*
1	Earth	1	1	1	288	1	1	1	1
2	Mars	0.532	0.713	0.449	227	0.8643	0.7394	0.7994	0.7
3	Mercury	0.382	0.984	0.379	440	0.8773	0.6143	0.7341	0.6
4	Moon	0.272	0.606	0.212	220	0.7509	0.6235	0.6842	0.56
5	Venus	0.949	0.95	0.925	730	0.9849	0.4381	0.6568	0.44
6	Io	0.285	0.639	0.228	130	0.766	0.3941	0.5494	0.36
7	Callisto	0.378	0.352	0.218	134	0.6916	0.4022	0.5274	0.34
8	Jupiter	10.973	0.24	5.379	152	0.48	0.4387	0.4589	0.29
9	Ganymede	0.412	0.352	0.244	110	0.7009	0.3375	0.4864	0.29
10	Ceres	0.074	0.376	0.045	167	0.5225	0.3447	0.4244	0.27
11	Europa	0.244	0.546	0.18	102	0.7194	0.2925	0.4587	0.26
12	Saturn	9.14	0.124	3.225	134	0.4049	0.4318	0.4181	0.25
13	Titan	0.404	0.341	0.235	94	0.6928	0.2836	0.4432	0.24
14	Uranus	3.98	0.23	1.91	76	0.5728	0.2604	0.3862	0.19
15	Neptune	3.864	0.297	2.105	72	0.62	0.2409	0.3865	0.18
16	Titania	0.123	0.301	0.069	60	0.5433	0.1295	0.2652	0.1
17	Enceladus	0.039	0.291	0.021	75	0.4271	0.1255	0.2315	0.094
18	Pluto	0.18	0.371	0.109	40	0.618	0.0878	0.2329	0.075
19	Triton	0.212	0.379	0.13	38	0.6402	0.0856	0.2341	0.074
20	HD69830 d	4.19	0.25	2.1	312	0.5818	0.8439	0.7007	0.6
21	55 Cnc c	5.68	0.25	2.84	310	0.551	0.8031	0.6652	0.56
22	55 Cnc f	4.91	0.39	3.06	310	0.6405	0.7918	0.7121	0.614
23	61 Vir d	3.68	0.46	2.5	375	0.7028	0.7093	0.706	
24	HIP 57050 b	6.64	0.32	3.78	250	0.5743	0.7217	0.6438	
25	Mu Ara d	8.38	0.28	4.45	327	0.5292	0.7059	0.6112	
26	HD 142 b	11.12	0.24	5.43	286	0.4787	0.7397	0.5951	
27	HD 96167 b	9.36	0.26	4.81	334	0.507	0.6833	0.5886	
28	HD 108874 b	12.49	0.22	5.89	294	0.4558	0.7197	0.5728	
29	HD 210277 b	11.98	0.23	5.72	275	0.4658	0.7109	0.5754	
30	HD 147513 b	10.99	0.24	5.38	263	0.4798	0.697	0.5783	
31	HD 69830 c	2.82	0.54	2.07	549	0.7636	0.5253	0.6333	

*Values reported by Schulze-Makuch et al.[1] for m=1 and T =0.8.

Table 2 Same as Table 1 with m=0.8 and T =0.9

S.No.	Body	ESI_I	ESI_S	ESI
1	Earth	1	1	1
2	Mars	0.9334	0.8671	0.8997
3	Mercury	0.9401	0.7945	0.8642
4	Moon	0.8735	0.8001	0.836
5	Venus	0.9928	0.6773	0.82
6	Io	0.8817	0.6442	0.7537
7	Callisto	0.8402	0.6505	0.7393
8	Jupiter	0.7071	0.6777	0.6923
9	Ganymede	0.8455	0.5988	0.7115
10	Ceres	0.736	0.6048	0.6672
11	Europa	0.856	0.5597	0.6922
12	Saturn	0.6525	0.6726	0.6625
13	Titan	0.8409	0.5515	0.681
14	Uranus	0.7686	0.5298	0.6381
15	Neptune	0.798	0.5106	0.6383
16	Titania	0.7497	0.3809	0.5344
17	Enceladus	0.6692	0.3753	0.5012
18	Pluto	0.7968	0.317	0.5026
19	Triton	0.8101	0.3133	0.5038
20	HD69830 d	0.7744	0.923	0.8454
21	55 Cnc c	0.7547	0.9016	0.8249
22	55 Cnc f	0.8103	0.8956	0.8519
23	61 Vir d	0.8466	0.8503	0.8484
24	HIP 57050 b	0.7696	0.8573	0.8123
25	Mu Ara d	0.7404	0.8483	0.7926
26	HD 142 b	0.7062	0.8673	0.7826
27	HD 96167 b	0.7256	0.8354	0.7786
28	HD 108874 b	0.6901	0.8562	0.7686
29	HD 210277 b	0.6971	0.8512	0.7703
30	HD 147513 b	0.707	0.8433	0.7721
31	HD 69830 c	0.8804	0.7379	0.806

Discussion

From Table 1, we have found that for each object, the present value of ESI, in general, larger than that of Schulze-Makuch et al.[1] It is due to the change in the value of m. We have also found that the exoplanet 55 Cnc f is within 10% limit from the threshold value T. The exoplanets HD 69830 d, 55 Cnc c, 55 Cnc f, 61 Vir d and HIP 57050 b have the ESI within 10% limit from the threshold value. This supports the opinion for existence of life in the universe.

Conclusion

We have derived the ESI with two free parameters, the threshold value T and m. After optimization that the ESI of Mars is equal to the threshold value T, we got the value of m as 0.8. In the paper of Schulze-Makuch et al.[1] the value of m is 1. For the present value of m=0.8, some exoplanets have come in the range up to the threshold value. It enhances the probability of finding life in the interstellar medium.

Acknowledgments

Conflicts of interest

Authors declare there is on conflicts of interest.

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