# 1. Appendix A

## 1.1 The dispersion relation additional elaboration

The equation (10) gives,

$$\frac{j(1-j\varepsilon_0\eta\omega).\upsilon_t^2}{-j(\omega-l\Omega)+\dfrac{\omega_{ce}^2}{\nu_{en}-j(\omega-l\Omega)}+j\dfrac{l^2\upsilon_t^2\Big/R^2}{\omega-l\Omega}+\nu_{en}}=\frac{-j(1-j\varepsilon_0\eta\omega).C_s^2(\nu_{in}-j\omega)}{(\nu_{in}-j\omega)^2+\omega_{ci}^2}-\lambda_D^2\omega$$

Or

$$\frac{j(1-j\varepsilon_0\eta\omega).\upsilon_t^2}{\dfrac{\omega_{ce}^2}{\nu_{en}}+j\dfrac{l^2\upsilon_t^2\Big/R^2}{\omega-l\Omega}+\nu_{en}}=\frac{-j(1-j\varepsilon_0\eta\omega).C_s^2(\nu_{in}-j\omega)}{(\nu_{in}-j\omega)^2+\omega_{ci}^2}-\lambda_D^2\omega$$

$.\Rightarrow$

$$\frac{j(1-j\varepsilon_0\eta\omega).\upsilon_t^2.\nu_{en}(\omega-l\Omega)}{\omega_{ce}^2(\omega-l\Omega)+jl^2\upsilon_t^2\Big/R^2\nu_{en}+\nu_{en}^2(\omega-l\Omega)}=\frac{-j(1-j\varepsilon_0\eta\omega).C_s^2(\nu_{in}-j\omega)}{(\nu_{in}-j\omega)^2+\omega_{ci}^2}-\lambda_D^2\omega$$

$.\Rightarrow$

$$\frac{j\upsilon_t^2\nu_{en}(1-j\varepsilon_0\eta\omega).(\omega-l\Omega)}{(\omega-l\Omega)(\omega_{ce}^2+\nu_{en}^2)+jl^2\upsilon_t^2\Big/R^2\nu_{en}}=\frac{-C_s^2(1-j\varepsilon_0\eta\omega).(\omega+j\nu_{in})}{(\nu_{in}-j\omega)^2+\omega_{ci}^2}-\lambda_D^2\omega$$

# 2. Appendix B

By putting,

$$\omega^2\omega_i\left[\upsilon_t^2\nu_{en}\varepsilon_0\eta+\omega_{ce}^2\lambda_D^2\right](3l\Omega-4\omega)\cong\omega^2\omega_i\lambda_D^2(\omega_{pe}^2\nu_{en}\varepsilon_0\eta+\omega_{ce}^2)(3l\Omega-4\omega),$$

The last relation becomes,

$$\omega^2\omega_i\lambda_D^2(\omega_{pe}^2\nu_{en}\varepsilon_0\eta+\omega_{ce}^2)(3l\Omega-4\omega)$$

$$+\left[\upsilon_t^2\nu_{en}(\varepsilon_0\eta\omega_{ci}^2+2\nu_{in})+\omega_{ce}^2\lambda_D^2\left[\omega_{pi}^2(1+\varepsilon_0\eta\nu_{in})+\omega_{ci}^2\right]+\frac{l^2\upsilon_t^2}{R^2}\nu_{en}\lambda_D^2(\omega_{pi}^2\varepsilon_0\eta+2\nu_{in})\right].2\omega\omega_i$$

$$+\left[-\upsilon_t^2\nu_{en}l\Omega(\varepsilon_0\eta\omega_{ci}^2+2\nu_{in})-l\Omega\omega_{ce}^2.\lambda_D^2\left[\omega_{pi}^2(1+\varepsilon_0\eta\nu_{in})+\omega_{ci}^2\right]\right].\omega_i=$$

$$=\left[\upsilon_t^2\nu_{en}(2\varepsilon_0\eta\nu_{in}+1)+\omega_{ce}^2\lambda_D^2(\varepsilon_0\eta\omega_{pi}^2+2\nu_{in})+\frac{l^2\upsilon_t^2}{R^2}\nu_{en}\lambda_D^2\right].\omega^3$$

$$-\left[\upsilon_t^2\nu_{en}l\Omega(2\varepsilon_0\eta\nu_{in}+1)+l\Omega\omega_{ce}^2.\lambda_D^2(\varepsilon_0\eta\omega_{pi}^2+2\nu_{in})\right].\omega^2$$

$$-\left[\upsilon_t^2\nu_{en}\omega_{ci}^2+\omega_{ce}^2C_s^2\nu_{in}+\frac{l^2\upsilon_t^2}{R^2}\nu_{en}\lambda_D^2\left[\omega_{pi}^2(1+\varepsilon_0\eta\nu_{in})+\omega_{ci}^2\right]\right].\omega+l\Omega\left[\upsilon_t^2\nu_{en}\omega_{ci}^2+\omega_{ce}^2C_s^2\nu_{in}\right]$$

$\Rightarrow$

$$\omega^2\omega_i(\omega_{pe}^2\nu_{en}\varepsilon_0\eta+\omega_{ce}^2)(3l\Omega-4\omega)$$

$$+\left[\omega_{pe}^2\omega_{ci}^2\nu_{en}\varepsilon_0\eta+\omega_{ce}^2(\omega_{pi}^2+\omega_{ci}^2)+\frac{l^2\upsilon_t^2}{R^2}\nu_{en}\omega_{pi}^2\varepsilon_0\eta\right].2\omega\omega_i$$

$$+\left[-\omega_{pe}^2\nu_{en}\varepsilon_0\eta\omega_{ci}^2-\omega_{ce}^2.(\omega_{pi}^2+\omega_{ci}^2)\right].l\Omega\omega_i=$$

$$=\left[\omega_{pe}^2\nu_{en}+\omega_{ce}^2\omega_{pi}^2\varepsilon_0\eta+\frac{l^2\upsilon_t^2}{R^2}\nu_{en}\right].\omega^3-l\Omega.(\omega_{pe}^2\nu_{en}+\omega_{ce}^2.\omega_{pi}^2\varepsilon_0\eta).\omega^2$$

$$-\nu_{en}\left[\omega_{pe}^2\omega_{ci}^2+\frac{l^2\upsilon_t^2}{R^2}(\omega_{pi}^2+\omega_{ci}^2)\right].\omega+l\Omega.\omega_{pe}^2\nu_{en}\omega_{ci}^2$$

$\Rightarrow$

$$\omega^2\omega_i(\omega_{pe}^2\nu_{en}\varepsilon_0\eta+\omega_{ce}^2)(3l\Omega-4\omega)$$

$$+\left[\nu_{en}\varepsilon_0\eta\omega_{pe}^2(\omega_{ci}^2+\frac{l^2\upsilon_t^2}{R^2}\frac{\omega_{pi}^2}{\omega_{pe}^2})+\omega_{ce}^2(\omega_{pi}^2+\omega_{ci}^2)\right].2\omega\omega_i$$

$$-\left[\omega_{pe}^2\omega_{ci}^2\nu_{en}\varepsilon_0\eta+\omega_{ce}^2.(\omega_{pi}^2+\omega_{ci}^2)\right].l\Omega\omega_i=$$

$$=\left[\nu_{en}\omega_{pe}^2(1+\frac{l^2\lambda_D^2}{R^2})+\omega_{ce}^2\omega_{pi}^2\varepsilon_0\eta\right].\omega^3-l\Omega.(\omega_{pe}^2\nu_{en}+\omega_{ce}^2.\omega_{pi}^2\varepsilon_0\eta).\omega^2$$

$$-\nu_{en}\omega_{pe}^2\left[\omega_{ci}^2+\frac{l^2\lambda_D^2}{R^2}(\omega_{pi}^2+\omega_{ci}^2)\right].\omega+l\Omega.\omega_{pe}^2\omega_{ci}^2\nu_{en}$$