Review Article

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Checking the absolute value of the speed of light in 1D

Abstract

Einstein's theory of relativity establishes that the measurement of the speed of light is constant and invariant in any frame of reference. In this article, we will review that this postulate is fulfilled by applying the Einstein's velocity addition for the case of one spatial dimension (1D). This work may be of special usefulness for physics students and teachers of pre-university and first-year university levels to introduce relativistic theory.

Keywords: speed of light, velocity addition, relativity, Einstein, Galileo

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Introduction

At the beginning of the 20th century there were some experiments that could not be explained by the classical mechanics of Newton and Galileo,[1] for example, the experiments of Michelson-Morley[2] and Bertozzi,[3] which are well known by scientific community.[2,3] In order to explain these experiments, after the attempt of some scientist like Lorentz and Minkowsky, Einstein developed the special theory of relativity,[4–7] which is based on solely two postulates.[8] The first postulate states that the laws of physics are the same in any inertial frame of reference.[8] The second postulate states that the speed of light is the same as measured from any frame of reference, regardless of the relative speed between the observer and the source.[8] Moreover, the speed of light in the vacuum is the highest possible in the universe and it cannot be surpassed.[9] In this article, we will focus on the second postulate and verify that it holds in one dimension (1D) using Einstein's relativistic velocity addition law, which contrasts with Galileo's classic velocity addition theory and contradicts human common sense, as we will see in the following sections. Therefore, the main objective of this article consists of providing some interesting ideas and developments in order to improve the pedagogical field of secondary and higher education about the second postulate of Einstein following the same line of recent works about teaching relativity.[10–12]

Relativistic addition of velocities

The velocity addition theory in relativistic terms developed by Einstein is well known to the scientific community. This theory establishes the velocity perceived by two systems moving with relative velocities between them considering that none of these systems can surpass the speed of light. This theory is a generalization of Galileo's velocity addition rule that is used in classical physics when velocities are much less than the speed of light. Consider an inertial frame of reference O and two systems A and B that move along the X axis according to O Figure 1.

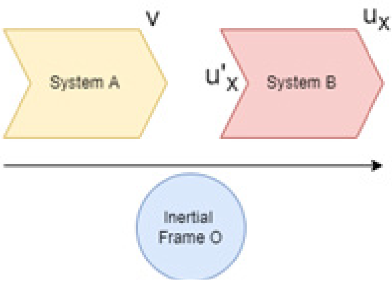

Figure 1 Addition of velocities scheme.

The velocity addition theory in relativity establishes that:[5]

$$u_x' = \frac{u_x - v}{1 - \frac{v}{c^2} u_x} \quad (1)$$

in which u_x is the speed at which the inertial frame of reference O observes system B is moving; v is the speed at which the inertial frame of reference O observes system A is moving; c is the speed of light; and u_x' is the speed at which A observes B is moving.

From this point on, we consider that system B is a photon that moves at the speed of light (c) according to the inertial frame of reference O in the positive direction of the x axis. Therefore, $u_x = c$, and the expression (1) can be rewritten as:

$$u_x' = \frac{c - v}{1 - \frac{v}{c}} \quad (2)$$

In the following sections we will study the speed of B (a photon) perceived by the observer A $\left(u_x'\right)$ according to the relative movement that these two systems maintain. We will also compare and discuss the results with the classical Galileo velocity addition.

Speed of light with observer moving at v_x

Consider that the observer A is moving in the positive X axis with velocity $v_x < c$ according to the inertial frame of reference O in such a way that A is going after B Figure 2.

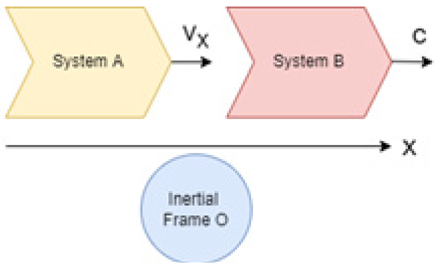

Figure 2 System A going after system B at speed v_x according to the inertial frame O.

Then, the system B is moving according to the observer A with this velocity:

$$u_x' = \frac{c - v_x}{1 - \frac{v_x}{c}} = \frac{c - v_x}{c - v_x} . c = c \quad (3)$$

Therefore, B is moving away from A at the speed of light c and the second postulate of Einstein's theory is satisfied in this case. This result is incongruent with the classical conception of Galileo's addition of velocities because this velocity should be $u_x^{'} = c - v_x$ as we were taught in the school and our common sense dictates (because if, for example, I am the system A moving at 6 km/h and I am going after system B, which is moving at 10 km/h, my perception should be that the system B is moving away from me at 4 km/h).

Speed of light with observer moving at -v_x

Consider now that the observer A is moving in the negative X axis with velocity $v_x < c$ according to the inertial frame of reference O in such a way that A is moving away from B Figure 3.

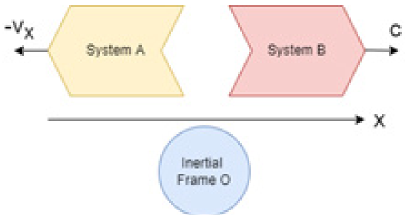

Figure 3 System A moving away from system B at speed v_x according to the inertial frame O.

Then, the system B is moving according to the observer A with this velocity:

$$u_x^{'} = \frac{c-(-v_x)}{1-\frac{(-v_x)}{c}} = \frac{c+v_x}{c+v_x}.c = c \qquad (4)$$

So, as the previous case, B is moving away from A at the speed of light c and the second postulate of Einstein theory is satisfied too. This result is incongruent with the classical conception of Galileo's addition of velocities because this velocity should be $u_x^{'} = c + v_x$, i.e. this velocity should be higher than speed of light (which is absolutely logical for us because if, for example, I am the system A moving at 6 km/h and I am moving away from system B, which is moving at 10 km/h, my perception should be that the system B is moving away from me at 16 km/h).

Speed of light with observer moving at c

Consider that the observer A is moving in the positive X axis with the higher possible velocity, the speed of light c, according to the inertial frame of reference O in such a way that A is going after B with the same velocity that B Figure 4.

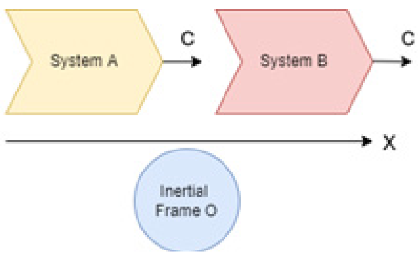

Figure 4 System A going after system B at speed c according to the inertial frame O.

Then, the system B is moving according to the observer A with this velocity:

$$u_x^{'} = \frac{c-c}{1-\frac{c}{c}} = \frac{c-c}{c-c}.\ c = \frac{0}{0} \qquad (5)$$

In this case we obtain and indetermination that can be solved applying limits when $v_x \to c$ and using the L'Hôpital's rule:

$$u_x^{'} = \lim_{v_x \to c} \left(\frac{c-v_x}{1-\frac{v_x}{c}} \right) = c \cdot \lim_{v_x \to c} \left(\frac{c-v_x}{c-v_x} \right) c \cdot \frac{-1}{-1} = c \qquad (6)$$

Thus, as the previous cases, B is moving away from A at the speed of light c and the second postulate is satisfied in this case too. This result is completely incongruent with the classical conception of Galileo's addition of velocities because this velocity should be $u_x^{'} = c - c = 0$ (velocity should be null because both systems goes at the same speed in the same direction and sense).

Speed of light with observer moving at -c

Consider that the observer A is moving in the negative X axis with the higher possible velocity, the speed of light c, according to the inertial frame of reference O in such a way that A is moving away from B with the same velocity that B Figure 5.

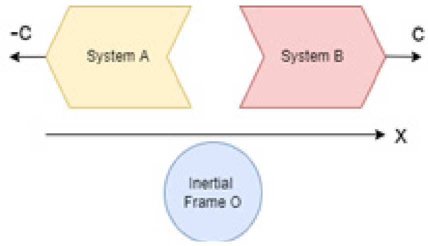

Figure 5 System A moving away from system B at speed c according to the inertial frame O.

Then, the system B is moving according to the observer A with this velocity:

$$u_x^{'} \frac{c-(-c)}{1-\frac{(-c)}{c}} = \frac{c+c}{c+c}.\ c = c \qquad (7)$$

Hence, as the previous cases, B is moving away from A at the speed of light c and the second postulate is also satisfied. This result is also incongruent according to the classical conception of Galileo's addition of velocities because this velocity should be $u_x^{'} = c + c = 2c$ (because if, for example, I am the system A moving at 10 km/h and I am moving away from system B, which is moving at 10 km/h, my perception should be that the system B is moving away from me at 20 km/h).

Conclusion

In this review we have analyzed the measurement of the speed of one photon (system B) from different frames of reference in 1D. This study has been developed applying the Einstein's addition of velocities law. All the studied cases provide the same results, the velocity of the photon is always the speed of light c independently of the reference frame. Then, these results verify the second postulate of Einstein and the relativistic addition of velocities in one dimension. This work can be useful for teachers or students of science education to explain and understand easily the second postulate of Einstein's Special Relativity Theory.

Acknowledgments

None.

Conflicts of interest

The author declares there is no conflict of interest.

References

1. Levy J. *From Galileo to Lorentz…and Beyond*. Apeiron, Montreal, 2003.
2. Michelson AA, Morley EW. On the Relative Motion of the Earth and the Luminiferous Ether. *Am J Sci*. 1887;34 :333–345.
3. Bertozzi W. Speed and kinetic energy of relativistic electrons. *Am J Phys*. 1964;32:551–555.
4. Williams WSC. *Introducing Special Relativity*. Taylor and Francis, London, 2002. p. 264.
5. French AP. *Special Relativity*. Nelson, London, 1968. p. 1–174.
6. Rindler W. *Introduction to Special Relativity*. Clarendon Press, Oxford, 1991.
7. Zhang YZ. *Special Relativity and its Experimental Foundations*. World Scientific, Singapore, 1997.
8. Einstein A. *On the Electrodynamics of Moving Bodies, in The Principle of Relativity.* In: HA Lorentz, et al., editors. Dover Publications, New York, 1952.
9. Ives HE. The Measurement of the Velocity of Light by Signals Sent in One Direction. *Journal of the Optical Society of America*. 1948;38:879.
10. Alstein P, Krijtenburg-Lewerissa K, Van Joolingen WR. Teaching and learning special relativity theory in secondary and lower undergraduate education: A literature review. *Physical Review Physics Education Research*. 2021;17:023101.
11. Kamphorst F, Vollebregt MJ, Savelsbergh ER, et al. An Educational Reconstruction of Special Relativity Theory for Secondary Education. *Sci & Educ*. 2023;32:57–100.
12. Taşkın T, Kandil İngeç, Şebnem. Prospective Physics Teachers' Understanding of the Speed of Light within the Scope of Special Relativity. *International Journal of Physics and Chemistry Education*. 2019;11(2):45–53.