Review Article

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Solving numerically a seventh order boundary value problem by splitting coupled finite difference method

Abstract

In this present article we concerned with numerical solution of seventh order boundary value problem. We have proposed a novel finite difference method and derived proposed finite difference method by splitting coupled equations method. Under appropriate conditions, we have established the convergence of the proposed method. Also we have obtained a numerical value of derivative of solution of the problem which is practically useful in some modeling problem. We have applied proposed method for the numerical solution of model problems. Numerical results are in good agreement to the proposed theoretical results.

Keywords: boundary value problem, finite difference method, higher order convergence, seventh order differential equation, splitting couple method

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Introduction

In the present article we consider seventh order boundary value problem of the following form:

$$u^{(7)}(x) = f(x, u),\ a < x < b,\ (1.1)$$

Subject to the boundary conditions

$$u(a) = \alpha_1,\ u'(a) = \alpha_2,\ u''(a) = \alpha_3,\ u^{(3)}(a) = 4,$$

$$u(b) = \beta_1,\ u'(b) = \beta_2 \quad and \quad u''(b) = \beta_3,$$

Where $\alpha_1, \alpha_2, \alpha_3, \alpha_4,\ \beta_1, \beta_2$, and β_3 are real constant.

The problems in physical sciences can be modeled mathematically and formulated by differential equations. The problems in engineering sciences deal with the formulation and solution of higher order differential equation. The higher order differential equation and boundary value problem studied and discussed in.[1] In particular seventh order boundary value problems arise in mathematical modeling of induction motors with two rotor circuits.[2] To ensure the existence and uniqueness of the solution of the problem (1.1), we presume the smoothness of the forcing function $f(x, u)$. However for the detail discussion on the existence and uniqueness of the solution of higher order differential equations and corresponding BVPs, we can refer.[3] In the present article, we concerned with numerical solution of reference problem instead of the analytical solution. In the literature on the numerical solutions of BVPs, several numerical methods have been reported for seventh order boundary value problems. We can list some of them for instance Variational Iteration Method,[4] Variation of Parameters Method,[5] Differential Transformation Method,[6] Reproducing Kernel Space,[7] Collocation Method using Sextic B- Splines,[8] Homotopy Analysis Method,[9] Optimal Homotopy Asymptotic Method[10] and references there in. Some advance numerical techniques for numerical solution of boundary value problems have been reported in the literature. These techniques are very satisfactory and yield a highly accurate numerical solution. Hence, the purpose of this article is to incorporate these advancements in developing numerical technique for numerical solution of seventh order boundary value problems (1.1). So we incorporated the those ideas in developing an accurate and convergent finite difference method for numerical solution of seventh order boundary value problem by splitting method, a system of boundary value problems. We hope that others may find the proposed method as an improvement in numerical technique to those existing techniques for the seventh order boundary value problems in the literature. We shall present our work in this article as follows: In Section 2 the finite difference method, in Section 3 the derivation of the proposed finite difference method. In Section 4, the convergence analysis of the proposed method under appropriate condition. The numerical experiment on model problems and short discussion on numerical results are presented in Section 5. A summary on the overall development and performance of the proposed method are presented in Section 6.

The difference method

Let us assume problem (1.1) posses solution and it will be $u(x)$ such that

$$u^{(4)}(x) = v(x),\ a < x < b\ (2.1)$$

And the boundary conditions are

$$u(a) = \alpha_1,\ u'(a) = \alpha_2,\ u(b) = \beta_1 \ and\ u'(b) = \beta_2$$

Where augment function $v(x)$ is regular and differentiable in [*a*, *b*]. Further we have following third order boundary value problem,

$$v^{(3)}(x) = f(x, u),\ a < x < b\ (2.2)$$

And the boundary conditions are

$$u''(a) = \alpha_3,\ u^{(3)}(a) = \alpha_3 \ and\ u''(b) = \beta_3$$

To incorporate these boundary conditions, let us define

$$v(x) = u^{(4)}(x) - \lambda u''(x)\ (2.3)$$

Where λ is coupling constant and $\lambda \in (0,\ 1)$. So we get problems (2.1)-(2.3), a system of boundary value problems by splitting method from problem (1.1). Thus the seventh order boundary value problem (1.1) has been transformed into a sys- tem of boundary value problems (2.1)-(2.3). Solving numerically problem (1.1) is equivalent to solve numerically system of problems (2.1)-(2.3). We partition the interval [a,b] in which the solution of problem (1.1) is desired to introduce finite number of mesh points. In these subintervals mesh points $a \le x_0 < x_1 < x_2 < \ldots\ldots < x_{N+1} \le b$ are generated by using uniform step length such that $x_i=a+ih, i=0,1,2,\ldots, N+1$. We wish to determine the numerical solution $u(x)$ of the problem (1.1) at these mesh points x_i. We denote the numerical approximation of $u(x)$ and $f(x,u(x))$ respectively by u_i and f_i at these mesh point $x=x_i, i=1,2,\ldots, N$. Also the boundary value problem (1.1) replaced by the system of boundary value problems (2.1)-(2.3) may be written as under

$$u_i^{(4)}=\upsilon_i, \quad (2.4)$$

$$\upsilon_i^{(3)}=f_i$$

At these node $x=x_i$, $i=0,\ldots,N+1$. Following the ideas in,[11,12] we propose our finite difference method for a numerical solution of problem (2.4),

$$-2\left(u_{i-1}-2u_{i+1}\right)+h\left(u'_{i+1}-u'_{i-1}\right)=\frac{h^4}{90}\left(\upsilon_{i+1}+13\upsilon_i+\upsilon_{i-1}\right), \quad (2.5)$$

$$-3\left(u_{i+1}-u_{i-1}\right)+h\left(u'_{i+1}+4u'_i+u'_{i-1}\right)=\frac{h^4}{60}\left(\upsilon_{i+1}-\upsilon_{i-1}\right), \quad (2.6)$$

$$-3\upsilon_{i-1}+4\upsilon_i-\upsilon_{i+1}=\frac{h^3}{6}\left(-3f_i+f_{i+1}\right), \quad i=1 \quad (2.7)$$

$$\upsilon_{i-2}-3\upsilon_{i-1}+3\upsilon_{i+1}\frac{h^3}{2}\left(-3f_i+f_{i+1}\right), \quad 2\le i\le N$$

If the source function $f(x,u)$ in problem (1.1) is linear then the system of equations (2.5)-(2.7) will be linear otherwise we will obtain nonlinear system of equations.

Derivation of the difference method

In this section we outline the derivation of the proposed method, we have followed the same approach as given in.[11,12] Let us write a linear combination of solution $u(x)$, $u^j(x)$ and $v(x)$ at nodes $x_{x\pm1}$, and x_i,

$$a_2u_{i+1}+a_1u_{i-1}+a_0u_i+h\left(b_2u^j{}_{i+1}+b_1u^j{}_{i-1}\right)+h^4\left(c_2v_{i+1}+c_0v_i+c_1v_{i-1}\right)=0 \quad (3.1)$$

where a_0-c_0 are constants to be determined. To determine these constants, we expanding each term on the left hand side of (3.1) in Taylor series about the point x_i. Using method of undetermined coefficients, compare the coefficients of h^p, $p=0,1,\ldots,7$ on both side we get a system of equations. Solving this system of equations, we get

$$(a_2,a_1,a_0,b_2,b_1,c_2,c_0,c_1)=\left(-2,-2,4,1,-1,\frac{1}{90},-\frac{13}{90}-\frac{1}{90}\right) \quad (3.2)$$

On substitution of these constants a_0-c_0 from (3.1) into (3.2) and simplify, we have

$$-(v_{i-1}+2v_i-v_{i+1})+h(u'_{i+1}-u'_{i-1})-\frac{h^4}{90}(v_{i+1}+13v_i+v_{i-1})+tu_i=0 \quad (3.3)$$

Where tu_i, $i=1,\ldots,N$ is local truncation error and equal to $-\frac{19h^8}{30240}u_i^{(8)}$. Similarly we can derive the following equations

$$-3(u_{i+1}-u_{i-1})+h(u'_{i+1}+u'_{i-1})-\frac{h^4}{60}(v_{i+1}-v_{i-1})+tu_i, \quad (3.4)$$

Where local error tu'_i is equal to $-\frac{h^5}{504}u_i^{(7)}, i=1,\ldots,N$ and

$$-3v_{i-1}+4v_i-v_{i+1}-2hv'_{i-1}-\frac{h^3}{6}(3f_i+f_{i+1})+tv_i, \quad i=1 \quad (3.5)$$

$$v_{i-2}-3v_{i-1}+3v_i-v_{i+1}-\frac{h^3}{2}(-3f_i+f_{i+1})+tv_i, \quad 2\le i\le N$$

Where local truncation error tv_i are respectively equal to $-\frac{3h^5}{20}v_i^{(5)}, i=1$ and $-\frac{h^5}{2}v_i^{(5)}, 2\le i\le N$.

Thus by neglecting the local error terms in (3.3)-(3.5), we will get our proposed difference method for the numerical solution of the problem (1.1). Moreover we are getting the numerical value of the derivative of the solution of the problem (1.1) as a byproduct of the method. Sometimes we need it which otherwise get approximated.

Convergence analysis

In this section we will discuss the convergence of the method proposed in section

Thus for the discussion of convergence let us consider following test equation.

$$u^{(7)}(x)=f(x,u), \quad a<x<b \quad (4.1)$$

$$u(a)=\alpha_1, u''(a)=\alpha_2 u''(a)=\alpha_3, u^{(3)}(a)=\alpha_4$$

$$u(a)=\beta_1, u''(b)=\beta_2 \text{ and } u''(a)=\beta_3$$

Let's be the approximate solution of difference method (2.4-2.5) for numerical solution of the problem (4.1), we can write this in the matrix form

Js = **Rh** (4.2)

where **J** is coefficient matrix, $s=\left[u,u^j,v\right]^T$ and $Rh=\left[rh_1,rh_2,rh_3\right]^T$. These matrix are

$$rh_3=\begin{pmatrix} 3\upsilon_0+2h\upsilon'_0+\frac{h^3}{6}(3f_1+f_2) \\ -\upsilon_0+\frac{h^3}{2}(-3f_2+f_3) \\ \frac{h^3}{2}(-3f_3+f_3) \\ \vdots \\ \upsilon N+1+\frac{h^3}{2}(-3fN+fN+1+\lambda\beta_3) \end{pmatrix}_{N\times1}, rh_2\begin{pmatrix} -3\alpha_1-h\alpha_2-\frac{h^4}{60}\upsilon_0 \\ 0 \\ \vdots \\ 3\beta_1-h\beta_2+\frac{h^4}{60}\upsilon N+1 \end{pmatrix}_{N+1}$$

$$rh_1\begin{pmatrix} 2\alpha_1-h\alpha_2-\frac{h^4}{90}\upsilon_0 \\ 0 \\ \vdots \\ 2\beta_1-h\beta_2+\frac{h^4}{90}\upsilon N+1 \end{pmatrix}_{N+1}, v=\begin{pmatrix}\upsilon_1\\ \vdots \\ \upsilon N\end{pmatrix}_{N+1}, u'=\begin{pmatrix}\upsilon_1\\ \vdots \\ \upsilon N\end{pmatrix}_{N+1}, u=\begin{pmatrix}\upsilon_1\\ \vdots \\ \upsilon N\end{pmatrix}_{N+1},$$

And let us define the coefficients matrix **J** in terms of block matrix

$$J=\begin{pmatrix} C_{1,1} & \vdots & C_{1,2} & \vdots & C_{1,3} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ C_{2,1} & \vdots & C_{2,2} & \vdots & C_{2,3} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ C_{3,1} & \vdots & C_{3,2} & \vdots & C_{3,3} \end{pmatrix}_{3N3N}$$

Where

$$C_{1,1}=2\begin{pmatrix} 2 & -1 & & & 0 \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ 0 & & & -1 & 2 \end{pmatrix}_{N\times N}, C_{1,2}=h\begin{pmatrix} 0 & 1 & & & 0 \\ -1 & 0 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 0 & 1 \\ 0 & & & -1 & 0 \end{pmatrix}_{N\times N},$$

$$C_{1,3}=-\frac{h^4}{90}\begin{pmatrix} 13 & 1 & & & 0 \\ 1 & 13 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & 13 & 1 \\ 0 & & & 1 & 13 \end{pmatrix}_{N\times N}, C_{2,1}=\frac{-3}{h}C1,2$$

$$C_{2,2}=h\begin{pmatrix} 4 & 1 & & & 0 \\ 1 & 4 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & 4 & 1 \\ 0 & & & 1 & 4 \end{pmatrix}_{N\times N}, C_{2,3}=\frac{-h3}{60}C_{1,2}$$

$$C_{3,3}=\begin{pmatrix} 4 & -1 & & & & 0 \\ -3 & 3 & -1 & & & \\ 1 & -3 & 3 & -1 & & \\ & \ddots & \ddots & \ddots & \ddots & \\ & & 1 & -3 & 3 & -1 \\ 0 & & & -1 & 3 & 3 \end{pmatrix}_{N\times N},$$

and matrices $\left(\mathbf{C}_{3,1}\right)_{N\times N}$ and $\left(\mathbf{C}_{3,2}\right)_{N\times N}$ depend on forcing function $f\left(x,u\right)$. These matrices are well defined. The exact solution $\mathbf{S}=\left[\mathbf{U},\mathbf{U}^{j},\mathbf{V}\right]^{T}$ of the difference method (2.4-2.5) will satisfy the following equation

$$\mathbf{JS}=\mathbf{Rh}+\mathbf{T} \qquad (4.3)$$

Where $T=[tu;\, tu',\, tv]^{T}$ is truncation error and will be defined as,

$$tu=\begin{pmatrix} \frac{19h^8}{30240}u_1^{(8)} \\ \vdots \\ \frac{19h^8}{30240}u_N^{(8)} \end{pmatrix}_{N\times 1}, tu'=\begin{pmatrix} \frac{h^7}{504}u_1^{(7)} \\ \vdots \\ \frac{19h^8}{30240}u_N^{(8)} \end{pmatrix}_{N\times 1}, tv=\begin{pmatrix} \frac{3h^5}{20}\upsilon_1^{(5)} \\ \frac{h^5}{2}\upsilon_2^{(5)} \\ \vdots \\ \frac{h^5}{2}\upsilon_N^{(5)} \end{pmatrix}_{N\times 1}$$

Let us define an error function the difference between approximate and exact solution of the difference method (2.4-2.5) i.e. E = s .. S.. To introduce and calculate so defined error function let subtract (4.3) from (4.2), we will obtain following error equation

$$\mathbf{JE}=\ -\mathbf{T} \qquad (4.4)$$

Thus from (4.4), we observe that the convergence of the proposed method depends on the properties of coefficients matrix **J**. We will prove under appropriate as- sumptions that the coefficient matrix J is invertible. Let us test the inevitability of coefficient matrix J. The diagonal matrices $C_{1,1}$, $C_{2,2}$ and $C_{3,3}$ of matrix J have different structure. The matrix $C_{1,1}$ is invertible.[13] Matrix $\mathbf{C}_{2,2}$ is strictly diagonally dominant so it will invertible. For matrix $C_{3,3}$, we have to rely on computation of explicit inverse. Let explicit inverses of $C_{3,3}$ *be* $C_{3,3}^{-1}=\ (k_{i,j})N\times N$, where

$$k_{i,j}=\begin{cases} \dfrac{i^2(N-j+1)(N-j+2)}{2(N+N)^2}, & i\le j\le N \\ \dfrac{(N-1)(N-j+1)}{2},k_{N-1,j}-\left((N-1)^2-1\right)kN,j, & j\le N \end{cases} \qquad (4.5)$$

$$k_{N,j}=\begin{cases} \dfrac{4N(N+2)(2N-1)-(N-2j)\left((N-2)^2(N-2j+2)+8N\right)}{32(N+1)^2}, & j\le\dfrac{N}{2} \\ \dfrac{N(2N^2+3N+2)+(N-2j+2)((2N+1))(2j-N)-2N}{8(N+1)^2}, & \dfrac{N}{2}<j \end{cases}$$

$$k_{N-1,j}=\begin{cases} \dfrac{N^3-2N+2-(N-2j)\left(N(N-j2+2)-2N\right)}{2(N+1)^2}, & j\le\dfrac{N}{2} \\ \dfrac{N^3-2N-2+(N-2j+2)\left(N(2j-N)+2\right)}{2(N+1)^2}, & \dfrac{N}{2}<j \end{cases}.$$

Thus from (4.5) we can verify that matrix $C_{3,3}$ is invertible. Let us define following terms,[14]

$$\upsilon_k^{up}=\max_{j=1,2,\ldots,k-1}\|A_{jk}A_{kk}^{-1}\|,\ k=2,3 \quad , \quad \upsilon_k^{low}=\max_{j=k+1,3}\|A_{jk}A_{kk}^{-1}\|,\ k=1,2,$$

$$M^{*}=\prod_{2\le k\le 3}\left(1+\upsilon_k^{up}\right)\ \ and\ \ M_{*}=\prod_{1\le k\le 2}\left(1+\upsilon_k^{low}\right).$$

Let us assume

$$M^{*}M_{*}<M_{*}+M^{*}\quad and\quad \max_{p=1,2,3}\|C_{p,p}^{-1}\|$$

Then matrix J is invertible[14] and moreover

$$\|J^{-1}\|\le\frac{MM_{*}M^{*}}{M_{*}+M^{*}-M_{*}M^{*}}. \qquad (4.6)$$

Thus from (4.4) and (4.6), we have

$$\|E\|\,\|\mathrm{TJ}^{-1}\|\le\|T\|\frac{MM_{*}M^{*}}{M_{*}+M^{*}-M_{*}M^{*}} \qquad (4.7)$$

It is easy to prove that $\frac{MM_{*}M^{*}}{M_{*}+M^{*}-M_{*}M^{*}}$ is finite. Thus $\|E\|$ is bounded. Also it is easy to prove $\|E\|$ tends to zero as $h\to 0$ .So we can conclude that finite difference method (2.5-2.7) converge. The order of the convergence of the difference method (2.5-2.7) is at least $O\left(h^2\right)$.

Numerical results

To test the computational efficiency of method (2.5-2.7), we have considered four model problems. In each model problem, we took uniform step size h. In Table 1 and Table 2, we have shown $MAEU$ and $MAEV$ the maximum absolute error in the solution $u(x)$ and derivatives of solution $v(x)$ of the problems (1.1) for different values of N. We have used the following formulas in computation of $MAEU$ and $MAEV$:

$$MAEU = \max_{1 \le i \le N} \left| u(x_i) - u_i \right|$$

$$MAEV = \max_{1 \le i \le N} \left| u'(x_i) - v_i \right|$$

We have used Gauss Seidel iterative method to solve linear system of equations (2.5-2.7). All computations were performed on a Windows 2007 Ultimate operating system in the GNU FORTRAN environment version 99 compiler (2.95 of gcc) on Intel Core i3-2330M, 2.20 GHz PC. The solutions are computed on N nodes and iteration is continued until either the maximum difference between two successive iterates is less than 10^{-6} or the number of iteration reached 10^3.

Problem 1 The model linear problem given by

$$u^{(7)}(x) = -u(x) - \left(35+12x+2x^2\right)\exp(x), \quad 0<x<1$$

Subject to boundary conditions

$$u(0) = 1, \; u'(0) = 0, \; u''(0) = -1, \; u'''(0) = -3 \;,$$

$$u(1) = 0, \; u'(1) = -\exp(1) \; and \; u''(1) = -4\exp(1)$$

The analytical solution of the problem is $u(x) = x(1-x)\exp(x)$. The $MAEU$ and $MAEV$ computed by method (2.5-2.7) for coupling constant C = .40199 and different values of N are presented in Table 1.

Table 1 Maximum absolute error (Problem 1)

	N		
	32	64	128
MAEU	.28539286(-2)	.14184146(-5)	.55249515(-7)
MAEV	.10026446(-1)	.32737342(-4)	.99397312(-5)

Problem 2 The model linear problem given by

$$u^{(7)}(x) = u(x)u'(x) + \left(2-3x+x^2+(x-8)\exp(x)\right)\exp(-2x), \quad 0<x<1$$

Subject to boundary conditions

$$u(0) = 1, \; u'(0) = 0, \; u''(0) = -1, \; u'''(0) = 2 \;,$$

$$u(1) = 2\exp(-1), \; u'(1) = -\exp(-1) \; and \; u''(1) = 0.$$

The analytical solution of the problem is $u(x) = x(1-x)\exp(-x)$. The $MAEU$ and $MAEV$ computed by method (2.4-2.5) for coupling constant C =:4099 and different values of N are presented in Table 2.

Table 2 Maximum absolute error (Problem 2)

	N		
	16	32	64
MAEU	.10952180(-3)	.11572996(-6)	.97807437(-7)
MAEV	.44546052(-3)	.30212423(-5)	.56484023(-5)

The numerical results obtained in numerical experiment on considered model problems are satisfactory. The error in numerical result decreases as step size h decreases. In our result, we have estimated the value of the coupling constant by guess and simulation. However accurate value of the coupling constant may possible increase the accuracy of the method. If we do not take an appropriate coupling constant then in this situation proposed method may not converge. We get numerical approximation of the fist derivative of solution of problem as a byproduct the proposed method (2.5-2.7).

Conclusion

In the present article, we have developed the numerical solutions of seventh order differential equations and corresponding boundary value problem by method of finite differences and splitting. We transformed the problem into system of problems by introducing a smooth augment function. The system of problems at nodal points $x = x_i$, $i = 1, 2.., N$ reduced to a system of algebraic equations (2.5-2.7). The system of algebraic equations is linear if source function $f(x, u)$ is linear otherwise nonlinear. The propose method in numerical experiments has shown its performance; also we get numerical approximation of first derivative of the solution as an intermediate result. In future work, we shall work with an improvement in present idea. Work in this direction is in progress.

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Conflicts of interest

The auhtor declares there is no conflict of interest.

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