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On M-asymmetric preirresolute multifunctions

Abstract

In this manuscript, we introduce and investigate a new form of mappings known as M-asymmetric preirresolute multifunctions defined on weaker form of sets in biminimal structure spaces. The relationship between such mappings in our sense and other form of asymmetric precountinuous and preirresolute mappings defined on biminimal structure spaces are discussed and their results established.

Keywords: m-space, biminimal structure spaces, m-asymmetric preopen sets, multifunction, upper and lower M-asymmetric preirresolute multifunctions and M-asymmetric preirresolute multifunctions

Volume 4 Issue 1 - 2023

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Received: August 11, 2023 | **Published:** September 07, 2023

Introduction

Multifunctions and its the concept in the theory of classical point set topology and its classical relation to continuity and irresoluteness, plays an important role in both functional analysis some branches of applied science, such as; engineering, mathematical economics and fuzzy (asymmetric)-topology. Considerable, this notion has received attention by many scholars and in this regard, several generalizations of continuity and irresoluteness of multifunctions using some weaker forms of sets namely; semiopen and preopen sets both in topological spaces, bitopological spaces and minimal structures has be done. The fundamental notion of semiopen sets and semicontinuity of mappings on such sets was first introduced by Levine. Maheshwari and Prasad then generalized and investigated several properties of semiopen sets and semicontinuity in the realm of bitopological spaces. They established that, openness and continuity implies semiopenness and semicontinuity however, not conversely. Minimal structures, a generalization of a topological spaces on a given nonempty set without a summed structure were first introduced and studied by Popa and Noiri. The concept of M-continuity of a function defined between a minimal structure and a topological space was investigated and they proved that M-continuous functions have properties similar to those of continuous functions between topological spaces.

In 1963 on the other hand, Berge introduced a new form of point-to-set mappings called upper and lower continuous multifunctions and investigated some characterizations and relationship of such mappings to other well-known continuous mappings. Lately, Popa extended this concept of point-to-set mappings to bitopological spaces and investigated how such mappings preserved the conserving properties of connectedness, compactness and paracompactness. Multifunctions has fruther been extended to the settings of minimal structure by various authors, for instance, Noiri and Popa in studied upper and lower M-continuous multifunctions, a generalization of results deal to Berge and Popa and Noiri respectively. It has observed in that, upper and lower M-continuous multifunctions have properties similar to those of upper and lower continuous functions and multifunctions in topological setting. Recently, Matindih and Moyo have generalized the ideas in to studied M-asymmetric semicontinuous multifunctions and proved that, such mappings have properties similar to those of upper and lower continuous functions and M-continuous multifunctions between topological spaces, with the difference that, the semiopen sets in use are asymmetric.

In relation to works by Levine, a new form of openness and continuity namely; preopen sets and precontinuity in the realm of topological spaces was first introduced and investigated by Mashhour et al. They established that every openness and continuity of a function implies preopenness and precontinuity but not conversely in general. Min and Kim in on the other hand have generalized the concepts in and respectively to study properties of the so called m-preopen sets and M-precontinuous functions defined between a minimal structure and a topological space. They proved that, every M-continuous function is M-precontinuous but not conversely. On the other hand, Boonpok et al. Has gone further to generalizing the results by studying $(\mathcal{T}_1,\mathcal{T}_2)$ -precontinuous multifunctions in bitopological spaces and obtained several characterizations.

In 1972, irresolute maps and their fundamental properties, were first introduced and investigated by Crossley and Hildebrand. They observed that, irresolute mappings are generally not continuous and neither are continuous mappings necessarily irresolute. Since then, several authors have extended this concept to the study pf irresolute multifunctions. Ewert and Lipski in for instance have studied upper and lower irresolute multivalued mappings, followed by Popa who investigated some properties of upper and lower irresolute multifunctions in topological spaces. As a generalization of ideas in and, Matindih et al. in and respectively, have established in the realm of bitopological spaces that, (upper and lower)-M-asymmetric irresolute multifunctions relates to those of (upper and lower)-M-irresolute multifunctions and note that, (upper and lower)-M-asymmetric irresolute multifunctions are (upper and lower)-M-asymmetric-continuous and not in general conversely. As an extended of results in on the other hand, Popa et-al in have study upper and lower preirresolute multifunctions and established their relation to precontinuous functions. And more recently, Matindih et al. in have generalized Popa et al. ideas to study upper and lower M-asymmetric preirresolute multifunctions defined on asymmetric preopen and preclosed sets satisfying certain minimal conditions.

As an extension of the results by Matindih et al. in and generalize of some results deal to Mahmoud, this manuscript introduces and investigate a new form of mappings, M-asymmetric preirresolute multifunctions defined between biminimal structure spaces. Their relationship to M-precontinuous and M-preirresolute-multifunctions will be discussed.[1–23]

Preliminaries and basic properties

In this section, we give some important properties and notations to be used in this paper. For more details, we refer the reader to,, , , , .

In the sense of Kelly, a bitopological space is a nonempty set X on which are defined two topologies $\mathcal{T}_1$ and $\mathcal{T}_2$ called the left and right topologies respectively. In the sequel, $(X,\mathcal{T}_1,\mathcal{T}_2)$ or in shorthand X will denote a bitopological space unless clearly stated. For a bitopological space $(X,\mathcal{T}_i,\mathcal{T}_j)$, $i,j=1,2$; $i\neq j$, the interior and closure of a subset E of X with respect to the topology $\mathcal{T}_i=\mathcal{T}_j$ shall respectively be denote by $Int_{\mathcal{T}_i}(E)$ and $Cl_{\mathcal{T}_i}(E)$

Definition 1: Let $(X,\mathcal{T}_i,\mathcal{T}_j)$, $i,j=1,2$; $i\neq j$ be a bitopological space.

1. A subset E of X is said to be $\mathcal{T}_i\mathcal{T}_j$ -open if, $E=E_i\cup E_j$ where $E_i\in\mathcal{T}_i$ and $E_j\in\mathcal{T}_j$. The complement of an $\mathcal{T}_i\mathcal{T}_j$ -open set is a $\mathcal{T}_i\mathcal{T}_j$ -closed set.
2. The $\mathcal{T}_i\mathcal{T}_j$ -interior of E denoted by $Int_{\mathcal{T}_i}\left(Int_{\mathcal{T}_j}(E)\right)$ is the union of all $\mathcal{T}_i\mathcal{T}_j$ -open subsets of X contained in A. Clearly if $E=Int_{\mathcal{T}_i}\left(Int_{\mathcal{T}_j}(E)\right)$, then E is $\mathcal{T}_i\mathcal{T}_j$ -open.
3. The $\mathcal{T}_i\mathcal{T}_j$ -closure of E denoted by $Cl_{\mathcal{T}_i}\left(Cl_{\mathcal{T}_j}(E)\right)$ is defined to be the intersection of all $\mathcal{T}_i\mathcal{T}_j$ -closed subsets of X containing A. Note that asymmetrically, $Cl_{\mathcal{T}_i}\left(Cl_{\mathcal{T}_j}(E)\right)\subseteq Cl_{\mathcal{T}_i}(E)$ and $Cl_{\mathcal{T}_i}\left(Cl_{\mathcal{T}_j}(E)\right)\subseteq Cl_{\mathcal{T}_j}(E)$.

Definition 2: A subfamily m_X of a power set 2^X of a nonempty set X is said to be a minimal structure (briefly m -structure) on X if both $\varnothing$ and X are members of m_X. The pair (X,m_X) is called an m -space and the members of (X,m_X) are said to be m_X -open.

Definition 3: Let $(X,\mathcal{T}_i,\mathcal{T}_j)$, $i,j=1,2$; $i\neq j$, be a bitopological space and m_X a minimal structure on X generated with respect to m_i and m_j. We shall call the ordered pair $\left((X,\mathcal{T}_i,\mathcal{T}_j),m_X\right)$ a minimal bitopological space.

Since the minimal structure m_X is determined by the left and right minimal structures m_i and m_j, $i,j=1,2$; $i\neq j$, we shall denote it by $m_{ij}(X)$ (or simply m_{ij}) and call the pair (X,m_{ij})) a minimal bitopological space as in the sense of Matindih and Moyo or a biminimal structure space as in the sense of Boonpok unless explicitly defined.

Definition 4: A biminimal structure $m_{ij}(X)$, $i,j=1,2$; $i\neq j$, on X is said to have property $(\mathcal{B})$ of Maki if the union of any collection of $m_{ij}(X)$ -open subsets of X belongs to $m_{ij}(X)$.

Definition 5: Let $(X,m_{ij}(X))$, $i,j=1,2$; $i\neq j$ be a biminimal structure space. Then a subset E of X is said to be:

1. $m_{ij}(X)$ -preopen if there exists an m_i -open set O such that $E\subseteq O\subseteq Cl_{m_j}(E)$ or equivalently, $E\subseteq Int_{m_i}\left(Cl_{m_j}(E)\right)$
2. $m_{ij}(X)$ -preclosed if there exists an m_i -open set O such that $Cl_{m_j}(O)\subseteq E$ whenever $E\subseteq O$, that is, $Cl_{m_i}\left(Int_{m_j}(E)\right)\subseteq E$.

The collection of all m_{ij} -preopen and m_{ij} -preclosed sets in $(X,m_{ij}(X))$ will be denote by $m_{ij}pO(X)$ and $m_{ij}pC(X)$ respectively.

Remark 6: Let $(X,m_{ij}(X))$, $i,j=1,2$; $i\neq j$ be a biminimal structure space.

1. if $m_i=\mathcal{T}_i$ and $m_j=\mathcal{T}_j$, the any $m_{ij}(X)$ -preopen set is $\mathcal{T}_i\mathcal{T}_j$ -preopen.
2. every $m_{ij}(X)$ -open set is $m_{ij}(X)$ -preopen, however, the converse is not necessarily true.

We should note that, the m_{ij} -open sets and m_{ij} -preopen sets, are not stable under the union operation. However, for certain m_{ij} -structures, the class of m_{ij} -preopen sets are stable under union of sets, as in the Lemma below.

Lemma 7: *Let* $(X,m_{ij}(X))$, $i,j=1,2$; $i\neq j$ *be a biminimal structure space and* $\{E_\gamma:\gamma\in\Gamma\}$ *be a family of subsets of* X. *The following properties holds:*

1. $\bigcup_{\gamma\in\Gamma}E_\gamma\in m_{ij}pO(X)$ *provided for all* $\gamma\in\Gamma$, $E_\gamma\in m_{ij}pO(X)$.

2. $\bigcap_{\gamma\in\Gamma}E_\gamma\in m_{ij}pC(X)$ *provided for all* $\gamma\in\Gamma$, $E_\gamma\in m_{ij}pC(X)$.

Remark 8: In a biminimal structure space $(X,m_{ij}(X))$, it may happen that, the intersection of some two m_{ij} -preopen sets is not m_{ij} -preopen.

Definition 9: Let $(X,m_{ij}(X))$, $i,j=1,2$; $i\neq j$ be a biminimal structure space. A subset N of X is said to be an m_{ij} -preneighborhood of a:

1. point x in X if there exists an m_{ij} -preopen subset O of X such that $x\in O\subseteq N$.
2. subset E of X if there exists an m_{ij} -preopen subset O of X such that $E\subseteq O\subseteq N$.

Citation: Matindih LK, Manyika DK, Mukonda D. On M-asymmetric preirresolute multifunctions. *Open Acc J Math Theor Phy.* 2023;4(1):15–23.
DOI: 10.15406/oajmtp.2023.04.00061

Definition 10: Let $(X, m_{ij}(X))$, $i, j = 1,2$; $i \neq j$ be a biminimal structure space. We defined and denoted the m_{ij} -preinterior and m_{ij} -preclosure of a nonempty subset E of X respectively by:

1. \$m_{ij}(X)pInt(E) = \bigcup\{U : U \subseteq E \hspace{0.3cm} and \hspace{0.3cm} U \in m_{ij}pO(X) \}\$,
2. \$m_{ij}(X)pCl(E) = \bigcap\{F : E \subseteq F \hspace{0.2cm} and \hspace{0.2cm} X \setminus F \in m_{ij}pO(X)\}\$,

Remark 11: For any bitopological spaces $(X, \mathcal{T}_1, \mathcal{T}_2)$;

1. $\mathcal{T}_i\mathcal{T}_j pO(X)$ is a biminimal structure of X .
2. For a nonempty subset E of X , if $m_{ij}(X) = \mathcal{T}_i\mathcal{T}_j pO(X)$, then by Definition 10;

a. $m_{ij} Int(E) = \mathcal{T}_i\mathcal{T}_j pInt(E)$,

b. $m_{ij} Cl(E) = \mathcal{T}_i\mathcal{T}_j pCl(E)$.

Lemma 12: *Given a biminimal structure space* $(X, m_{ij}(X))$, $i, j = 1,2$; $i \neq j$ *and subsets* E *and* B *of* X *, the following properties concerning* m_{ij} *-preinterior and* m_{ij} *-preclosure holds:*

1. $m_{ij}(X)pInt(E) \subseteq E$ *and* $E \subseteq m_{ij}pCl(E)$.
2. $m_{ij}(X)pInt(E) \subseteq m_{ij}pInt(B)$ *and* $m_{ij}(X)pCl(E) \subseteq m_{ij}pC(B)$ *provided* $E \subseteq B$.
3. $m_{ij}(X)pInt(\varnothing) = \varnothing$, $m_{ij}(X)pInt(X) = X$, $m_{ij}(X)pCl(\varnothing) = \varnothing$ *and* $m_{ij}(X)pCl(X) = X$.
4. $E = m_{ij}(X)pInt(E)$ *provided* $E \in m_{ij}pO(X)$ *and* $E = m_{ij}(X)pCl(E)$ *provided* $X \setminus E \in m_{ij}pO(X)$.
5. $m_{ij}(X)pInt\left(m_{ij}(X)pInt(E)\right) = m_{ij}(X)pInt(E)$ *and* $m_{ij}(X)pCl\left(m_{ij}(X)pCl(E)\right) = m_{ij}(X)pCl(E)$

Lemma 13: *Let* $(X, m_{ij}(X))$, $i, j = 1,2$; $i \neq j$ *be biminimal structure space and* E *a nonempty subset of* X *. Then,* $U \cap E \neq \varnothing$ *for each* $U \in m_{ij}pO(X)$ *containing* x_o *, if and only if* $x_o \in m_{ij}pCl(E)$.

Lemma 14: *Given a biminimal structure space* $(X, m_{ij}(X))$, $i, j = 1,2$; $i \neq j$ *and subsets* E *of* X *, the following properties holds:*

1. $m_{ij}(X)pCl(X \setminus E) = X \setminus \left(m_{ij}(X)pInt(E)\right)$,
2. $m_{ij}(X)pInt(X \setminus E) = X \setminus \left(m_{ij}(X)pCl(E)\right)$.
3. $m_{ij}(X)pCl(E) = Cl_{m_i}\left(Int_{m_j}(E)\right)$ *provided* $E \in m_{ij}pO(X)$*. The converse to this assertion is not necessarily true.*

Remark 15: For a bitopological space $(X, \mathcal{T}_i, \mathcal{T}_j)$, $i, j = 1,2$; $i \neq j$ the families $\mathcal{T}_i\mathcal{T}_j O(X)$ and $\mathcal{T}_i\mathcal{T}_j pO(X)$ are all m_{ij} -structures of X satisfying property β .

Lemma 16: *Let* $(X, m_{ij}(X))$, $i, j = 1,2$; $i \neq j$ *be a biminimal structure space satisfying property* β *and* E *and* F *be subsets of* X *. Then, the properties below holds:*

1. $m_{ij}(X)pInt(E) = E$ *provided* $E \in m_{ij}pO(X)$.
2. $X \setminus F \in m_{ij}pO(X)$ *provided* $m_{ij}(X)pCl(F) = F$.
3. *Further, if* X *satisfy property* β *, then*
4. $E = m_{ij}(X)pInt(E)$ *if and only if* A *is an* $m_{ij}(X)$ *-preopen set.*
5. $E = m_{ij}(X)pCl(E)$ *if and only if* $X \setminus E$ *is an* $X \setminus Em_{ij}(X)$ *-preopen set.*
6. $m_{ij}(X)pInt(E)$ *is* $m_{ij}(X)$ *-preopen and* $m_{ij}(X)pCl(E)$ *is* $m_{ij}(X)$ *-preclosed.*

Lemma 17: *Let* $(X, m_{ij}(X))$, $i, j = 1,2$; $i \neq j$ *be a biminimal structure space satisfying the property* β *and* $\{E_\gamma : \gamma \in \Gamma\}$ *be an arbitrary collection of subsets of* X *. Then,* $\bigcup_{\gamma \in \Gamma} E_\gamma \in m_{ij}pO(X)$ *provided* $E_\gamma \in m_{ij}pO(X)$ *for every* $\gamma \in \Gamma$.

DOI: 10.15406/oajmtp.2023.04.00061

Lemma 18: *Given a biminimal structure space* $(x, m_{ij}(x))$, $i, j = 1,2$; $i \neq j$ *satisfying property* $\mathcal{B}$ *and a nonempty subset* E *of* X *, the following properties holds:*

1. $m_{ij}(X)\,pInt(E) = E \cap Int_{m_i}\left(Cl_{m_j}(E)\right)$, *and*
2. $m_{ij}(X)\,pCl(E) = E \cup Cl_{m_i}\left(Int_{m_j}(E)\right)$ *holds.*

And the equality does not necessarily hold if the property $\mathcal{B}$ *of Make is removed. Further, for any subset* U *of* X *,*

3. $m_{ij}(X)\,pInt(U) \subseteq Int_{m_i}\left(Cl_{m_j}\left(m_{ij}\,pInt(U)\right)\right) \subseteq Int_{m_i}\left(Cl_{m_j}(U)\right)$.
4. $Cl_{m_i}\left(Int_{m_j}(U)\right) \subseteq Cl_{m_i}\left(Int_{m_j}\left(m_{ij}(X)pCl(U)\right)\right) \subseteq m_{ij}(X)\,pCl(U)$.

Definition 19: A point-to-set correspondence $F: X \to Y$ between two topological spaces X and Y such that for each point x of X , $F(x)$ is a none-void subset of Y is called a multifunction.

In the sense of Berge, we shall denote and define the upper and lower inverse of a nonempty subset G of Y with respect to a multifunction F respectively by:

$$F^{+}(G) = \{x \in X : F(x) \subseteq G \} \hspace{0.3cm} \text{and} \hspace{0.3cm} F^{-}(G) = \{x \in X : F(x) \cap G \neq \emptyset\}.$$

Generally, F^{-} and F^{+} between Y and the power set 2^X , $F^{-}(y) = \{x \in X : y \in F(x)\}$ provided $y \in Y$. Clearly for a nonempty subset G of Y , $F^{-}(G) = \bigcup\{F^{-}(y) : y \in G\}$ and also,

$$F^{+}(G) = X \setminus F^{-}(Y \setminus G) \hspace{0.2cm} \text{and} \hspace{0.2cm} F^{-}(G) = X \setminus F^{+}(Y \setminus G)$$

For any non-void subsets E and G of X and Y respectively, $F(E) = \bigcup_{x \in E} F(x)$ and $E \subseteq F^{+}(F(E))$ and also, $F(F^{+}(G)) \subseteq G$.

Definition 20: Let $(X, m_{ij}(X))$ and $(Y, m_{ij}(Y))$, $i, j = 1,2$; $i \neq j$ be biminimal structure spaces. A multifunction $F:(X, m_{ij}(X)) \to (Y, m_{ij}(Y))$ is said to be:

1. Upper m_{ij} -semi-continuous at some point $x_o \in X$ provided for any $m_{ij}(Y)$ -open set V satisfying $V \supseteq F(x_o)$, there is an $m_{ij}(X)$ -semiopen set O with $x_o \in U$ for which $V \supseteq F(O)$, whence, $F^{+}(V) \supseteq O$.
2. Lower m_{ij} -semi-continuous at some point $x_o \in X$ provided for each $m_{ij}(Y)$ -open set V satisfying $V \cap F(x_o) \neq \varnothing$, we can find an $m_{ij}(X)$ -semi open set O with $x_o \in O$ such that for all $x_o \in O$, $V \cap F(x) \neq \varnothing$.
3. Upper (resp Lower) m_{ij} -semi continuous if it is Upper (resp Lower) m_{ij} -semi continuous at each and every point of X .

Definition 21: A multifunction $F:(X, m_{ij}(X)) \to (Y, m_{ij}(Y))$ $i, j = 1,2$; $i \neq j$, between biminimal structure spaces X and Y is said to be M -Asymmetric irresolute at a point $x_o \in X$ if for any $m_{ij}(Y)$ -semiopen sets H_1 and H_2 such that $H_1 \supseteq F(x_o)$ and $H_2 \cap F(x_o) \neq \varnothing$, there exists an $m_{ij}(X)$ -semiopen set O containing x_o such that $H_1 \supseteq F(O)$ and $H_2 \cap F(x) \neq \varnothing$ for every $x \in O$.

The multifunction F is M -Asymmetric irresolute if it is M -Asymmetric irresolute at every point $x_o \in X$.

Definition 22: A multifunction $F:(X, m_{ij}(X)) \to (Y, m_{ij}(Y))$, $i, j = 1,2$; $i \neq j$ between biminimal sturcture spaces X and Y is called:

1. upper M -asymmetric preirresolute at a point $x_o \in X$ provided for any $m_{ij}(Y)$ -preopen subset G such that $F(x_o) \subseteq G$, there exists an $m_{ij}(X)$ -preopen set O with $x \in O$ such that $F(O) \subseteq G$ whence $O \subseteq F^{+}(G)$.
2. Lower M -asymmetric preirresolute at a point $x_o \in X$ provided for any $m_{ij}(Y)$ -preopen set G such that $G \cap F(x_o) \neq \varnothing$, there exists a $m_{ij}(X)$ -preopen set O with $x \in O$ such that $F(x) \cap G \neq \varnothing$ for all $x \in O$ whence $O \subseteq F^{-}(G)$.
3. Upper (resp lower) M -asymmetric preirresolute provided it is upper (resp lower) M -Asymmetric preirresolute at each and every point x_o of X .

Some characterization on M-asymmetric preirresolute multifunctions

We now introduce an M -Asymmetric-preirresolute multifunction F and investigate some of its characterizations, as well look at some of it's relationship to M -Asymmetric-precontinuous multifunctions.

DOI: 10.15406/oajmtp.2023.04.00061

Definition 23: Let $\left(X, m_{ij}(X)\right)$ and $\left(Y, m_{ij}(Y)\right)$, $i, j = 1,2$; $i \neq j$ be biminimal structure spaces. A multifunction $F : X \to Y$ is said to be:

1. M -Asymmetric preirresolute at a point x_o of X provided for any $m_{ij}(Y)$ -preopen sets H_1 and H_2 such that $F(x_o) \subseteq H_1$ and $H_2 \cap F(x_o) \neq \varnothing$, there is some m_{ij} -preneighborhood O of x_o with $F(O) \subseteq H_1$ and, for any other $x \in O$, $H_2 \cap F(x) \neq \varnothing$.
2. M -Asymmetric irresolute if it is M -Asymmetric irresolute at every point x_o of X.

Remark 24: Obviously, any M -asymmetric preirresolute multifunction is both upper and lower M -Asymmetric preirresolute and vice-versa

Theorem 25: A multifunction $F : \left(X, m_{ij}(X)\right) \to \left(Y, m_{ij}(Y)\right)$, $i, j = 1,2$; $i \neq j$ where Y satisfies property $\mathcal{B}$ is M -asymmetric preirresolute at a point x_o in X if and only if for any $m_{ij}(Y)$-preopen sets H_1 and H_2 such that $F(x_o) \subseteq H_1$ and $H_2 \cap F(x_o) \neq \varnothing$, the relation $x_o \in Int_{m_i}(Cl_{m_j}\left(F^+(H_1) \cap F^-(H_2)\right)$ holds.

Proof: Suppose the multifunction F is M -asymmetric preirresolute at the point x_o in X. Let H_1 and H_2 be $m_{ij}(Y)$-preopen sets satisfying $F(x_o) \supseteq H_1$ and $F(x_o) \cap H_2 \neq \varnothing$. Then, there is some $m_{ij}(X)$-preopen neighbourhood O of x_o for which $F(O) \subseteq H_1$ and $F(x) \cap H_2 \neq \varnothing$ for every $x \in O$. Thus, $F(O) \subseteq H_1 \subseteq Int_{m_i}\left(Cl_{m_j}(H_1)\right)$ and $F(x_o) \subseteq F(x_o) \cap H_2 \subseteq Int_{m_i}\left(Cl_{m_j}(F(x) \cap H_2)\right)$. By the hypothesis of F, $x_o \in O \subseteq F^+(H_1)$ and $x_o \in O \subseteq F^-(H_2)$ giving $x_o \in O \subseteq F^+(H_1) \cap F^-(H_2)$. Since $O \in m_{ij}pO(X)$, $O = m_{ij}(X)\, pInt(O) \subseteq Int_{m_i}\left(Cl_{m_j}(O)\right)$. Since Y satisfies property $\mathcal{B}$, as a result, then Lemmas 16 and 17 gives, $x_o \in O = m_{ij}pInt(O) \subseteq Int_{m_i}\left(Cl_{m_j}(O)\right) \subseteq Int_{m_i}(Cl_{m_j}\left(F^+(H_1) \cap F^-(H_2)\right)$.

Conversely, assume $x_o \in Int_{m_i}(Cl_{m_j}\left(F^+(H_1) \cap F^-(H_2)\right)$ for $H_1, H_2 \in m_{ij}pO(Y)$ satisfying $F(x_o) \subseteq H_1$ and $F(x_o) \cap H_2 \neq \varnothing$. By Lemma 13, there exists an $m_{ij}(X)$-preopen neighborhood O of x_o with $x_o \in O \subseteq \left(F^+(H_1) \cap F^-(H_2)\right)$. So $x_o \in O \subset (F^+(H_1)$ and $x_o \in O \subseteq F^-(H_2)$. Since the sets H_1 and H_2 are $m_{ij}(Y)$-preopen, we get that $F(O) \subseteq H_1 \subseteq Cl_{m_j}\left(Int_{m_i}(H_1)\right)$ and $H_2 \cap F(x) \neq \varnothing$ for all $x \in O$. But $O \in m_{ij}pO(X)$, as a result, we conclude that, F is M -asymmetric preirresolute at x_o in X.

Theorem 26: A multifunction $F : \left(X, m_{ij}(X)\right) \to \left(Y, m_{ij}(Y)\right)$ $i, j = 1,2$; $i \neq j$ with Y satisfy property $\mathcal{B}$ is M -asymmetric preirresolute at a point x_o in X if and only if for any m_{ij} -preneighborhood O of x_o and any $m_{ij}(Y)$ -semiopen sets H_1 and H_2 such that $F(x_o) \subseteq H_1$ and $H_2 \cap F(x_o) \neq \varnothing$, there exists an $m_{ij}(X)$-open set $U_O \subseteq O$ such that $F(U_O) \subseteq H_1$ and for each $x \in U_O$, $H_2 \cap F(x) \neq \varnothing$.

***Proof*:** Assume that F is M -asymmetric preirresolute at x_o a point in X. Let H_1 and H_2 be $m_{ij}(Y)$-preopen sets satisfying $F(x_o) \subseteq H_1$ and $F(x_o) \cap H_2 \neq \varnothing$ respectively. Then, by Theorem 25, $x_o \in Int_{m_i}(Cl_{m_j}\left(F^+(H_1) \cap F^-(H_2)\right)$. Let O be an $m_{ij}(X)$-preopen neighbourhood of x_o. By Remark 6(2), $F(O) \subseteq H_1$ and $F(O) \subseteq H_2$, whence $O \subseteq F^+(G)$ and $O \subseteq F^-(H_2)$, so that, $O \cap Int_{m_i}\left(Int_{m_j}\left(F^+(H_1) \cap F^-(H_2)\right)\right) \neq \varnothing$. It is a well known that, $Int_{m_i}\left(Int_{m_j}\left(F^+(H_1) \cap F^-(H_2)\right)\right) = Int_{m_i}\left(Int_{m_j}\left(F^+(H_1)\right)\right) \cap Int_{m_i}\left(Int_{m_j}\left(F^-(H_2)\right)\right)$ and as a result, $O \cap \left[Int_{m_i}\left(Int_{m_j}\left(F^+(H_1)\right)\right) \cap Int_{m_i}\left(Int_{m_j}\left(F^+(H_2)\right)\right)\right] \neq \varnothing$.

Since $Int_{m_i}\left(Int_{m_j}\left(F^+(H_1) \cap F^-(H_2)\right)\right) = Int_{m_i}\left(Int_{m_j}\left(F^+(H_1)\right)\right) \cap Int_{m_i}\left(Int_{m_j}\left(F^-(H_2)\right)\right) \subseteq F^+(H_1) \cap F^-(H_2)$
$\subseteq Int_{m_i}\left(Cl_{m_j}\left(F^+(H_1) \cap F^-(H_2)\right)\right)$,

Lemma 1 3then implies $O \cap Cl_{m_j}\left(Int_{m_i}\left(F^+(H_1) \cap F^-(H_2)\right)\right) \neq \varnothing$. Put $O \cap \left[Int_{m_i}\left(Int_{m_j}\left(F^+(H_1)\right)\right) \cap Int_{m_i}\left(Int_{m_j}\left(F^-(H_2)\right)\right)\right] = U_O$. Then, $U_O \neq \varnothing$, $U_O \subseteq O$, $U_O \subseteq F^+(H_1)$, $U_O \subseteq F^-(H_2)$ implying that, U_O is $m_{ij}(X)$-open subset of X. Consequently, $F(U_O) \subseteq H_1$ and $F(x) \cap H_2 \neq \varnothing$ for all $x \in U_O$.

Conversely, let $\{O_{x_O}\}$ be a space of all $m_{ij}(X)$-open neighbourhoods of a point x_o and let O be any member of $\{O_{x_O}\}$. Then, for any $m_{ij}(Y)$-preopen sets H_1 and H_2 satisfying $F(x_o) \subseteq H_1$ and $F(x_o) \cap H_2 \neq \varnothing$, there exists some $m_{ij}(X)$-open set U_O contained in O such that for all $x \in U_O$, $F(U_O) \subseteq H_1$ and $F(x) \cap H_2$. Set $Q = \bigcup_{O \in O_{x_o}} U_O$. Then Q is an $m_{ij}(X)$-open set, $x_o \in Cl_{m_i}\left(Cl_{m_j}(Q)\right)$ by Theorem 25, giving

Citation: Matindih LK, Manyika DK, Mukonda D. On M-asymmetric preirresolute multifunctions. *Open Acc J Math Theor Phy.* 2023;4(1):15‒23.
DOI: 10.15406/oajmtp.2023.04.00061

$F(Q)\subseteq H_1$ and $F(q)\cap H_2\neq\varnothing$ for all $q\in Q$. Set $W=\{x_o\}\cup Q$, then $Q\subseteq W\subseteq Cl_{m_i}\left(Cl_{m_j}(Q)\right)$. So, Q is $m_{ij}(X)$-preopen, $x_o\in W$, $F(W)\subseteq H_1$ and $F(w)\cap H_2\neq\varnothing$ for all $w\in W$, whence, $x_o\in W\subseteq F^+(H_1)$ and $x_o\in W\subseteq F^-(H_2)$. Consequently, F is an $\mathcal{M}$-asymmetric preirresolute multifunction at x_o in X.

Remark 27: It should be noted from Theorem 26 that, an $\mathcal{M}$ -asymmetric preirresolute multifunction is generally $\mathcal{M}$ -asymmetric precontinuous and not generally conversely as we see in Example 28.

Example 28: Define the minimal structures on a set $X=\{a,b,c,d,e\}$ by $m_1(X)=\{X,\varnothing,\{a\},\{c\},\{c,d,f\}\}$ and $m_2(X)=\{X,\varnothing,\{b\},\{d\},\{c,e,f\},X\}$ and on $Y=\{-1,0,1,2\}$ by $m_1(Y)=\{Y,\varnothing,\{-1\},\{2\},\{-1,2\},\{-1,0,2\},Y\}$ and $m_2(Y)=\{Y,\varnothing,\{1\},\{-1,0\},\{1,2\},\{-1,0,1\}\}$.

Define a multifunction $F:\left(X,m_{ij}(X)\right)\to\left(Y,m_{ij}(Y)\right)$ by $F(a)=\{2\}$, $F(c)=\{-1,0\}$ and $F(d)=\{1,2\}$. Clearly, F is $\mathcal{M}$ -asymmetric preirresolute $a\in X$. Indeed, $F(a)\subseteq\{2\}\in m_{ij}sO(Y)$ and $F(a)\cap\{-1,2\}\neq\varnothing$ for the $m_{ij}(Y)$-preopen sets $\{2\}$ and $\{-3,1,3\}$ whence, $F(\{a\})\subset\{-1,2,3\}$ and $F(\{a\})\cap\{-1,2\}\neq\varnothing$. This property respectively holds for some $m_{ij}(X)$-preopen sets containing c and d. Since $\left(Y,m_{ij}(Y)\right)$ as defined satisfies property $\mathcal{B}$, it follows that, F is $\mathcal{M}$ -asymmetric precountinuous.

Theorem 29: Let the m_{ij} -space $\left(Y,m_{ij}(Y)\right)$, $i,j=1,2$; $i\neq j$ satisfy property $\mathcal{B}$. Then for any multifunction $F:\left(X,m_{ij}(X)\right)\to\left(Y,m_{ij}(Y)\right)$, the following properties are equivalent:

1. F is $\mathcal{M}$ -asymmetric irresolute,
2. For if $H_1,H_2\in m_{ij}pO(Y)$, then $F^+(H_1)\cap F^-(H_2)\in m_{ij}pO(X)$.
3. For if $K_1,K_2\in m_{ij}pC(Y)$, then $F^-(K_1)\cup F^+(K_2)\in m_{ij}pC(X)$.
4. The set inclusion $Cl_{m_i(X)}\left(Int_{m_j(X)}\left(F^-(E_1)\cup F^+(E_2)\right)\right)\subseteq F^-\left(m_{ij}(Y)pCl(E_1)\right)\cup F^+\left(m_{ij}(Y)pCl(E_2)\right)$, holds for any $E_1,E_2\subseteq Y$.
5. The set inclusion $m_{ij}(X)pCl\left(F^-(V_1)\cup F^+(V_2)\right)\subseteq F^-\left(m_{ij}(Y)pCl(V_1)\right)\cup F^+\left(m_{ij}(Y)pCl(V_2)\right)$; holds for any given $V_1,V_2\subseteq Y$.
6. The set inclusion $m_{ij}(X)pInt\left(F^-(Q_1)\cap F^+(Q_2)\right)\supseteq F^-\left(m_{ij}(Y)pInt(Q_1)\right)\cap F^+\left(m_{ij}(Y)pInt(Q_2)\right)$, holds for any given $Q_1,Q_2\subseteq Y$.

Proof:

1. (1) $\Rightarrow$ (2): Assume (1) holds. Let H_1 and H_2 be any $m_{ij}(Y)$-preopen sets satisfying $F(x_o)\subseteq H_1$ and $F(x_o)\cap H_1\neq\varnothing$, for x_o a point in X. Then, for some $m_{ij}(X)$-preopen neighborhood O of x_o, $F(O)\subseteq H_1$ and $F(x)\cap H_2\neq\varnothing$ for all x contained in O. Thus, $F(x_o)\subseteq F(O)\subseteq H_1$ and $F(x_o)\subseteq F(O)\subseteq H_2$ and by hypothesis, $x_o\in O\subset F^+(H_1)\cap F^-(H_2)$. Theorem 3.3 and 3.9 of implies, $x_o\in F^+(H_1)\subseteq Int_{m_i}\left(Cl_{m_j}\left(F^+(H_1)\right)\right)$ and $x_o\in F^-(H_2)\subseteq Int_{m_i}\left(Cl_{m_j}\left(F^-(H_2)\right)\right)$ respectively. From Theorem 25, it follows that, $x_o\in Int_{m_i}\left(Cl_{m_j}\left(F^+(H1)\cap F^-(H1)\right)\right)$

   Consequently, $F^+(H_1)\cap F^-(H_2)$ is an $m_{ij}(X)$-preopen set as the point x_o was arbitrarily chosen.
2. (2) $\Rightarrow$ (3): Assume (2) holds. Since by Lemma 16, $Y\setminus K_1$ and $Y\setminus K_2$ are $m_{ij}(Y)$-preopen sets for any $m_{ij}(Y)$-preclosed sets K_1 and K_2 and as well $F^+(Y\setminus K_1)=X\setminus F^-(K_1)$ and $F^-(Y\setminus K_2)=X\setminus F^+(K_2)$, we have by Lemma 14 that;

$$\begin{aligned}X\setminus F^-(K_1)=F^+(Y\setminus K_1)&\supseteq m_{ij}(X)pInt\left(F^+(Y\setminus K_1)\right)\\&=m_{ij}(X)pInt\left(X\setminus F^-(K_1)\right)\\&=X\setminus m_{ij}(X)pCl\left(F^-(K_1)\right)\end{aligned}$$

giving, $F^-(K_1)\supseteq m_{ij}(X)pCl\left(F^-(K_1)\right)$. Similarly,

$$\begin{aligned}X\setminus F^+(K_2)=F^-(Y\setminus K_2)&\supseteq m_{ij}(X)pInt\left(F^-(Y\setminus K_2)\right)\\&=m_{ij}(X)pInt\left(X\setminus F^+(K_2)\right)\\&=X\setminus m_{ij}(X)pCl\left(F^+(K_2)\right)\end{aligned}$$

giving $F^{+}\left(K_2\right)\supseteq m_{ij}(X)\,pCl\left(F^{+}\left(K_2\right)\right)$. As a result,

$$\begin{aligned} F^{-}\left(K_1\right)\cup F^{+}\left(K_2\right) &\supseteq m_{ij}(X)\,pCl\left(F^{-}\left(K_1\right)\right)\cup m_{ij}(X)\,pCl\left(F^{+}\left(K_2\right)\right)\\ &= m_{ij}(X)\,pCl\left(F^{-}\left(K_1\right)\cup F^{+}\left(K_2\right)\right)\end{aligned}$$

But then, $F^{-}\left(K_1\right)\cup F^{+}\left(K_2\right)\subseteq m_{ij}(X)\,pCl\left(F^{-}\left(K_1\right)\cup F^{+}\left(K_2\right)\right)$. As a result, by Lemma 7 and 16, $F^{-}\left(K_1\right)\cup F^{+}\left(K_2\right)$ is an $m_{ij}(X)$-preclosed set.

3. (3) $\Rightarrow$ (4): Assume (3) holds. Since $m_{ij}(Y)\,pCl\left(E_1\right)$ and $m_{ij}(Y)\,pCl\left(E_2\right)$ are $m_{ij}(Y)$-preclosed sets for any arbitrary subsets E_1 and E_2 of Y, then $F^{-}\left(m_{ij}(Y)\,pCl\left(E_1\right)\right)\cup F^{+}\left(m_{ij}(Y)\,pCl\left(E_2\right)\right)$ is an $m_{ij}(X)$-preclosed set by Lemma 7. But then, $E_1\subseteq m_{ij}(Y)\,pCl\left(E_1\right)$ and $E_2\subseteq m_{ij}(Y)\,pCl\left(E_2\right)$, it follows that $F^{-}\left(E_1\right)\subseteq F^{-1}\left(m_{ij}(Y)\,pCl\left(E_1\right)\right)$ and $F^{+}\left(E_2\right)\subseteq F^{+}\left(m_{ij}(Y)\,pCl\left(E_2\right)\right)$. From Definition 5, we get

$$\begin{aligned} &F^{-}\left(m_{ij}(Y)\,pCl\left(E_1\right)\right)\cup F^{+}\left(m_{ij}(Y)\,pCl\left(E_2\right)\right)\\ &\supseteq Cl_{m_i(X)}\left(Int_{m_j(X)}\left(F^{-}\left(m_{ij}(Y)\,pCl\left(E_1\right)\right)\cup F^{+}\left(m_{ij}(Y)\,pCl\left(E_2\right)\right)\right)\right)\\ &\supseteq Cl_{m_i(X)}\left(Int_{m_j(X)}\left(F^{-}\left(E_1\right)\cup F^{+}\left(E_2\right)\right)\right).\end{aligned}$$

4. (4) $\Rightarrow$ (5): Let V_1 and V_2 be any subsets of Y and assume that (4) holds. Then, $m_{ij}(Y)\,pCl\left(V_1\right)$ and $m_{ij}(Y)\,pCl\left(V_1\right)$ are all $m_{ij}(Y)$-preclosed sets. But then, by Lemma 18, for any $B\subseteq X$, $m_{ij}(X)\,pCl(B)=B\cup Cl_{m_i(X)}(Int_{m_j(X)}(B)$, we consequently obtain;

$$\begin{aligned} &m_{ij}(X)\,pCl\left(F^{-}\left(V_1\right)\cup F^{+}\left(V_2\right)\right)=\left[F^{-}\left(V_1\right)\cup F^{+}\left(V_2\right)\right]\cup Cl_{m_i(X)}\left(Int_{m_j(X)}\left(F^{-}\left(V_1\right)\cup F^{+}\left(V_2\right)\right)\right)\\ &\subseteq\left[F^{-}\left(V_1\right)\cup F^{+}\left(V_2\right)\right]\cup\left[F^{-}\left(m_{ij}(Y)\,pCl\left(V_1\right)\right)\cup F^{+}\left(m_{ij}(Y)\,pCl\left(V_2\right)\right)\right]\\ &\subseteq F^{-}\left(m_{ij}(Y)\,pCl\left(V_1\right)\right)\cup F^{+}\left(m_{ij}(Y)\,pCl\left(V_2\right)\right)\end{aligned}$$

Hence, the inclusion holds.

5. (5) $\Rightarrow$ (6): Assume (5) is holds and let Q_1 and Q_2 be any subsets of Y. Since $m_{ij}(Y)\,pInt\left(Q_1\right)$ and $m_{ij}(Y)\,pInt\left(Q_2\right)$ are $m_{ij}(Y)$-preopen sets and, by Lemma 14, $m_{ij}(Y)\,pInt\left(Q_1\right)=Y\setminus m_{ij}(Y)\,pCl\left(Y\setminus Q_1\right)$ and $m_{ij}(Y)\,pInt\left(Q_2\right)=Y\setminus m_{ij}(Y)\,pCl\left(Y\setminus Q_2\right)$, we then obtain,

$$\begin{aligned} &X\setminus[F^{-}\left(m_{ij}(Y)\,pInt\left(Q_1\right)\right)\cap F^{+}\left(m_{ij}(Y)\,pInt\left(Q_2\right)\right)]\\ &=\left[X\setminus F^{-}\left(m_{ij}(Y)\,pInt\left(Q_1\right)\right)\right]\cup\left[X\setminus F^{+}\left(m_{ij}(Y)\,pInt\left(Q_2\right)\right)\right]\\ &=F^{+}\left(Y\setminus m_{ij}(Y)\,pInt\left(Q_1\right)\right)\cup F^{-}\left(Y\setminus m_{ij}(Y)\,pInt\left(Q_2\right)\right))\\ &=F^{+}\left(m_{ij}(Y)\,pCl\left(Y\setminus Q_1\right)\right)\cup F^{-}\left(m_{ij}(Y)\,pCl\left(Y\setminus Q_2\right)\right))\\ &\supseteq m_{ij}(X)\,pCl\left(\left(F^{+}\left(Y\setminus Q_1\right)\right)\cup F^{-}\left(Y\setminus Q_2\right)\right))\\ &=m_{ij}(X)\,pCl\left(\left(X\setminus F^{-}\left(Q_1\right)\right)\cup\left(X\setminus F^{+}\left(Q_2\right)\right)\right)\\ &=m_{ij}(X)\,pCl\left(X\setminus\left[F^{-}\left(Q_1\right)\cap F^{+}\left(Q_2\right)\right]\right)\\ &=X\setminus m_{ij}(X)\,pInt\left(F^{-}\left(Q_1\right)\cap F^{+}\left(Q_2\right)\right)\end{aligned}$$

Henceforth, $m_{ij}(X)\,pInt\left(F^{-}\left(Q_1\right)\cap F^{+}\left(Q_2\right)\right)\supseteq F^{-}\left(m_{ij}(Y)\,pInt\left(Q_1\right)\right)\cap F^{+}\left(m_{ij}(Y)\,pInt\left(Q_2\right)\right)$.

6. (6) $\Rightarrow$ (1): Assume that (6) holds and let H_1 and H_2 be any $m_{ij}(Y)$-preopen sets satisfying $F\left(x_o\right)\subseteq H_1$ and $F\left(x_o\right)\cap H_2\neq\varnothing$ for an arbitrary point x_o in X. From the assumption, part (2) and the fact that Y satisfies property $\mathcal{B}$, we obtain,

$$\begin{aligned} F^{+}\left(H_1\right)\cap F^{-}\left(H_2\right) &= F^{+}\left(m_{ij}(Y)\,pInt\left(H_1\right)\right)\cap F^{-}\left(m_{ij}(Y)\,pInt\left(H_2\right)\right)\\ &\subseteq m_{ij}(X)\,pInt\left(F^{+}\left(H_1\right)\cap F^{-}\left(H_2\right)\right)\\ &\subseteq Int_{m_i}\left(Cl_{m_j}\left(F^{+}(G)\cap F^{-}\left(H_2\right)\right)\right)\end{aligned}$$

$$Int_{m_i}\left(Cl_{m_j}\left(F^{+}(G)\cap F^{-}(H_2)\right)\right) \supseteq m_{ij}(X)\,pInt\left(F^{+}(H_1)\cap F^{-}(H_2)\right)$$
$$\supseteq F^{+}\left(m_{ij}(Y)\,pInt(H_1)\right)\cap F^{-}\left(m_{ij}(Y)\,pInt(H_2)\right)$$
$$=F^{+}(H_1)\cap F^{-}(H_2).$$

Thus, $F^{+}(H_1)\cap F^{-}(H_2)$ is an $m_{ij}(X)$-preopen set containing x_o. Set $O=F^{+}(H_1)\cap F^{-}(H_2)$. Then $F(O)\subseteq H_1$ and for all $x\in O$, $F(x)\cap H_2\neq\varnothing$. Therefore, F is an m_{ij}-asymmetric preirresolute at x_o in X as x_o was chosen arbitrarily.

Theorem 30: Let the m_{ij}-space, $(Y,m_{ij}(Y))$, $i,j=1,2;$ $i\neq j$ satisfy property B and $(X,m_{ij}(X))\rightarrow(Y,m_{ij}(Y))$ be an M-asymmetric preirresolute multifunction at an arbitrary point x_o of X. The following properties holds:

7. For an arbitrary $m_{ij}(Y)$-preneighbourhood H_1 of $F(x_o)$ with x_o a point of X, and each $m_{ij}(Y)$-preopen set H_2 satisfying $F(x_o)\cap H_2\neq\varnothing$, the subset $F^{+}(H_1)\cap F^{-}(H_2)$ of X is an $m_{ij}(X)$-preneighbourhood of the point x_o.
8. For every $m_{ij}(Y)$-preneighbourhood H_1 of $F(x_o)$ with x_o a point of X, and each $m_{ij}(Y)$-preopen set H_2 satisfying $F(x_o)\cap H_2\neq\varnothing$, there is some $F(x_o)\cap H_2\neq\varnothing$-preneighborhood O of x_o such that $F(O)\subseteq H_1$ and $F(x_o)\cap H_2\neq\varnothing$ for every $x\in O$.

Proof:

9. Let H_1 and H_2 be $m_{ij}(Y)$-preneighbourhoodS of $F(x_o)$ satisfying $F(x_o)\subseteq H_1$ and $F(x)\cap H_1\neq\varnothing$ for a point x_o of X. Then, there exists $m_{ij}(Y)$-preopen sets V_1 and V_2 such that $F(x_O)\subseteq V_1\subseteq H_1$ and $F(x_O)\cap V_2\subseteq F(x_O)\cap H_2\neq\varnothing$ whence, $x_o\in F^{+}(V_1)$ and $x_o\in F^{-}(V_2)$ giving, $x_o\in F^{+}(V_1)\cap F^{-}(V_2)$. By Theorem 29 (2) and assumption of F, $F^{+}(V_1)\cap F^{-}(V_2)$ is an $m_{ij}(X)$-preopen set, as a result,

$$F^{+}(H_1)\cap F^{-}(H_2)\supseteq m_{ij}(X)\,pInt\left[F^{+}(H_1)\cap F^{-}(H_2)\right]$$
$$\supseteq m_{ij}(X)\,pInt\left(F^{+}(V_1)\cap F^{-}(V_2)\right)$$
$$\supseteq F^{+}\left[m_{ij}(Y)\,pInt(V_1)\right]\cap F^{-}\left[m_{ij}(Y)\,pInt(V_2)\right]$$
$$=F^{+}(V_1)\cap F^{-}(V_2)\ni x_O$$

Consequently, $F^{+}(H_1)\cap F^{-}(H_2)$ is an $m_{ij}(X)$-preopen neighbourhood of x_O.

10. Let H_1 and H_2 be $m_{ij}(Y)$-preopen neighbourhoods of $F(x_0)$ satisfying $F(x_0)\subseteq H_1$ and $F(x_O)\cap H_2\neq\varnothing$. Put $O=F^{+}(H_1)\cap F^{-}(H_2)$. Then from part (1), O is an $m_{ij}(X)$-preopen-neighbourhood of x_O and by hypothesis, $F(O)\subseteq H_1$ and $F(x)\cap H_2\neq\varnothing$ for every x in O.

Conclusion

A new form of point-to-set mapping called M-asymmetric preirresolute multifunctions on biminimal structural spaces containing weaker form of sets has been introduced and some of its properties investigated. We have established that provided a multifunction is both lower and upper M-asymmetric preirresolute, then it is M-asymmetric preirresolute and vice-versa, and that any M-asymmetric preirresolute multifunction is M-asymmetric precontinuous but not conversely.

Acknowledgments

The authors wish to acknowledge the refereed authors for their helpful work towards this paper and the anonymous reviewers for their valuable comments and suggestions towards the improvement of the original manuscript.

Conflicts of Interest

Regarding the publication of this paper, the authors declare no conflict of interest.

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Citation: Matindih LK, Manyika DK, Mukonda D. On *M*-asymmetric preirresolute multifunctions. *Open Acc J Math Theor Phy.* 2023;4(1):15–23.
DOI: 10.15406/oajmtp.2023.04.00061