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Building (1+1) holographic superconductors in the presence of non-linear Electrodynamics

Abstract

In the framework of the gauge/gravity duality, and in particular of the correspondence, we study one-dimensional superconductors analysing the dual (1+2)-dimensional gravity in the presence of the Einstein-power-Maxwell non linear electrodynamics. In the probe limit we compute the critical temperature of the transition as a function of the mass of the scalar field. The computation is performed analytically employing the Rayleigh-Ritz variational principle. The comparison with the (1+3)-dimensional Einstein-Maxwell theory for two-dimensional superconductors is made as well.

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Introduction

The Superconductor-Normal transition is one of the most exciting research areas in condensed matter physics. Superconductivity was discovered in the Leiden laboratories by H. K. Onnes in 1911,[1] when he observed that the electrical resistivity of mercury dropped at an unmeasurably low value at a transition temperature of $T_c \simeq 4K$. The microscopic theory that explains the underlying mechanism was finally formulated in 1957 by Bardeen, Cooper and Schrieffer (BCS).[2,3] In 1986-1987, however, new superconducting materials were designed in which the transition temperature was found to be significantly larger than what was expected from the BCS theory.[4,5] In particular, in conventional superconductors the highest known transition temperature is $T_c = 23.2K$ in Nb_3Ge, while in the high-T_c oxides the observed transition temperature was in the $T_c = 30K$ range in 1986 in the Ba-La-Cu-O system,[4] and $T_c \simeq 90K$ in the following year in the Y-Ba-Cu-O system.[5] The discoveries of low and high temperature superconductors as well as the formulation of the BCS theory have been awarded with the Nobel prize in Physics in 1913,[6] in 1987[7] and in 1972,[8], respectively.

One of the most remarkable consequences of Superstring Theory[9,10] has been the AdS/CFT correspondence,[11] which was later extended to gauge/gravity duality.[12] The main idea is that a strongly coupled conformal field theory in d dimensions can be understood by solving a weakly coupled gravitational system in $d+1$ dimensions. The aforementioned equivalence raises the hope and the expectation that strongly coupled condensed matter systems may be explained from black hole physics. The original conjecture posits an equivalence between type IIB string theory on $AdS_5 \times S_5$ and a supersymmetric $\mathcal{N}=4$ Yang-Mills $SU(N)$ theory in (1+3) dimensions. There are at least two arguments pointing to this kind of equivalence: First symmetry counting, namely a conformal theory in $(1+3)$ dimensions has *15* degrees of freedom, while anti-de Sitter in $(1+4)$ dimensions has the isometries of $SO(1,4)$ with 15 generators. In addition, a stuck of N parallel D3-branes[13,14] can be viewed in two different ways as follows: From the one hand it naturally supports a four-dimensional supersymmetric gauge field theory based on *SU(N)* gauge group with $\mathcal{N}=4$ supersymmetric generators.[15] On the other hand the stuck of D-branes generates a gravitational field which in the near-horizon limit becomes $AdS_5 \times S_5$.[15]

High-T_c superconductivity cannot be described by the BCS theory, and it is one of the most enigmatic areas of condensed matter physics. The pioneer works of[16,17] in 2008 marked the birth of the field of holographic superconductors, and by now it is a very active one. For reviews see e.g.[18,19] To build holographic superconductors the minimal ingredients are a) gravity with a negative cosmological constant, b) the Maxwell potential A_μ and c) an electrically charged massive scalar field.[15] However, in principle the electromagnetic theory may be a non-linear one (NLE), such as Born-Infeld (BI) or the so called Einstein-power-Maxwell (EpM).

A special attention is devoted to NLE, which has a long history and it has been studied over the years in several different contexts. Maxwell's classical theory is based on a system of linear equations, but when quantum effects are taken into account, the effective equations become non-linear. The first models go back to the 30's when Euler and Heisenberg obtained QED corrections,[20] while Born and Infeld obtained a finite self-energy of point-like charges.[21] Furthermore, a straightforward generalization of Maxwell's theory leads to the so called Einstein-power-Maxwell (EpM) theory,[22–27] described by a Lagrangian density of the form $\mathcal{L}(F) = F^q$, where F is the Maxwell invariant, and q is an arbitrary rational number. This class of theories maintain the nice properties of conformal invariance in any number of space time dimensionality D if $q = D/4$. Finally, assuming appropriate non-linear electromagnetic sources, which in the weak field limit are reduced to the standard Maxwell's linear theory, one can generate a new class of solutions (Bardeen-like solutions,[28] see also[29]) to Einstein's field equations,[30–37] which on the one hand have a horizon, and on the other hand their curvature invariants, such as the Ricci scalar R, are regular everywhere, as opposed to the standard Reissner-Nordström solution.[38]

Holographic superconductors in the presence of EpM have been studied e.g. in[39–41] for $D \geq 4$ (see also[42–44] for works on holographic superconductors in the Einstein-Gauss-Bonnet gravity in higher dimensions), and in the presence of BI in[45–53] In the present work we wish to build one-dimensional holographic superconductors in the presence of EpM NLE, which to the best of our knowledge has not been done yet, with a twofold goal. On the one hand to fill a gap in the literature, and on the other hand, upon comparison to the $D=4$ Einstein-Maxwell theory, to see how the dimensionality of the system affects the critical temperature of the condensate. Our work is organized as follows: In the next section we present the model and the field equations, while the critical temperature of the transition is discussed in section 3, where our numerical results are shown. Finally, we conclude in the last section.

Model and field equations

We consider a gravitational system described by the action

$$q = 3/4 \tag{1}$$

setting $8G=1$, where the gravitational part S_G consists of the Einstein-Hilbert term with a negative cosmological constant $\ddot{E}_3 = -1/l^2$, while the matter part S_M consists of the (non-linear) electromagnetic theory and a massive charged scalar field with mass *m*. The covariant derivative D_μ and the Maxwell's invariant *F* are given by

$$D_\mu = \partial_\mu - ieA_\mu \tag{2}$$

$$F = F_{\mu\nu}F^{\mu\nu} \tag{3}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{4}$$

where A_μ is the Maxwell potential, and *e* is the electric charge of the scalar field.

In the following we consider the EpM theory $\mathcal{L}(F)_{EM} = -\beta F^q$, with $q=3/4$ for which the electromagnetic stress-energy tensor is traceless. What is more, we work in the probe limit neglecting the back reaction of the matter fields on the geometry. At least for temperatures close to the transition temperature this should be a good approximation. Therefore, in the following we consider a fixed gravitational background, which is no other than the Bañados-Teitelboim-Zanelli (BTZ) black hole[54,55]

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2dx^2 \tag{5}$$

with the metric function (setting the Newton's constant) *8G=1* given by

$$f(r) = -M + \frac{r^2}{l^2} = r^2 - r_H^2 \tag{6}$$

where the mass of the black hole M and the event horizon r_H are related via $r_H = l\sqrt{M}$.

Varying the action with respect to the scalar field and the Maxwell potential we obtain the following field equations (we set the electric charge *e=1*) for $\psi(r)$ and $A_0(r) \equiv \phi(r)$

$$\psi_{rr} + \left(\frac{f_r}{f} + \frac{1}{r}\right)\psi_r + \left(\frac{\phi^2}{f^2} - \frac{m^2}{f}\right)\psi = 0 \tag{7}$$

$$\phi_{rr} + 2\frac{\phi_r}{r} - \frac{16\psi^2\phi(\phi_r)^{1/2}}{3(-2)^{7/4}\beta f} = 0 \tag{8}$$

subjected to the following boundary conditions: At the event horizon, $r \to r_H$

$$\phi(r_H) = 0 \tag{9}$$

$$\psi(r_H) = \frac{2r_H}{m^2}\psi_r(r_H) \tag{10}$$

while at the boundary, $r \to \infty$ the solutions are required to behave like

$$\psi \sim \frac{\psi_-}{r^{\lambda_-}} + \frac{\psi_+}{r^{\lambda_+}} \tag{11}$$

$$\phi \sim \mu - \frac{\rho^2}{r} \tag{12}$$

where the power λ_+ is given by

$$\lambda_\pm = 1 \pm \sqrt{1+m^2} \tag{13}$$

and we may choose either $\psi_- = 0$ or $\psi_+ = 0$. The quantities μ and ρ are interpreted as the chemical potential and the energy density, respectively. In the following we set $\psi_- = 0$ and work with λ_+.

Study of the critical temperature: Analytical results

Since the scalar field plays the role of the order parameter in phase transitions, above the critical temperature the scalar field vanishes, and the equation for ϕ takes the simple form

$$\phi_{rr} + 2\frac{\phi_r}{r} = 0 \tag{14}$$

The solution that satisfies both the differential equations and the boundary conditions has the simple form

$$\phi(r) = \frac{\rho^2}{r_H}\left(1 - \frac{r_H}{r}\right) \tag{15}$$

If we define $z = r_H/r$ and $\xi = (\rho/r_H)^2$, the solution takes the form

$$\phi(z) = \xi r_H(1-z) \tag{16}$$

Next, the equation for the scalar field becomes

$$A\psi_{zz} + \left(\frac{f_z}{f} + \frac{1}{z}\right)\psi_z - \left(\frac{m^2 r_H^2}{z^4 f}\right)\psi = -\left(\frac{r_H^2\phi(z)^2}{z^4 f^2}\right)\psi \tag{17}$$

where from now on we use z as the independent variable instead of r. Taking into account the boundary condition for the scalar field at the boundary, $z \to 0$, we set

$$\psi(z) = z^\lambda F(z) \tag{18}$$

with some function that satisfies the conditions $F(0)=1$ and $F_z(0)=0$. Then we obtain the following equation for $F(z)$

$$-F_{zz} + AF_z + BF = \frac{\xi^2}{(1+z)^2}F \tag{19}$$

with the coefficients $A(z), B(z)$ given by

$$A(z) = \frac{1}{z}\left(\frac{2}{1-z^2} - 1 - 2\lambda\right) \tag{20}$$

$$B(z) = \frac{m^2}{z^2(1-z^2)} - \frac{\lambda(1-\lambda)}{z^2} + \frac{\lambda}{z}\left(\frac{2}{1-z^2} - \frac{1}{z}\right) \tag{21}$$

To solve in an analytical way the boundary value problem of the form

$$-\left(p(z)F_z\right)'+q(z)F(z)=\xi^2 w(z)F(z) \tag{22}$$

in the range $0\le z\le 1$, where ξ^2 is the eigenvalue, and $w(z),p(z),q(z)$ are given functions, we try a test function $F(z;a)$ with some unknown parameter a that minimizes the expression[56]

$$\xi(a)^2=\frac{\int_0^1 p(z)[F(z\,a)']^2+\int_0^1 q(z)[F(z;a)]^2}{\int_0^1[\mathrm{w(z)}\ F(\mathrm{z;a})]^2} \tag{23}$$

In our case we use as a test function $F(z)=1-az^2$, and it is easy to verify that the functions $w(z),p(z),q(z)$ are given by the following expressions

$$p(z)=\left(1-z^2\right)z^{2\lambda-1} \tag{24}$$

$$w(z)=\frac{p(z)}{(1+z)^2} \tag{25}$$

$$q(z)=p(z)B(z) \tag{26}$$

Upon minimization of the expression above at a_*, we determine the pair of values of (a_*,ξ_{min}), and finally the critical temperature is given by the corresponding Hawking temperature of the BTZ black hole

$$T_H=\frac{2r_H}{4\pi} \tag{27}$$

or

$$T_c=\frac{\rho}{2\pi\xi_{min}^{1/2}} \tag{28}$$

Considering a scalar field mass in the range $-1\le m^2\le 0$, the power λ_+ takes values in the range $1\le\lambda_+\le 2$. In Table 1 we show the critical temperature (for $\rho=1$) for several different values of λ_+. In the same table we also show T_c for the (1+3)-dimensional Einstein-Maxwell theory. For better visualization, we show our results graphically in Figure 1 and 2, and for a comparison we show them together in the same plot, Figure 3. We also show the fitting curves, $y(x)=0.111/x^{0.66}$ for the EpM case, and $y(x)=0.206/x^{0.81}$ for the Maxwell case. We see that the critical temperature of one-dimensional superconductors is lower than T_c of two-dimensional superconductors.

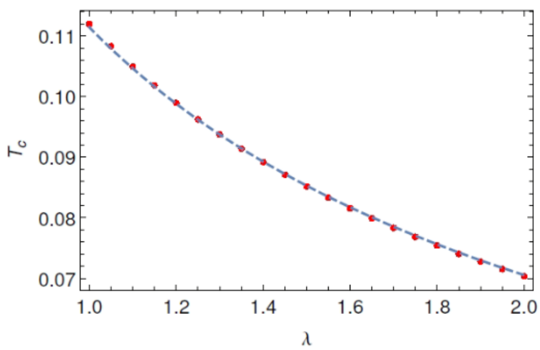

Figure 1 Critical temperature (in units of ρ) as a function of λ_+ in the $D=3,q=3/4$ case. The dashed curve corresponds to the fitting curve $y=0.111/x^{0.66}$.

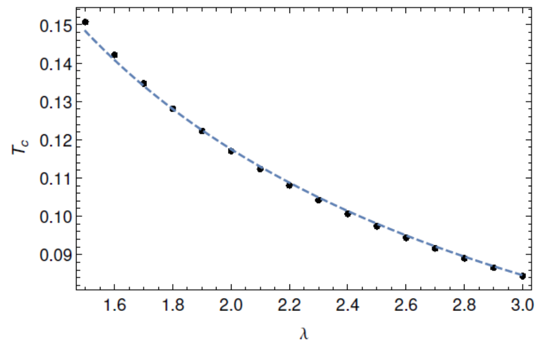

Figure 2 Critical temperature (in units of $\rho^{1/2}$) for different values of λ_+ in the $D=4,q=1$ case. The dashed curve corresponds to the fitting curve $y=0.206/x^{0.81}$.

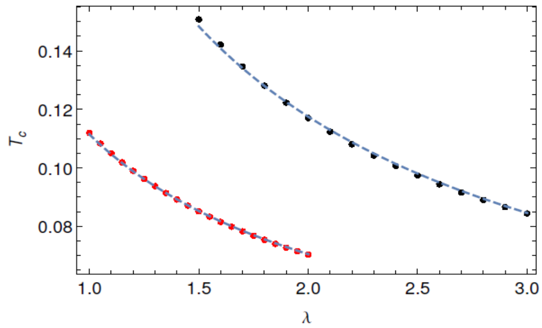

Figure 3 Comparison between 1D superconductors in the presence of EpM non-linear electrodynamics, and 2D superconductors in the presence of Maxwell's linear theory.

Critical temperature (for $\rho=1$) for different values of λ_+ for the EpM theory (second column) and Maxwell's theory (third column).

Power λ_+	**T_c (EpM)**	**T_c (Maxwell)**
.00	0.111964	-
.05	0.108324	-
.10	0.104964	-
.15	0.101854	-
.20	0.0989643	-
.25	0.0963727	-
.30	0.0937587	-
.35	0.0914047	-
.40	0.0891954	-
.45	0.0871174	-
.50	0.085159	0.150713
.55	0.0833099	-

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Table continue

Power λ_+	T_c (EpM)	T_c (Maxwell)
.60	0.0815606	0.142141
.65	0.0799032	-
.70	0.0783302	0.134672
.75	0.0768351	-
.80	0.075412	0.128097
.85	0.0740555	-
.90	0.0727609	0.122262
.95	0.0715239	-
.00	0.0703404	0.117042
.10	-	0.112343
.20	-	0.108087
.30	-	0.104212
.40	-	0.100666
.50	-	0.0974083
.60	-	0.0944027
.70	-	0.0916196
.80	-	0.0890341
.90	-	0.0866247
.00	-	0.0843729

Conclusion

We have studied one-dimensional superconductors analysing the dual (1+2)-dimensional gravitational system in the presence of non-linear electrodynamics. In particular, we have considered the Einstein-Power-Maxwell theory for $q = 3/4$ that corresponds to a traceless electromagnetic tensor. We have studied the critical temperature of the transition in the probe limit analytically employing the Reyleigh-Ritz variational principle. The critical temperature as a function of the mass of the scalar field has been obtained, and the comparison with the (1+3)-dimensional case of the Einstein-Maxwell theory has been made as well.

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Conflicts of interest

None.

References

1. K Onnes. Leiden Commun. 1911;120b,122b,124c.
2. J Bardeen, LN Cooper, JR Schrieffer. Microscopic Theory of Superconductivity. *Phys Rev.* 1957;106:162.
3. J Bardeen, LN Cooper, JR Schrieffer. Theory of Superconductivity. *Phys Rev.* 1957;108:1175.
4. JG Bednorz, KA Muller. Possible highT c superconductivity in the Ba−La−Cu−O system. *Zeitschrift für Physik B Condensed Matter.* 1986;64:189.
5. M Wu, JR Ashburn, CJ Torng, et al. Superconductivity at 93 K in a new mixed–phase Y–Ba–Cu–O compound system at ambient pressure. *Phys Rev Lett.* 1987;58:908.
6. Heike Kamerlingh O. The Nobel Prize in Physics 1913.
7. J Georg Bednorz, K Alex Müller. The Nobel Prize in Physics 1987.
8. John Bardeen, Leon N. Cooper, Robert Schrieffer. The Nobel Prize in Physics 1972.
9. MB Green, JH Schwarz, E Witten. Superstring Theory. Cambridge University Press, Cambridge, England, 2012.
10. J Polchinski. String Theory, Vol. 1 & 2, Superstring theory and beyond. Cambridge University Press, Cambridge, England, 2005.
11. JM Maldacena. The Large N Limit of Superconformal Field Theories and Supergravity. *Int J Theor Phys.* 1999;38(1):1113.
12. M K Benna, I R Klebanov. On Normal Modes of a Warped Throat. *JHEP.* 2008;0806:070.
13. CV Johnson, D–Branes. Introduction to String Theory and D–Branes. Cambridge University Press, Cambridge, England, 2003.
14. B. Zwiebach, A First Course in String Theory , Cambridge University Press, Cambridge, England, 2004.
15. Makoto Natsuume. AdS/CFT Duality User Guide. *Lect Notes Phys.* 2015;903.
16. SA Hartnoll, CP Herzog, GT Horowitz. Building a Holographic Superconductor. *Phys Rev Lett.* 2008;101:031601.
17. Sean A Hartnoll, Christopher P, Gary T. Horowitz. Holographic superconductors. *JHEP.* 2008;0812:015.
18. Sean A Hartnoll. Lectures on holographic methods for condensed matter physics. *Class Quant Grav.* 2009;26:224002.
19. CP Herzog. Lectures on holographic superfluidity and superconductivity. *J Phys.* 2009;42:343001.
20. W Heisenberg, H Euler. Consequences of Dirac Theory of the Positron. *Z Phys.* 1936;98:714.
21. M Born, L Infeld. Foundations of the new field theory. *Proc Roy Soc Lond.* 1934;144:425.
22. O Gurtug, SH Mazharimousavi, M Halilsoy. 2+1–dimensional electrically charged black holes in Einstein – power Maxwell Theory. *Phys Rev*. 2012;85:104004.
23. SH Mazharimousavi, O Gurtug, M Halilsoy, et al. Higher–order spin effects in the dynamics of compact binaries II. Radiation field. *Phys Rev.* 2011;84:124021.
24. W Xu, DC Zou. *Gen Rel Grav.* 2017;49(6):73.
25. Á Rincón, E Contreras, P Bargueño, et al. Black hole shadow of a rotating polytropic black hole by the Newman–Janis algorithm without complexification. *Eur Phys J.* 2017;77(7):494.

26. G Panotopoulos, Á Rincón. Int J Mod Phys. 2017;27(3):1850034.
27. Á Rincón, G Panotopoulos. Greybody factors and quasinormal modes for a nonminimally coupled scalar field in a cloud of strings in (2+1)–dimensional background. *Phys Rev.* 2018;97:024027.
28. J Bardeen. Presented at GR5, Tiflis, U.S.S.R., and published in the conference proceedings in the U.S.S.R. 1968.
29. A Borde. Regular black holes and topology change. *Phys Rev.* 1997;55:7615.
30. Eloy Ayón–Beato, Alberto García. Regular Black Hole in General Relativity Coupled to Nonlinear Electrodynamics. *Phys Rev Lett.* 1998;80:5056.
31. Eloy Ayón–Beato, Alberto García. Non–Singular Charged Black Hole Solution for Non–Linear Source. *Gen Rel Grav.* 1999;31:629. Eloy Ayón–Beato, Alberto García. New Regular Black Hole Solution from Nonlinear Electrodynamics. *Phys Lett.* 1999;464:25.
32. KA Bronnikov. Regular magnetic black holes and monopoles from nonlinear electrodynamics. *Phys Rev.* 2001;63:044005.
33. Irina Dymnikova. Regular electrically charged structures in Nonlinear Electrodynamics coupled to General Relativity. *Class Quant Grav.* 2004;21:4417–4429.
34. Sean A Hayward. Formation and evaporation of non–singular black holes. *Phys Rev Lett.* 2006;96:031103.
35. Leonardo Balart, Elias C Vagenas. Regular black hole metrics and the weak energy condition. *Phys Lett.* 2014;730:14.
36. Leonardo Balart, Elias C Vagenas. Regular black holes with a nonlinear electrodynamics source. *Phys Rev.* 2014;90:124045.
37. H Reissner. *Annalen Phys.* 1916;355:106–120.
38. Jing Q Pan, S Chen. *JHEP.* 2011;1111:045.
A. Sheykhi HR, A Montakhab. *JHEP.* 2016;1604:058.
39. Seyed Hossein Hendi, Gu–Qiang Li, Jie–Xiong Mo, et al. New perspective for black hole thermodynamics in Gauss–Bonnet–Born–Infeld massive gravity. *Eur Phys J C.* 2016;76(10):575.
40. Ruth Gregory, Sugumi Kanno, Jiro Soda. Holographic Superconductors with Higher Curvature Corrections. *JHEP.* 2009;0910:010.
41. Qiyuan Pan, Bin Wang, Eleftherios Papantonopoulos, et al. Holographic Superconductors with various condensates in Einstein–Gauss–Bonnet gravity. *Phys Rev.* 2010;D81:106007.
42. Luke Barclay, Ruth Gregory, Sugumi Kanno, et al. Gauss–Bonnet Holographic Superconductors. *JHEP.* 2010;1012:029.
43. Mahya Mohammadi, Ahmad Sheykhi, Mahdi Kord Zangeneh. Analytical and numerical study of backreacting one–dimensional holographic superconductors in the presence of Born–Infeld electrodynamics. *Eur Phys J C.* 2018;78:654.
44. Jiliang Jing, Songbai Chen. Holographic superconductors in the Born–Infeld electrodynamics. *Phys Lett.* 2010;686:68.
45. J Jing L, Wang Q Pan, S Chen. *Phys Rev.* 2011;83:201066010.
46. D Roychowdhury. Effect of external magnetic field on holographic superconductors in presence of nonlinear corrections. *JHEP.* 2012;1205.
47. S Gangopadhyay, D Roychowdhury. Effect of external magnetic field on holographic superconductors in presence of nonlinear corrections. *JHEP.* 2012;1205:156.
48. Weiping Yao, Jiliang Jing. Analytical study on holographic superconductors for Born–Infeld electrodynamics in Gauss–Bonnet gravity with backreactions. *JHEP.* 2013;1305:101.
49. Chuyu Lai, Qiyuan Pan, Jiliang Jing, et al. On analytical study of holographic superconductors with Born–Infeld electrodynamics. *Phys Lett.* 2015;749:437.
50. D Ghorai, S Gangopadhyay. Higher dimensional holographic superconductors in Born–Infeld electrodynamics with back–reaction. *Eur Phys J.* 2016;76(3):146.
51. Sheykhi A, F Shaker. Analytical study of holographic superconductor in Born–Infeld electrodynamics with back reaction. *Phys Lett.* 2016;754:281.
52. Banados M, C Teitelboim, J Zanelli. The Black hole in three–dimensional space–time. *Phys Rev Lett.* 1992;69:1849–1851.
53. Maximo Banados, Marc Henneaux, Claudio Teitelboim, et al. Geometry of the (2+1) black hole. *Phys Rev.* 1993;48:1506–1525.
54. G Siopsis, J Therrien. Holographic superconductors near the Breitenlohner–Freedman bound. *JHEP.* 2010;1005.

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