Open Access Journal of Mathematical and Theoretical Physics

Research Article

Open Access CrossMark

Motion analysis of particles using the hyper-complex numbers representation

Abstract

The present contribution uses the Hyper-Complex Numbers model to describe and analyze the kinematics and dynamics of ideal particles. The analysis extends to super-luminal particles and tachyons. The pure particles and pure forces are also defined and discussed. The behavior of pure tardyons and pure tachyons is further presented.

Keywords: H and VH- numbers representation, geometrized unit system, big bang, pseudo-rotation, Lorenz transformations, equations of the motion, source, tardyons, tachyons, mass-momentum relationships.

Volume 2 Issue 1 - 2019

Vlad L Negulescu
Dresdener Ring 31 61130 Nidderau, Germany

Correspondence: Vlad L. Negulescu, Dresdener Ring 31 61130 Nidderau, Germany, Tel 004961872092820, Email Andrej.coer@ob-valdoltra.si

Received: February 05, 2019 | **Published:** February 25, 2019

Introduction and preliminary

In various articles,[1–3] the author developed a mathematical model showing interesting applications in mechanics, electromagnetism and quantum behavior. The present paper uses the model to propose a new theory concerning the tachyons.

The material particle as a VH-number

According to the article,[3] an ideal particle is represented by a Vector-Hyper-Complex number, or VH-number, which is formally written as:

$$p = t + iz + j\overline{y} + k\overline{x} \qquad 1.1$$

where, $t + iz$ represents the scalar part of this number and $j\overline{y} + k\overline{x}$ is a vector part. Its geometrical correspondence is a point in an eight-dimensional space. The symbols i, j and k are fundamental units of H-numbers as defined in references.[1,2] The Table 1 shows the multiplication rules of these units.

Table 1 Units multiplication table

x	1	i	j	k
1	1	i	j	k
i	i	-1	-k	j
j	j	-k	-1	i
k	k	j	i	1

The four parameters of the particle's representation are time (t), mass (z), momentum (y) and space(x). The space and momentum are vectors and they are accordingly labeled in 1.1. The geometrized system of units[4] enables to express all these parameters in the common unit meter as shown in the Table 2 presented below.

Table 2 The direct and reversed conversion from international system of units (SI) to geometrized system of units (GU)

	GU	SI	SI→GU	GU→SI
Length	m	m	1	1
Time	m	s	c	c^{-1}
Velocity	dimensionless	ms^{-1}	c^{-1}	c
Mass	m	Kg	Gc^{-2}	$G^{-1}c^{2}$
Momentum	m	$Kgms^{-1}$	Gc^{-3}	$G^{-1}c^{3}$
Force	dimensionless	N	Gc^{-4}	$G^{-1}c^{4}$

Where G=6.6742× 10^{-11} $m^3kg^{-1}s^{-2}$ is Gravitation Constant and c=2.998x10^8 m s^{-1} represents the velocity of light in vacuum.

The physical world and non static ST

The eight- dimensional space mentioned in the precedent paragraph represents the "physical world" as was defined in the reference.[3] The four-dimensional continuous Space Time (ST), or Minkowski space modified, is a subset of the physical world.

The geometry of ST was mathematically expressed, and described in the reference paper.[2] Our universe generally is a vacuum with some celestial bodies at sufficiently large distance, and we can consider it as a practically empty space. The universe is, according to the last measurements, flat i.e. Euclidean. The estimated density of the universe is approximately $9.9x10^{-27} Kg / m^3$. The physical world should be seen as a continuous ST with its cosmic space S containing tinny islands of masses along with their moments. We should be aware that the continuous ST is permanently expanding since the Big Bang, which happened 13.8 billion years ago. A simplified graphical representation of the two-dimensional ST is presented in the Figure 1. The common unit is the meter and the light velocity is 1. The horizontal axis represents the time which is a scalar. The vertical axis represents the space x multiplied by the anti-real unit k. The space is actually a vector in a Euclidean three-dimensional space. The red line and the blue line represent the motion of light particles. In the four-dimension ST they build a super-cone of light which separates the time-like zone and the space-like zone. The green line represents the motion of a subluminal particle, and the orange line shows a superluminal one.

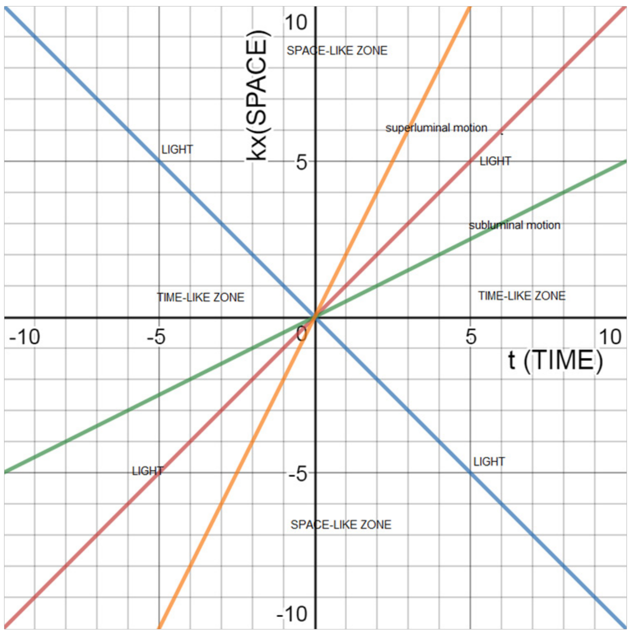

Figure 1 The ST-plane.

On the above diagram we labeled the time-like zone and the space-like zone.

Lorenz Coordinate Transformations in the two-dimensional ST

The ST- plane was described as the 3rd fundamental plane (t, x) of the set H of the Hyper-complex Numbers, as you can see in the reference paper.[2]

The equations 3.1 and 3.2, presented in the same article,[2] express the Lorenz Coordinate Transformations from the reference frame S (t, x) to the frame $S'(t', x')$ as the result of a pseudo-rotation with the argument $-\chi$.

$$\begin{aligned} t' &= t\cosh\chi - x\sinh\chi \\ x' &= x\cosh\chi - t\sinh\chi \end{aligned} \qquad 1.2$$

The Figure 2 represents the Lorenz transformations produced by a pseudo-rotation with $\chi = 0.549$,i.e. $\tanh\chi = 0{,}5$.

$$\begin{aligned} t' &= 1.155t - 0.577x \\ x' &= 1.155x - 0.577t \end{aligned} \qquad 1.3$$

Note that the axis t is the real axis and the space axis is an anti-real one, i.e., the length x is multiplied with k, the anti-real unit.[2]

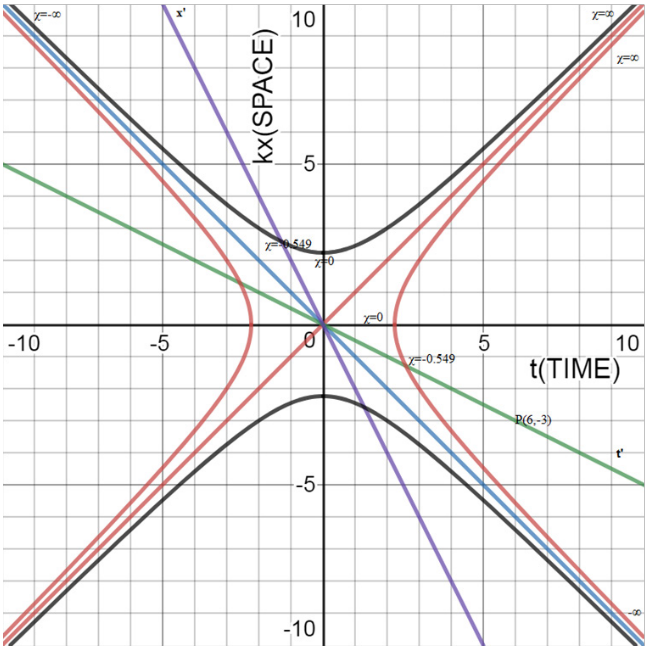

Figure 2 The illustration of a lorenz transformation in the plane ST.

The green line, labeled t', represents the transformation of the axis t. The violet line, labeled x', is the image of the axis x after the transformation.

We may locate a point in the ST plane using, besides Cartesian coordinate, a kind of special hyperbolic "polar coordinates." The conversion between these coordinate systems is shown below:

For time like zone, $t + kx = \pm\rho e^{k\chi}$

For space like zone, $t + kx = \pm\rho k e^{k\chi}$, where $\rho = \sqrt{\left|(t^2 - x^2)\right|}$ and $|a|$ is the absolute value of a.

In the Figure 2, are also represented two conjugate hyperbolas, colored black and respectively red.

Their equations are written below:

$$\begin{aligned} t + kx &= \pm 2.236 e^{-k\chi} = \pm 2.236(\cosh\chi + k\sinh\chi) \\ t + kx &= \pm 2.236 k e^{-k\chi} = \pm 2.236(\sinh\chi + k\cosh\chi) \end{aligned} \qquad 1.4$$

All points belonging to these hyperbolas have the same norm.[2]

$\rho = 2.236.$

Alternate and more familiar equations of the hyperbolas above are as it follows:

$$\begin{aligned} x^2 - y^2 &= 5 \\ y^2 - x^2 &= 5 \end{aligned} \qquad 1.5$$

On these curves it was marked different values of argument χ. For example, on the right side of the diagram, the argument χ goes from $-\infty$ through -0.549 and 0, reaching finally $+\infty$.

Processing the equation 1.2 it obtains the velocity of the frame S' relative to S.

$$v = \tanh\chi \qquad 1.6$$

Consequently we may write:

$$\begin{aligned} \cosh\chi &= \frac{1}{\sqrt{1 - v^2}} \\ \sinh\chi &= \frac{v}{\sqrt{1 - v^2}} \end{aligned} \qquad 1.7$$

Converting from GU to SI units, by multiplying time values with c (speed of light in ms^{-1}) and dividing v by c, it obtains the familiar forms known from every manual[5] of Special Relativity

$$\begin{aligned} t' &= \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{x\frac{v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \\ x' &= \frac{x}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{vt}{\sqrt{1 - \frac{v^2}{c^2}}} \end{aligned} \qquad 1.8$$

Using the Lorenz factor $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$, it obtains an alternate form

$$\begin{aligned} t' &= \gamma\left(t - x\frac{v}{c^2}\right) \\ x' &= \gamma(x - vt) \end{aligned} \qquad 1.9$$

The pseudo-rotation and Lorenz transformations in the four-dimensional ST

Let us make a coordinates transformation from the system $S(t, \overline{x})$ to $S'(t', \overline{x}')$.

The transformation is performed using a vector pseudo-rotation with a constant argument as can be seen in the reference paper.[3]

$$p' = pe^{-k\overline{\chi}} \qquad 1.10$$

Where, $p = t + k\overline{u}_1 x$

The space vector has the magnitude x. Its versor, or unit vector, is

$$\overline{u}_1 = \frac{x_1\overline{1} + x_2\overline{2} + x_3\overline{3}}{x}$$

$\overline{1}, \overline{2}, \overline{3}$ are the unit vectors along the cartesian axes and x_1, x_2, x_3

are the components relative to those axes.

The co-real argument $\overline{\chi}$, is a vector with the magnitude χ and the unit vector $\overline{u}_2 = \overline{1}\cos\varepsilon_1 + \overline{2}\cos\varepsilon_2 + \overline{3}\cos\varepsilon_3$, where $\cos^2\varepsilon_1 + \cos^2\varepsilon_2 + \cos^2\varepsilon_3 = 1$

$$\overline{\chi} = \overline{u}_2\chi \qquad 1.11$$

$$e^{-k\overline{\chi}} = \cosh\chi - k\overline{u}_2\sinh\chi \qquad 1.12$$

If it performs the multiplication with a pseudo-rotor, then the product between vectors is by definition a scalar (dot) product. We may write :

$$\begin{aligned} &\overline{u}_1 \bullet \overline{u}_2 = \cos\Phi \\ &\overline{u}_1 = \overline{u}_2\cos\Phi + (\overline{u}_1 - \overline{u}_2\cos\Phi) \end{aligned} \qquad 1.13$$

It is easy to see that the two components of the versor $\overline{u}$ are orthogonal. The rules of the vector pseudo-rotation be as follows:[1]

The vector pseudo-rotation fully acts on the scalar part and on the vector part(of the number p) which is parallel with $\overline{u}_2$.

The vector part of the number p, which is perpendicular on $\overline{u}_2$, remains unchanged after the pseudo-rotation.

Let us write the number p split in three parts:

$$\begin{aligned} &p = t + k\overline{u}_2 x\cos\Phi + kx(\overline{u}_1 - \overline{u}_2\cos\Phi) \\ &p = \text{scalar} + \text{parallelvector} + \text{perpendicularvector} \end{aligned} \qquad 1.15$$

If we apply the above multiplication rules , then it gets:

$$\begin{aligned} &p'_1 = t\cosh\chi - k\overline{u}_2 t\sinh\chi \\ &p'_2 = k\overline{u}_2 x\cos\Phi\cosh\chi - k^2\overline{u}_2 \bullet \overline{u}_2 x\cos\Phi\sinh\chi = -x\cos\Phi\sinh\chi + k\overline{u}_2 x\cos\Phi\cosh\chi \\ &p'_3 = kx(\overline{u}_1 - \overline{u}_2\cos\Phi) \\ &p' = t' + k\overline{x}' \end{aligned} \qquad 1.16$$

Finnaly we can get, the four-dimensional Lorenz transformations:

$$\begin{aligned} &t' = t\cosh\chi - x\cos\Phi\sinh\chi \\ &\overline{x}' = \overline{u}_1 x + \overline{u}_2\left[x\cos\Phi(\cosh\chi - 1) - t\sinh\chi\right] \end{aligned} \qquad 1.17$$

The transformations (1.17) can be written also in the matrix format:

$$\begin{pmatrix} t' \\ x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} = \begin{pmatrix} \cosh\chi & -\sinh\chi\cos\varepsilon_1 & -\sinh\chi\cos\varepsilon_2 & -\sinh\chi\cos\varepsilon_3 \\ -\sinh\chi\cos\varepsilon_1 & 1+(\cosh\chi-1)\cos^2\varepsilon_1 & (\cosh\chi-1)\cos\varepsilon_1\cos\varepsilon_2 & (\cosh\chi-1)\cos\varepsilon_1\cos\varepsilon_3 \\ -\sinh\chi\cos\varepsilon_2 & (\cosh\chi-1)\cos\varepsilon_2\cos\varepsilon_1 & 1+(\cosh\chi-1)\cos^2\varepsilon_2 & (\cosh\chi-1)\cos\varepsilon_2\cos\varepsilon_3 \\ -\sinh\chi\cos\varepsilon_3 & (\cosh\chi-1)\cos\varepsilon_3\cos\varepsilon_1 & (\cosh\chi-1)\cos\varepsilon_3\cos\varepsilon_2 & 1+(\cosh\chi-1)\cos^2\varepsilon_3 \end{pmatrix} \begin{pmatrix} t \\ x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

The Lorenz Transformations presented in (1.17) are valid for both zones of ST, time-like and space- like. The formula of velocity's transformation can be obtained by differentiation:

$$\frac{d\overline{x}'}{dt'} = \frac{\overline{u}_1\dfrac{dx}{dt} + \overline{u}_2\left[\dfrac{dx}{dt}\cos\Phi(\cosh\chi - 1) - \sinh\chi\right]}{\cosh\chi - \dfrac{dx}{dt}\cos\Phi\sinh\chi} \qquad 1.18$$

Let us make the following replacements:

$$\begin{aligned} &\frac{dx}{dt} = v \\ &\left(\frac{d\overline{x}'}{dt'}\right)^2 = v'^2 \end{aligned} \qquad 1.19$$

Performing the calculations it obtains:

$$v'^2 - 1 = \frac{v^2 - 1}{\left(\cosh\chi - v\cos\Phi\sinh\chi\right)^2} \qquad 1.20$$

That means:

-the subluminal velocities transform in subluminal ones after a vector pseudo-rotation;

-the superluminal velocities remain superluminal after the same transformation;

-the light velocity remains unchanged after the transformation.

The Generalized Lorenz Transformations (GLT) are mentioned in the standard work[6] of Recami[6] (see equations 43 from this paper). If GLT exist, then they are only valid in two-dimensional ST. In a paper,[7] published in 2012, Ricardo S Vieira[7] has proposed a six-dimensional ST (3 space dimensions and 3 time dimensions) in order to avoid the difficulties which appear when it tries to extend the Lorenz transformations to superluminal frames in the four-dimensional ST.

We consider that the coordinate transformations in ST are permitted only using vector pseudo-rotation, i.e., the Lorenz Transformations (as known from the Special Relativity) are the only valid ones. Subluminal inertial reference frames cannot be converted in superluminal inertial reference frames and vice versa.

Motion of the particles

The Motion of a particle from s-rest frame

How we generally define the motion? A simple and logical definition could be the following: "the change in position of an object over the time". In the present work it is forbidden any motion backwards in time. The particles involved in the further analysis are only forward ones. The actual analysis is developed for the simple case of one dimensional lengths and momentum.

Let us consider a material particle located on the time axis in ST space. An observer attached to this particle "flows "with the time, in one direction, along to this axis. We may say that the particle is at "space rest" or s-rest. The reference frame attached to particle will be called s-rest frame. In the Special Relativity (SR) this frame is known as the proper or commoving frame. The H number associated to the particle is purely complex:

$$p_0 = t_0 + iz_0 \qquad 2.1$$

The parameter t_0 is s-rest time and z_0 is s-rest mass (alternate denominations in according to SR are proper time and proper mass).

The reference paper[2] postulates that the motion is a "conservative" mapping of p_0 on the set of H-numbers, H:

$$\begin{aligned} &dp = e^{k\delta}dp_0 = dt + idz + jdy + kdx \\ &\delta(t_0 + iz_0) = U(t_0, z_0) + iV(t_0, z_0) \end{aligned} \qquad 2.2$$

Expressions U, V and δ are respectively called rapidity, vigor and source.[1,2] They are in general vectors but, as we mentioned above, this time they are considered scalars.

With all these we may write: $e^{kU} = \cosh U + k\sinh U$ 2.3

$$e^{jV} = \cos V + j\sin V$$

By processing the expressions 2.2 and 2.3 it obtains the general equations of the motion of a particle initially located on the s-rest axis.

$$\begin{aligned} dt &= \cosh U \cos V dt_0 - \sinh U \sin V dz_0 \\ dz &= \sinh U \sin V dt_0 + \cosh U \cos V dz_0 \\ dy &= \cosh U \sin V dt_0 + \sinh U \cos V dz_0 \\ dx &= \sinh U \cos V dt_0 - \cosh U \sin V dz_0 \end{aligned} \qquad 2.4$$

The implicit form of those equations is written below:

$$\begin{aligned} dt^2 - dz^2 + dy^2 - dx^2 &= dt_0^2 - dz_0^2 \\ dtdz - dxdy &= dt_0 dz_0 \end{aligned} \qquad 2.5$$

Pure particles and forces

A moving particle is called pure, when the particle s-rest mass is constant, i.e. $dz_0 = 0$. The force acting on a pure particle is also named pure force

In the case of the pure particle the equations 2.4 become:

$$\begin{aligned} dt &= \cosh U \cos V dt_0 \\ dz &= \sinh U \sin V dt_0 \\ dy &= \cosh U \sin V dt_0 \\ dx &= \sinh U \cos V dt_0 \end{aligned} \qquad 2.6$$

From 2.6 it obtains the velocity, the mass flow or power and the force acting on the particle. The power can be written as the product of force and velocity.

$$\begin{aligned} v &= \frac{dx}{dt} = \tanh U \\ P &= \frac{dz}{dt} = \tanh U \tan V \\ F &= \frac{dy}{dt} = \tan V \\ P &= Fv \end{aligned} \qquad 2.7$$

Another interesting relation deduced from 2.6 is:

$$dz = \frac{dy}{dt} dx = F dx \qquad 2.8$$

The right member of equation 2.8 represents the work done by the pure force F when moving its point of application through dx.

Integrating 2.8 and converting to SI units it obtains the Einstein's formula $E = mc^2$ as was shown in reference.[2]

As we see this particle is a pure bradyon (tardyon), because its absolute velocity is less than 1 (velocity of the light), i.e. a particle with a constant s-rest mass moves only in the time-like zone of the ST diagram.

If V is zero (which implies a constant U), then we have the case of a typical uniform motion, in a force free space. In this special case the motion parameters are obtained by using a simple pseudo-rotation[2] with a constant argument $\delta = kU$, and relationships 2.4 become:

$$\begin{aligned} t &= t_0 \cosh U \\ x &= t_0 \sinh U \\ z &= z_0 \cosh U \\ y &= z_0 \sinh U \end{aligned} \qquad 2.9$$

As was shown in the basic paper[2] the last two equations above keep their validity for small s-rest mass, and small forces. They represent the relativistic expressions of mass and momentum of a material particle moving in the time-like zone. Now we have to be aware that the conditions mentioned above (small masses and forces) are fulfilled by practically all subluminal objects in universe. As an example let us consider an object with an s-rest mass of 1000Kg. This mass expressed in meter (Geometrical Unit System[2]) is 7.47×10^{-23} m and thus the following approximations are allowed:

$$\begin{aligned} U(t_0 + iz_0) &= U + iV \approx U(t_0, 0) + iz_0 U'(t_0, 0) \\ \sin V &\approx z_0 \frac{dU}{dt_0} \\ \cos V &\approx 1 \end{aligned} \qquad 2.10$$

Introducing the sinus and the cosines expressions shown in 2.10 into the last two equations of 2.4 it obtains, after integration, the mass and momentum expressions having the same forms as in 2.9.

Processing it obtains the mass-momentum equation, in the time-like zone:

$$z^2 - y^2 = z_0^2 \qquad 2.11$$

If the formula 2.11 is converted to SI, then we obtain the familiar energy-momentum relation:

$$E^2 - (pc)^2 = m_0^2 c^4 \qquad 2.12$$

This relation above is generally valid for subluminal particles. For the light particles (photons), were the s-rest mass is zero, the equation 2.12 reduces to:

$$E = pc \qquad 2.13$$

This relation belongs to the classical electro-magnetism and is also well known.

The s-rest mass of a tardyon is an invariant for all reference frames, not only the inertial ones, located in the time-like zone of ST.

In the **Figure 3** are presented the graphs for the energy (red) and momentum (blue) as functions of the particle's velocity.

Motion in the space-like zone

Now we consider a point placed on the space axis. We may say that its associated particle is at "time rest" (t-rest) or is in the t-rest frame. The corresponding H number is:

$$p_0 = kx_0 + jy_0 = k(x_0 + iy_0) \qquad 2.14$$

The parameters x_0 and y_0 are called t-rest space and respectively t-rest momentum.

There is no motion, according to the definition stated at the beginning of the paragraph 2.1, because the time parameter is missing. The motion is defined, through extension of the Axiom 2,[2] as the mapping of p_0 on the set H.

$$dp = e^{k\delta}dp_0 = dt + idz + jdy + kdx$$
$$\delta(x_0 + iy_0) = U(x_0, y_0) + iV(x_0, y_0) \qquad 2.15$$

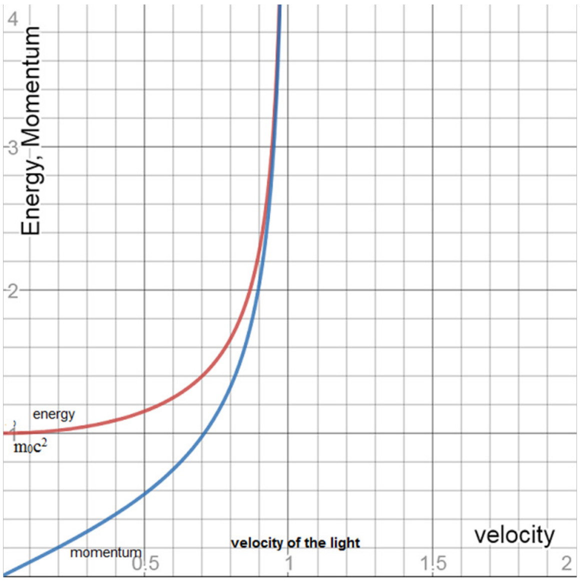

Figure 3 Energy and momentum of a tardyon versus its velocity.

The expressions 2.15 yield to the equations of the motion in space-like zone:

$$dt = dx_0 \sinh U \cos V - dy_0 \cosh U \sin V$$
$$dx = dx_0 \cosh U \cos V - dy_0 \sinh U \sin V$$
$$dz = dx_0 \cosh U \sin V + dy_0 \sinh U \cos V \qquad 2.16$$
$$dy = dx_0 \sinh U \sin V + dy_0 \cosh U \cos V$$

Their implicit form is as it follows:

$$dx^2 - dy^2 + dz^2 - dt^2 = dx_0^2 - dy_0^2$$
$$dxdy - dtdz = dx_0 dy_0 \qquad 2.17$$

The both implicit forms 2.5 and 2.17 can be easily extended for vector space and momentum, making following replacements:

$$dx^2 = dx_1^2 + dx_2^2 + dx_3^2$$
$$dy^2 = dy_1^2 + dy_2^2 + dy_3^2$$
$$dxdy = dx_1 dy_1 + dx_2 dy_2 + dx_3 dy_3$$

Pure particles and forces in the space-like zone

A particle is called pure particle when the t-rest momentum is constant, i.e. dy_0 =0. The force acting on it is, of course, a pure one.

For a constant t-rest momentum, the equations 2.16 become:

$$dt = \sinh U \cos V dx_0$$
$$dz = \cosh U \sin V dx_0$$
$$dy = \sinh hU \sin V dx_0 \qquad 2.18$$
$$dx = \cosh U \cos V dx_0$$

The equation 2.8 keeps its validity, i.e. the Einstein's formula is also valid in the space-like zone.

The velocity, the mass-flow and the force are presented bellow. The relationship between them remain unchanged when compare to 2.7.

$$v = \frac{dx}{dt} = \coth U$$
$$P = \frac{dz}{dt} = \coth U \tan V \qquad 2.19$$
$$F = \frac{dy}{dt} = \tan V$$
$$P = Fv$$

If this particle exists, then it is a pure tachyon because its absolute velocity is higher than 1(velocity of the light). The particle is alternatively named a pure supra-luminal particle.

We should mention that photons or luxons which move with the light velocity are typical pure particles.

If V is zero (U is a constant), then we have again the case of the uniform motion, this time with super-luminal velocity, in a force free space. By proceeding similarly to the equation 2.9 we obtain:

$$t = x_0 \sinh U$$
$$x = x_0 \cosh U$$
$$z = y_0 \sinh U \qquad 2.20$$
$$y = y_0 \cosh U$$

The last two equations keep their validity for small forces and momenta, i.e. for non-uniform motions. This can be easily shown using the procedure presented in the equation 2.10.

It is interesting to mention that for both tardyons and tachyons the following relation is valid:

$$y = zv \qquad 2.21$$

were v is the particle's velocity.

Now we are able to write the relation mass-momentum for tachyons:

$$y^2 - z^2 = y_0^2 \qquad 2.22$$

Converting to SI it obtains the energy-momentum relation:

$$(pc)^2 - E^2 = (p_0 c)^2 \qquad 2.23$$

The above equation shows that the mass (energy) of a tachyon moving with an infinite velocity is zero.

In the following diagram, presented in the Figure 4, are shown the energy (red curve) and the momentum (blue curve) of a tachyon as functions of its velocity. They go asymptotically to infinity when velocity approaches to 1(light velocity)

The proper mass is considered in SR as a universal invariant, but this is valid only for tardyons, and makes no sense in the case of tachyons. In this presentation we have shown that a particle in its t-rest frame is characterized by t-rest space x_0 and respectively momentum y_0. The universal invariant for a tachyon, moving in the space-like zone of ST, is its proper momentum y_0. Now let us try to put together kinematics and dynamics of different moving particles.

Citation: Negulescu VL. Motion analysis of particles using the hyper-complex numbers representation. *Open Acc J Math Theor Phy.* 2019;2(1):15‒20. DOI: 10.15406/oajmtp.2019.02.00047

Table 3 Comparison between different types of pure particles. Relationships are written in SI

	Pure Tardyon	Pure Tachyon	Photon
Velocity	$v < c$	$v > c$	$v = c$
Energy	$E = mc^2$	$E = mc^2$	$E = mc^2$
Momentum	$p = m$	$p = vm$	$p = cm$
Force	$F = \frac{dp}{dt}$	$F = \frac{dp}{dt}$	$F = \frac{dp}{dt}$
Power	$P = Fv$	$P = Fv$	$P = Fc$
Frame invariant parameter	m_0	p_0	$m_0 = p_0 = 0$
Energy-momentum relation	$E^2 - (pc)^2 = m_0^2 c^4$	$(pc)^2 - E^2 = (p_0 c)^2$	$E = pc$

In the next table are presented a comparison between tardyons, tachyons and light particles.

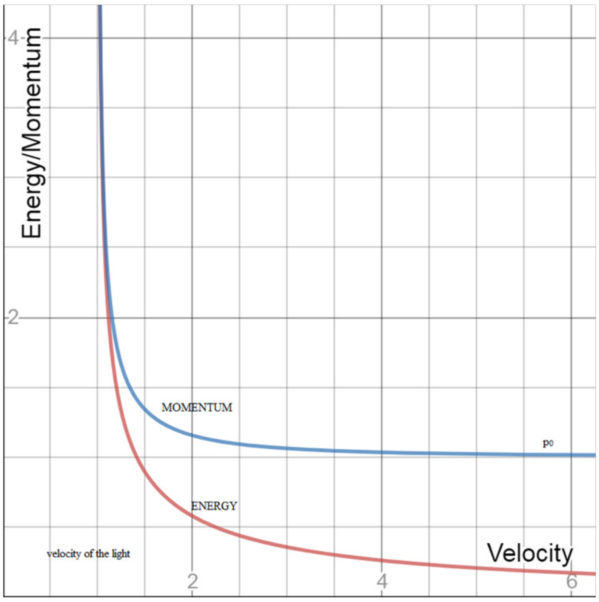

Figure 4 The Energy and Momentum of a tachyon as function of velocity.

Conclusion

The classic theory of tachyons considers the energy-momentum relation, known from SR and universally accepted, as still valid and concludes that the rest mass of a tachyon must be imaginary. In some modern works[6,7] the mass of tachyons is regarded as real. Our present contribution demonstrates, in a simple way, that the imaginary mass concept is superfluous.

Acknowledgments

None.

Conflicts of interest

Author declare that there is no conflict of interest.

References

1. Vlad L Negulescu. Hypernumbers and their applications in Mechanics. *Romanian Journal of Physics*. 1997;42:3–4.
2. Vlad L Negulescu. Hyper-Complex Numbers in Physics. *International Journal of Theoretical and Mathematical Physics*. 2015;5(2): 28–43.
3. Vlad L Negulescu. Addition of Velocities, Forces and Powers using Vector H-number Representation. *International Journal of Theoretical and Mathematical Physics*. 2017;7(3):57–60.
4. Geometrized Units System. 2018.
5. Lorenz Transformation. 2018.
6. Recami E. Classical Tachyons and Possible Applications. *Rivista Del Nuovo Cimiento*. 1986;9(6):1–176.
7. Ricardo S Vieira. An introduction to the theory of Tachyons;2012.

Citation: Negulescu VL. Motion analysis of particles using the hyper-complex numbers representation. *Open Acc J Math Theor Phy.* 2019;2(1):15–20. DOI: 10.15406/oajmtp.2019.02.00047