

A new study of energy levels of hydrogenic atoms and some molecules for new more general exponential screened coulomb potential

Abstract

In present research paper, the solutions of the modified Schrodinger (MSE) with new more general exponential screened coulomb (NMGESC) potential, have been presented by means generalized Bopp's shift method and standard perturbation theory, in the noncommutative three dimensional space phase (NC: 3D-RSP). The bound state energy eigenvalues, in terms of the generalized the hypergeometric function, the discrete atomic quantum numbers ($j = |l - s|, \dots, (l + s)(n, l)$ and m), two infinitesimal parameters which are induced by position-position, in addition to, the dimensional parameters (Θ, χ) of NMGESC potential and the corresponding noncommutative Hamiltonian operator were obtained for hydrogenic atoms and the molecules (CO, NO) . We have also shown that, the total complete degeneracy of energy levels of NMGESC potential equals the new values $2n^2$. Furthermore, the global group symmetry (NC: 3D-RSP) corresponding NMGESC potential reduce to the new subgroup (NC: 3D-RS) symmetries.

Keywords: schrödinger equation, hydrogenic atoms, more general exponential screened coulomb potential, noncommutative space and phase, star product and generalized Bopp's shift method

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Abbreviations: NMGESC, new more general exponential screened coulomb potential; NC: 3D-RSP, noncommutativity three dimensional real space phase; CCRs, canonical commutations relations; NNCCRs, new noncommutative canonical commutations relations; SP, Schrödinger picture; HP, Heisenberg picture; MSE, modified Schrödinger equation

Introduction

The more general exponential screened coulomb (MGESC) potential is known to describe adequately the effective potential of a many-body system of a variety of fields such as the atomic, solid state, plasma and quantum field theory.¹⁻⁴ In particularity, this potential used to calculate the bounded state eigen values of molecules (CO, NO) . The noncommutativity of space-time, which known firstly by Heisenberg and was formalized by Snyder in 1947, suggest by the physical recent results in string theory. Very recently, several authors have attempted to obtain either the exact or approximate solutions of the non-relativistic Schrodinger equation or two relativistic (Klein-Gordon and Dirac) equations for different potentials in NC space. We want to extended, the study of Ita et al.,³ to the case of extended quantum mechanics to the possibility of finding other applications and more profound interpretations in the sub-atoms scales on based to the works⁵⁻¹⁹ and our previously works²⁰⁻⁴⁰ in this context. The no relativistic energy levels for hydrogenic atoms and molecules

(CO, NO) , which interacted with NMGESC potential in the context of NC space have not been obtained yet. The purpose of the present paper is to attempt study the MSE with NMGESC potential (see below):

$$V_{mg}(r) = -\frac{V_0}{r}(1 + (1 + ar)\exp(-2ar)) \rightarrow V_{mg}(\hat{r}) = V_0\alpha - \frac{V_0}{\hat{r}} - 2V_0\alpha^3\hat{r}^2 \quad (1)$$

in (NC: 3D-RSP) symmetries using the generalized Bopp's shift method which depend on the concepts that we present below in the third section. The new structure of extended quantum mechanics based to new NC canonical commutations relations (NNCCRs) in both Schrödinger and Heisenberg pictures ((SP) and (HP)), respectively, as follows (Throughout this paper, the natural units $c = \hbar = 1$ will be used):⁵⁻²³

$$\begin{cases} [x_i, p_j] = [x_i(t), p_j(t)] = i\delta_{ij} \\ [x_i, x_j] = [x_i(t), x_j(t)] = 0 \\ [p_i, p_j] = [p_i(t), p_j(t)] = 0 \end{cases} \Rightarrow \begin{cases} [\hat{x}_i, \hat{p}_j] = [\hat{x}_i(t), \hat{p}_j(t)] = i\delta_{ij} \\ [\hat{x}_i, \hat{x}_j] = [\hat{x}_i(t), \hat{x}_j(t)] = i\theta_{ij} \\ [\hat{p}_i, \hat{p}_j] = [\hat{p}_i(t), \hat{p}_j(t)] = i\bar{\theta}_{ij} \end{cases} \quad (2)$$

However, the new operators $\hat{\xi}(t) = [\hat{x}_i(t) \vee \hat{p}_i(t)]$ in (HP) are depending to the corresponding new operators $\hat{\xi} = [\hat{x}_i \vee \hat{p}_i]$ in (SP) from the following projections relations:²⁰

$$\xi(t) = \exp(i\hat{H}_{mg}(t-t_0))\xi \exp(-i\hat{H}_{mg}(t-t_0)) \Rightarrow \hat{\xi}(t) = \exp(i\hat{H}_{nc-ni}(t-t_0)) * \hat{\xi} * \exp(-i\hat{H}_{nc-ni}(t-t_0)) \quad (3)$$

Here $\xi = (x_i \vee p_i)$ and $\hat{\xi}(t) = (x_i(t) \vee p_i(t))$, while the dynamics of new systems $\frac{d\xi(t)}{dt}$ are described from the following motion equations in extended quantum mechanics:²⁰

$$\frac{d\hat{\xi}(t)}{dt} = [\hat{\xi}(t), \hat{H}_{mg}] \Rightarrow \frac{d\hat{\xi}(t)}{dt} = [\hat{\xi}(t), \hat{H}_{nc-mg}] \quad (4)$$

the two operators \hat{H}_{mg} and \hat{H}_{nc-mg} are presents the ordinary and new quantum Hamiltonian operators for NMGESC potential in the

quantum mechanics and its extension, respectively, while $\frac{d\hat{\xi}(t)}{dt}$ are describe the dynamics of systems in (NC: 3D-RSP). The very small two parameters $\theta^{\mu\nu}$ and $\bar{\theta}^{\mu\nu}$ (compared to the energy) are elements

$$(f.g)(x,p) \rightarrow (\hat{f}, \hat{g})(\hat{x}, \hat{p}) \equiv (f * g)(x,p) = \left(fg - \frac{i}{2} \theta^{\mu\nu} \partial_\mu^x f \partial_\nu^x g - \frac{i}{2} \bar{\theta}^{\mu\nu} \partial_\mu^p f \partial_\nu^p g \right) (x,p) \quad (5)$$

where the notion $(\partial_\mu^x, \partial_\mu^p) f(x,p)$ denote to the $\left(\frac{\partial}{\partial p^\mu}, \frac{\partial}{\partial x^\mu} \right) f(x,p)$. The effects of (space-space) and (phase-phase) noncommutativity properties, respectively induce the second and the third terms in the above equation. The organization scheme of the recently work is given as follows: In next section, we briefly review the ordinary SE with MGESC potential on based to ref.³ The Section 3 is devoted to studying the MSE by applying the generalized Bopp's shift method for NMGESC potential. In the next subsection, by applying standard perturbation theory to find the quantum spectrum of n^{th} excited levels in for spin-orbital interaction in the framework of the global group (NC-3D: RSP) and then, we derive the magnetic spectrum for NMGESC potential. In the fourth section, we resume the global spectrum and corresponding NC Hamiltonian operator for NMGESC potential and corresponding energy levels of hydrogenic atoms and the molecules (CO, NO). Finally, the concluding remarks have been presented in the last section.

Overview of the eigenfunctions and the energy eigenvalues for MGESC potential for hydrogenic atoms and molecules (CO, NO):

In this section, we shall recall here the time independent SE for a MGESC potential $V_{mg}(r)$, which studied by Ita et al.,² and generalized to new form by Ita et al.,³ also in ref.:^{3,4}

$$V_{mg}(r) = \left(-\frac{a}{r} \right) (1 + (1+b)\exp(-2b)) \rightarrow V_{mg}(r) = -\frac{V_0}{r} (1 + (1+\alpha r)\exp(-2\alpha r)) \quad (6)$$

where $a \rightarrow V_0$ and $b \rightarrow \alpha r$ are the strength coupling constant (the potential depth of the MGESC potential) and the screened parameter (adjustable positive parameter), respectively. The part with exp. term of eq. (6) can be expanded in the power series of r up to the second term:

$$\frac{1}{r} e^{-2\alpha r} \cong \frac{1}{r} (1 - 2\alpha r + 2\alpha^2 r^2) = -2\alpha + \frac{1}{r} + 2\alpha^2 r \quad (7)$$

Inserting eq. (7) into eq. (6), explicit form of MGESC potential is obtained as:

$$V(r) \cong V_0 \alpha - \frac{2V_0}{r} - 2V_0 \alpha^3 r^2 \quad (8)$$

If we insert this potential into the Schrödinger equation (3):

$$\left(\frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} - \frac{l(l+1)}{r^2} \right) R_{nl}(r) + 2\mu \left[E_{nl} + \frac{V_0}{r} (1 + (1+\alpha r)\exp(-2\alpha r)) \right] R_{nl}(r) = 0 \quad (9)$$

Here μ is the reduced mass of molecules (CO, NO) or the reduced mass of electron and its nucleus for hydrogenic atoms. The electronic radial wave functions are shown as a function of the Laguerre polynomial in terms of some parameters:³

$$R_{nl}(v) = N_{n,l} (2\beta)^{\frac{1}{2}-\alpha} \exp\left(-\frac{v}{2}\right) v^{\alpha-\frac{1}{2}} L_n^{2\alpha+1}(v) \quad (10)$$

of two anti symmetric real matrixes $(\theta^{\mu\nu}, \bar{\theta}^{\mu\nu}) = -(\theta^{\nu\mu}, \bar{\theta}^{\nu\mu})$ and $(*)$ denote to the new star product, which is generalized between two arbitrary functions $(f,g)(x,p) \rightarrow (\hat{f}, \hat{g})(\hat{x}, \hat{p})$ to the new form $\hat{f}(\hat{x}, \hat{p}) \hat{g}(\hat{x}, \hat{p}) \equiv (f * g)(x,p)$ in ordinary 3-dimensional space-phase:⁶⁻²¹

where $r = (2\beta)^{-1} v$, therefore, the complete wave function $\Psi(r, \theta, \phi)$ and the energy E_{nl} of the potential in eq. (6) are given by:³

$$\Psi(r, \theta, \phi) = N_{nl} (2\beta)^{\alpha-1/2} \exp\left(-\frac{v}{2}\right) v^{\alpha-1/2} L_n^{2\alpha+1}(v) Y_l^m(\theta, \phi) \quad (11)$$

$$E_{nl} = -V_0 e^{-\alpha r_0} + 2\mu \left(\frac{V_0 + V_0 e^{-\alpha r_0}}{n+l+1} \right)^2 \quad (12)$$

With $r_0 = 1.21282$ and $r_0 = 1.1508$ for (CO and NO) molecules, for hydrogenic atoms, r_0 can be present the average dimension between the electron and the nucleus, N_{nl} is the normalization constant, $\alpha = \frac{1}{2} \sqrt{4l(l+1)+1}$, $\beta^2 = -\frac{2\mu E_{nl}}{\alpha^2}$ and $Y_l^m(\theta, \phi)$ are the well-known spherical harmonic functions.

Method and theoretical approach

In this section, we shall give an overview or a brief preliminary for a NMGESC potential $V_{nc-mg}(r)$, in (NC: 3D-RSP) symmetries. To perform this task the physical form of modified Schrödinger equation (MSE), it is necessary to replace ordinary three-dimensional Hamiltonian operators $\hat{H}(p_i, x_i)$, ordinary complex wave function $\Psi(\vec{r})$ and ordinary energy E_{nl} by new three Hamiltonian operators $\hat{H}_{nc-mg}(\hat{p}_i, \hat{x}_i)$, new complex wave function $\hat{\Psi}(\vec{\hat{r}})$ and new values E_{nc-mg} , respectively. In addition to replace the ordinary old product by new star product $(*)$, which allow us to constructing the MSE in (NC-3D: RSP) symmetries as:²¹⁻²⁸

$$\hat{H}_{mg}(p_i, x_i) \Psi(\vec{r}) = E_{nl} \Psi(\vec{r}) \Rightarrow \hat{H}(\hat{p}_i, \hat{x}_i) * \hat{\Psi}(\vec{\hat{r}}) = E_{nc-mg} \hat{\Psi}(\vec{\hat{r}}) \quad (13)$$

The Bopp's shift method employed in the solutions enables us to explore an effective way of obtaining the modified potential in extended quantum mechanics, it based on the following new commutators:²⁸⁻³⁴

$$[\hat{x}_i, \hat{x}_j] = [\hat{x}_i(t), \hat{x}_j(t)] = i\theta_{ij} \quad \text{and} \quad [\hat{p}_i, \hat{p}_j] = [\hat{p}_i(t), \hat{p}_j(t)] = i\bar{\theta}_{ij} \quad (14)$$

The new generalized positions and momentum coordinates (\hat{x}_i, \hat{p}_i) in (NC: 3D-RSP) are depended with corresponding usual generalized positions and momentum coordinates (x_{ii}, p_i) in ordinary quantum mechanics by the following, respectively:³⁰⁻³⁶

$$(x_i, p_i) \Rightarrow (\hat{x}_i, \hat{p}_i) = \left(x_i - \frac{\theta_{ij}}{2} p_j, p_i + \frac{\bar{\theta}_{ij}}{2} x_j \right) \quad (15)$$

The above equation allows us to obtain the two operators \hat{r}^2 and \hat{p}^2 in (NC-3D: RSP):³⁵⁻³⁸

$$(r^2, p^2) \Rightarrow (\hat{r}^2, \hat{p}^2) = \left(r^2 - \vec{L}\vec{\Theta}, p^2 + \vec{L}\vec{\bar{\Theta}} \right) \quad (16)$$

The two couplings $\vec{L}\vec{\Theta}$ and $\vec{L}\vec{\bar{\Theta}}$ are $(L_x\Theta_{12} + L_y\Theta_{23} + L_z\Theta_{13})$ and $(L_x\bar{\theta}_{12} + L_y\bar{\theta}_{23} + L_z\bar{\theta}_{13})$, respectively and $(L_x, L_y, \text{and } L_z)$ are the

three components of angular momentum operator \vec{L} while $\Theta_{ij} = \theta_{ij} / 2$. Thus, the reduced Schrödinger equation (without star product) can be written as:

$$\hat{H}(\hat{p}_i, \hat{x}_i) * \Psi(\vec{r}) = E_{nc-mg} \Psi(\vec{r}) \Rightarrow H(\hat{p}_i, \hat{x}_i) \psi(\vec{r}) = E_{nc-mg} \psi(\vec{r}) \quad (17)$$

the new operator of Hamiltonian $H_{nc-mgi}(\hat{p}_i, \hat{x}_i)$ can be expressed as:

$$H_{mg}(p_i, x_i) \Rightarrow H_{nc-mgi}(\hat{p}_i, \hat{x}_i) \equiv H\left(\hat{x}_i = x_i - \frac{\theta_{ij}}{2} p_j, \hat{p}_i = p_i + \frac{\bar{\theta}_{ij}}{2} x_j\right) \quad (18)$$

Now, we want to find to the NMGESC potential $V_{mg}(\hat{r})$:

$$V_{mg}(r) \Rightarrow V_{mg}(\hat{r}) = V_0 \alpha - \frac{2V_0}{\hat{r}} - 2V_0 \alpha^3 \hat{r}^2 \quad (19)$$

After straightforward calculations, we can obtain the important term $\left(-\frac{V_0}{\hat{r}}\right)$, which will be use to determine the NMGESC potential in (NC: 3D- RSP) symmetries as:

$$-\frac{V_0}{r} \Rightarrow -\frac{V_0}{\hat{r}} = -\frac{V_0}{r} - \frac{V_0 \vec{L} \vec{\Theta}}{2r^3} \quad (20)$$

By making the substitution above equation into eq. (19), we find the global our working new Hamiltonian operator $H_{nc-mg}(\hat{r})$ satisfies the equation in (NC: 3D-RSP) symmetries:

$$H_{mg}(p_i, x_i) \Rightarrow H_{nc-mg}(\hat{r}) \Rightarrow H_{mg}(p_i, x_i) + \left(2V_0 \alpha^3 - \frac{V_0}{r^3}\right) \vec{L} \vec{\Theta} + \frac{\vec{L} \vec{\bar{\theta}}}{2\mu} \quad (21)$$

where the operator $H_{mg}(p_i, x_i)$ is just the ordinary Hamiltonian operator with MGESC potential in commutative space:

$$H_{mg}(p_i, x_i) = \frac{p^2}{2\mu} + V_0 \alpha - \frac{2V_0}{r} - 2V_0 \alpha^3 r^2 \quad (22)$$

while the rest two terms are proportional's with two infinitesimals parameters $H_{per-mg}(r)$ and then we can considered as a perturbations terms $H_{per-mg}(r)$ in (NC: 3D-RSP) symmetries as:

$$H_{per-mg}(r) = \left(2V_0 \alpha^3 - \frac{V_0}{r^3}\right) \vec{L} \vec{\Theta} + \frac{\vec{L} \vec{\bar{\theta}}}{2\mu} \quad (23)$$

The exact modified spin-orbital spectrum for NMGESC potential in global (NC: 3D- RSP) symmetries

$$\left(\frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} - \frac{l(l+1)}{r^2}\right) R_{nl}(r) + 2\mu \left[E_{nl} + \frac{V_0}{r} (1 + (1 + \alpha r) \exp(-2\alpha r)) - \left(2V_0 \alpha^3 - \frac{V_0}{r^3}\right) \vec{L} \vec{\Theta} - \frac{\vec{L} \vec{\bar{\theta}}}{2\mu} \right] R_{nl}(r) = 0 \quad (27)$$

The two terms which composed the expression of $H_{per-mg}(r)$ are proportional with two infinitesimals parameters (Θ and $\bar{\theta}$), thus, in what follows, we proceed to solve the modified radial part of the MSE that is, equation (27) by applying standard perturbation theory for their exact solutions at first order of two parameters Θ and $\bar{\theta}$.

The exact modified spin-orbital spectrum for NMGESC potential in extended global (NC: 3D- RSP) symmetries

The purpose here is to give a complete prescription for determine the energy level of n^{th} excited states, of hydrogenic atoms with NMGESC potential, we first find the corrections E_{u-mg} and E_{d-mg} for hydrogenic atoms which have $j = l + 1/2$ (spin up) and $j = l - 1/2$ (spin down), respectively, at first order of two parameters Θ and $\bar{\theta}$ obtained by applying the standard perturbation theory to find the following:

In this subsection, we apply the same strategy, which we have seen in our previously works,³⁶⁻⁴⁰ under such particular choice, one can easily reproduce both couplings $\left(\vec{L} \vec{\Theta} \text{ and } \vec{L} \vec{\bar{\theta}}\right)$ to the new physical forms $(\gamma \Theta L S \text{ and } \gamma \bar{\theta} L S)$, respectively, to obtain the new forms of $H_{so-mg}(r, \Theta, \bar{\theta})$ for 3D- NMGESC potential as follows:

$$H_{so-mg}(r, \Theta, \bar{\theta}) \equiv \gamma \left\{ \left(2V_0 \alpha^3 - \frac{V_0}{r^3}\right) \Theta + \frac{\bar{\theta}}{2\mu} \right\} L S \quad (24)$$

Here $\gamma \approx \frac{1}{137}$ is a new constant, which play the role of fine structure constant, we have chosen the two vectors $\vec{\Theta}$ and $\vec{\bar{\theta}}$ parallel to the spin S of hydrogenic atoms. Furthermore, the above perturbative terms $H_{per-mg}(r)$ can be rewritten to the following new form:

$$H_{so-mg}(r, \Theta, \bar{\theta}) = \frac{\gamma}{2} \left\{ \left(2V_0 \alpha^3 - \frac{V_0}{r^3}\right) \Theta + \frac{\bar{\theta}}{2\mu} \right\} \left(\vec{J} - \vec{L} - \vec{S} \right)^2 \quad (25)$$

This operator traduces the coupling between spin \vec{S} and orbital momentum $\vec{L} S$. The set $(H_{so-mg}(r, \Theta, \bar{\theta}), J^2, L^2, S^2 \text{ and } J_z)$ forms a complete of conserved physics quantities and for $\vec{S} = 1/2$, the eigen values of the spin orbital coupling operator are $k_{\pm} \equiv \frac{1}{2} \left\{ \left(l \pm \frac{1}{2} \right) \left(l \pm \frac{1}{2} + 1 \right) + l(l+1) - \frac{3}{4} \right\}$ corresponding: $j = l + 1/2$ (spin up) and $j = l - 1/2$ (spin down), respectively then one can form a diagonal

(3×3) matrix, with diagonal elements are $(H_{so-mg})_{11}, (H_{so-mg})_{22}$ and $(H_{so-mg})_{33}$ for NMGESC potential in (NC: 3D-RSP) symmetries, as:

$$\begin{aligned} (H_{so-mg})_{11} &= \gamma k_+ \left(\left(2V_0 \alpha^3 - \frac{V_0}{2r^3}\right) \Theta + \frac{\bar{\theta}}{2\mu} \right) \text{ if } j = l + 1/2 \\ (H_{so-mg})_{22} &= \gamma k_- \left(\left(2V_0 \alpha^3 - \frac{V_0}{2r^3}\right) \Theta + \frac{\bar{\theta}}{2\mu} \right) \text{ if } j = l - 1/2 \\ (H_{so-mg})_{33} &= 0 \end{aligned} \quad (26)$$

After profound calculation, one can show that, the new radial function $R_{nl}(r)$ satisfying the following differential equation for NMGESC potential:

$$\begin{aligned} E_{u-mg} &= \gamma N_{nl}^2 (2\beta)^{-2-2\alpha} k_+ \int_0^{+\infty} \exp(-\nu) \nu^{2\alpha+1} [L_n^{2\alpha+1}(\nu)]^2 \left(\left(2V_0 \alpha^3 - \frac{V_0}{r^3}\right) \Theta + \frac{\bar{\theta}}{2\mu} \right) d\nu \\ E_{d-mg} &= \gamma N_{nl}^2 (2\beta)^{-2\alpha-2} k_- \int_0^{+\infty} \exp(-\nu) \nu^{2\alpha+1} [L_n^{2\alpha+1}(\nu)]^2 \left(\left(2V_0 \alpha^3 - \frac{V_0}{r^3}\right) \Theta + \frac{\bar{\theta}}{2\mu} \right) d\nu \end{aligned} \quad (28)$$

Now, we can write the above two equations to the new form:

$$\begin{aligned} E_{u-mg} &= \gamma N_{nl}^2 (2\beta)^{-2\alpha-2} k_+ \left\{ \Theta T_1(n, \alpha) + \Theta T_2(n, \alpha) + \frac{\bar{\theta}}{2\mu} T_3(n, \alpha) \right\} \\ E_{d-mg} &= \gamma N_{nl}^2 (2\beta)^{-2\alpha-2} k_- \left\{ \Theta T_1(n, \alpha) + \Theta T_2(n, \alpha) + \frac{\bar{\theta}}{2\mu} T_3(n, \alpha) \right\} \end{aligned} \quad (29)$$

Moreover, the expressions of the three factors $T_1(n, \alpha), T_2(n, \alpha)$ and $T_3(n, \alpha)$ are given by:

$$T_1(n, \alpha) = 2V_0 \alpha^3 T_3(n, \alpha) = 2V_0 \alpha^3 \int_0^{+\infty} v^{(2\alpha+2)^{-1}} \exp(-v) [L_n^{2\alpha+1}(v)]^2 dv \quad (30)$$

$$T_2(n, \alpha) = -V_0 (2\beta)^3 \int_0^{+\infty} v^{(2\alpha-1)^{-1}} \exp(-v) [L_n^{2\alpha+1}(v)]^2 dv$$

To evaluate the above factors $T_1(n, \alpha)$, $T_2(n, \alpha)$ and $T_3(n, \alpha)$, we apply the following special integration:⁴¹

$$\int_0^{+\infty} t^{\varepsilon-1} \exp(-\omega t) L_m^\lambda(\omega t) L_n^\beta(\omega t) dt = \frac{\omega^{-\varepsilon} \Gamma(n-\varepsilon+\beta+1) \Gamma(m+\lambda+1)}{m! n! \Gamma(1-\varepsilon+\beta) \Gamma(1+\lambda)} {}_3F_2(-m, \varepsilon, \varepsilon-\beta; -n+\varepsilon, \lambda+1; 1) \quad (31)$$

where ${}_3F_2(-m, \varepsilon, \varepsilon-\beta; -n+\varepsilon, \lambda+1; 1)$ is obtained from the generalized hyper geometric function. ${}_pF_q(\alpha_1, \dots, \alpha_p, \beta_1, \dots, \beta_q, z)$ for $p=3$ and $q=2$ while $\Gamma(x) = \int_0^{+\infty} z^{x-1} e^{-z} dz$ denote to the usual Gamma function. After straightforward calculations, we can obtain the explicitly results:

$$T_1(n, \alpha) = 2V_0 \alpha^3 T_3(n, \alpha) = 2V_0 \alpha^3 \frac{\Gamma(n) \Gamma(n+2\alpha+2)}{n! \Gamma(0) \Gamma(2\alpha+2)} {}_3F_2(-n, 2\alpha+2, 1; -n+2\alpha+2, 2\alpha+2; 1)$$

$$T_2(n, \alpha) = -V_0 (2\beta)^3 \frac{\Gamma(n+1) \Gamma(n+2\alpha+2)}{n! \Gamma(3) \Gamma(2\alpha+2)} {}_3F_2(-n, 2\alpha-1, -2; -n+2\alpha-1, 2\alpha+2; 1) \quad (32)$$

We have $\Gamma(0) = (-1)! = \infty$, $\Gamma(n+1) = n!$ and $\Gamma(3) = 2$, allow us the two to obtain the exact modifications E_{u-mg} and E_{d-mg} of n^{th} excited states of hydrogenic atoms with NMGESC potential, which produced by modified spin-orbital effect $H_{so-mg}(r, \Theta, \bar{\theta})$ as:

$$E_{u-mg} = -\frac{1}{2} \gamma \Theta N_{nl}^2 (2\beta)^{1-2\alpha} V_0 k_- f(n, \alpha) \quad (33)$$

$$E_{d-mg} = -\frac{1}{2} \gamma \Theta N_{nl}^2 (2\beta)^{1-2\alpha} V_0 k_+ f(n, \alpha)$$

Where $f(n, \alpha)$ is given by:

$$f(n, \alpha) = \frac{\Gamma(n+2\alpha+2)}{n! \Gamma(2\alpha+2)} {}_3F_2(-n, 2\alpha-1, -2; -n+2\alpha-1, 2\alpha+2; 1) \quad (34)$$

Thus, the extended global quantum group symmetry (NC: 3D-RSP) reduce to new quantum subgroup symmetry (NC: 3D-RS).

The exact modified magnetic spectrum for NMGESC potential in extended global (NC:3D-RSP) symmetries:

Further to the important previously obtained results, now, we consider another physically meaningful phenomena produced by the effect of NMGESC potential related to the influence of an external uniform magnetic field \vec{B} , to avoid the repetition in the theoretical calculations, it's sufficient to apply the following replacements:

$$\begin{cases} \bar{\Theta} \rightarrow \chi \vec{B} \\ \bar{\theta} \rightarrow \bar{\sigma} \vec{B} \end{cases} \Rightarrow \left(\left(2V_0 \alpha^3 - \frac{V_0}{2r^3} \right) \Theta + \frac{\bar{\theta}}{2\mu} \right) \Rightarrow \left(\left(2V_0 \alpha^3 - \frac{V_0}{2r^3} \right) \chi + \frac{\bar{\sigma}}{2\mu} \right) \vec{B} \vec{L} \quad (35)$$

Here χ and $\bar{\sigma}$ are two infinitesimal real proportional's constants, and we choose the arbitrary external magnetic field \vec{B} parallel to the (Oz) axis, which allow us to introduce the new modified magnetic Hamiltonian H_{m-mg} in (NC: 3D-RSP) symmetries as:

$$H_{m-mg} = \left(\left(2V_0 \alpha^3 - \frac{V_0}{2r^3} \right) \chi + \frac{\bar{\sigma}}{2\mu} \right) \mathfrak{S}_{\text{mod-}z} \quad (36)$$

Here $\mathfrak{S}_{\text{mod-}z} \equiv \vec{B} \cdot \vec{J} - \mathfrak{S}_z$ denote to the modified Zeeman effect while $\mathfrak{S}_z \equiv -\vec{S} \cdot \vec{B}$ is the ordinary Hamiltonian operator of Zeeman Effect. To obtain the exact noncommutative magnetic modifications of energy $E_{\text{mag-mg}}(n, m, \alpha)$, we just replace k_+ and Θ in the eq. (33) by the following parameters: m and χ , respectively:

$$E_{\text{mag-mg}}(n, m, \alpha) = -\frac{1}{2} \gamma \chi N_{nl}^2 (2\beta)^{1-2\alpha} V_0 k_- f(n, \alpha) B m \quad (37)$$

We have $-l \leq m \leq +l$, which allow us to fixing $(2l+1)$ values for discrete number m .

Results

In the light of the results of the preceding sections, let us resume the modified eigenenergies $E_{nc-umg}(n, j, l, s, m, \alpha)$ and $E_{nc-dmg}(n, j, l, s, m, \alpha)$ of a hydrogenic atoms with spin $S=1/2$ for MSE with NMGESC potential obtained in this paper, the total energies corresponding n^{th} excited states in (NC: 3D-RSP) symmetries are determined on based to our original results presented on the Eqs. (33) and (37), in addition to the ordinary energy E_{nl} MGESC potential, which presented in the eq. (13):

$$E_{nc-umg}(n, j, l, s, m, \alpha) = E_{nl} - \frac{1}{2} \gamma \chi N_{nl}^2 (2\beta)^{1-2\alpha} V_0 k_+ f(n, \alpha) (\Theta k_+ + B m \chi)$$

$$E_{nc-dmg}(n, j, l, s, m, \alpha) = E_{nl} - \frac{1}{2} \gamma \chi N_{nl}^2 (2\beta)^{1-2\alpha} V_0 k_- f(n, \alpha) (\Theta k_- + B m \chi) \quad (38)$$

This is the main goal of this work, It's clearly, that the obtained eigenvalues of energies are real's and then the noncommutative diagonal Hamiltonian H_{nc-mg} is Hermitian, furthermore it's possible to writing the three elements: α as follows:

$$(H_{nc-mg})_{11} = -\frac{\Delta_{nc}}{2\mu} + H_{\text{int-umg}}$$

$$(H_{nc-mg})_{22} = -\frac{\Delta_{nc}}{2\mu} + H_{\text{int-dmg}} \quad (39)$$

$$(H_{nc-mg})_{33} = -\frac{\Delta_{nc}}{2\mu} - \frac{V_0}{r} (1 + (1+\alpha r) \exp(-2\alpha r))$$

Where

$$\frac{\Delta_{nc}}{2\mu} = \frac{\Delta - \bar{\theta} \vec{L} - \bar{\sigma} \vec{L}}{2\mu}$$

$$H_{\text{int-umg}} = -\frac{V_0}{r} (1 + (1+\alpha r) \exp(-2\alpha r)) + \gamma (k_+ \Theta + \chi \mathfrak{S}_{\text{mod-}z}) \left(2V_0 \alpha^3 - \frac{V_0}{r^3} \right)$$

$$H_{\text{int-dmg}} = -\frac{V_0}{r} (1 + (1+\alpha r) \exp(-2\alpha r)) + \gamma (k_- \Theta + \chi \mathfrak{S}_{\text{mod-}z}) \left(2V_0 \alpha^3 - \frac{V_0}{r^3} \right) \quad (40)$$

Thus, the ordinary kinetic term for MGESC potential $-\frac{\Delta}{2\mu}$ and ordinary interaction $-\frac{V_0}{r} (1 + (1+\alpha r) \exp(-2\alpha r))$ are replaced by new modified form of kinetic term $\frac{\Delta_{nc}}{2\mu}$ and new modified interactions

modified to the new form ($H_{\text{int-umg}}$ and $H_{\text{int-dmg}}$). On the other hand, it is evident to consider the quantum number m takes $(2l+1)$ values and we have also two values for $2(2l+1)$, thus every state in usually three dimensional space of energy for NMGESC potential will be $2(2l+1)$ sub-states. To obtain the total complete degeneracy of energy level of the NMGESC potential in noncommutative three-dimension spaces-phases, we need to sum for all allowed values of l . Total degeneracy is thus,

$$2 \sum_{i=0}^{n-1} (2l+1) \equiv 2n^2 \quad (41)$$

$$E_{\text{nc-mg}}(n, j, l, s, m, \alpha) = -V_0 e^{-\alpha r} + 2\mu \left(\frac{V_0 + V_0 e^{-\alpha r}}{n+l+1} \right)^2 - \frac{1}{2} \gamma \chi N_{nl}^2 (2\beta)^{1-2\alpha} V_0 f(n, \alpha) (\Theta k(j, l, s) + Bm\chi) \quad (42)$$

Paying attention to the behavior of the spectrums (38) and (42) ($E_{\text{nc-umg}}(n, j, l, s, m, \alpha)$ and $E_{\text{nc-dmg}}(n, j, l, s, m, \alpha)$), it is possible to recover the results of commutative space (12) when we consider $(\Theta, \chi) \rightarrow (0, 0)$. Finally, we can say that the results we have obtained in our recently research are more profound than the results listed in our reference.²⁰

Conclusion

In this paper three-dimensional MSES for NMGESC potential has been solved via Bopp's shift method and standard perturbation theory in (NC: 3D-RSP) symmetries, we resume the main obtained results:

1. The exact energy spectrum ($E_{\text{nc-umg}}(n, j, l, s, m, \alpha)$, $E_{\text{nc-dmg}}(n, j, l, s, m, \alpha)$) for n^{th} excited levels, for hydrogenic atoms,
2. Ordinary interaction ($-\frac{V_0}{r}(1+(1+\alpha r)\exp(-2\alpha r))$) were replaced by new modified interactions 3) ($H_{\text{int-umg}}$ and $H_{\text{int-dmg}}$) for hydrogenic atoms.
3. The ordinary kinetic term $-\frac{\Delta}{2\mu}$ modified to the new form $\frac{\Delta_{\text{nc}}}{2\mu} = \frac{\Delta - \vec{\theta} \cdot \vec{L} - \vec{\sigma} \cdot \vec{L}}{2\mu}$ for NMGESC potential,
4. Spectra of bounded diatomic molecules (CO, NO) were studied analytically. NMGESC potential has been used to model the molecules (CO, NO)
5. We have shown that, the group symmetry (NC: 3D-RSP) corresponding NMGESC potential reduce to the symmetry subgroup (NC: 3D-RS).
6. It has been shown that, the MSE presents useful rich spectrums for improved understanding of hydrogenic atoms and molecules (CO, NO) influenced by the NMGESC potential and we have seen also that the modified of spin-orbital and modified Zeeman effect were appears du the presence of the two infinitesimal parameters (Θ, χ) which are induced by position-position noncommutativity property of space.

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None.

Conflicts of interest

Author declares that there is no conflicts of interest.

Note that the obtained new energy eigen values ($E_{\text{nc-umg}}(n, j, l, s, m, \alpha)$ and $E_{\text{nc-dmg}}(n, j, l, s, m, \alpha)$) and $E_{\text{nc-mg}}(n, j, l, s, \alpha, m)$ now depend to new discrete atomic quantum numbers (n, j, l, s) and m in addition to the parameter α of the potential. It is pertinent to note that when the molecules (CO, NO) have $\vec{S} \neq 1/2$, the total operator can be obtains from the interval $|l-s| \leq j \leq |l+s|$, which allow us to obtaining the eigenvalues of the operator $(\vec{J}^2 - \vec{L}^2 - \vec{S}^2)$ as $k(j, l, s) \equiv j(j+1) + l(l+1) - s(s+1)$ and then the non-relativistic energy spectrum $E_{\text{nc-mg}}(n, j, l, s, m, \alpha)$ reads:

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