Review Article

A systematic analysis for various threshold policies in queuing systems

Abstract

This analysis dedicates for estimating various threshold policies in queuing as well as machining systems. Such types of models deal with a very important class of real life congestion situations encountered in day-to-day as well as industrial scenario. A number of queuing/machine models have been developed by numerous researchers time to time by incorporating various thresholds namely N-policy, F-policy and D-policy. The concept of threshold gains a tremendous significance in present circumstances. The overview of the works done, methodology, some key aspects and ramifications concerning queuing systems with threshold has been outlined. The review of the relevant research work starting from the advent of threshold queuing systems up to the recent contributions has been provided. The main objective of this analysis is to provide sufficient evidence to queuing theorists and machine analysts who are interested and want to locate such phenomena in their research.

Keywords: queuing models, n-policy, f-policy, d-policy

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Introduction

In recent years queuing models under various thresholds have been the subject of great interest for the queue theorists. There is no need to discuss about the increasing graph of queuing theory due to its significant role in performance prediction of various congestion systems under N-policy, F-policy and D-policy. Queuing system under N-policy, F-policy and D-policy has been studied extensively since the late 1963's. A prominent work in this area was done in the early 90's. Queuing models under various thresholds policies consider the most common concern of controlling influxes and in reducing down the total cost. This was due to their wide demand in the modeling purpose of construction system, manufacturing system, telecommunication system and many more. Our main objective in this study is to provide an outline on the features for various thresholds policies for queuing systems in different frame-works. Theses thresholds policies for queuing systems are categorized as (i) N-policy, (ii) F-policy and (iii) D-policy.

In N-policy queuing system, the server (repairman) starts service (repair) to arriving customers or items when the number reaches up to some fixed value say 'N. On the other hand, in F-policy queuing system, no more customers or items are allowed to join the queuing system if the number of arriving customers or items reaches to its fixed capacity say 'L'. Arriving customers or items are allowed to join the queuing system as queue length decrease to its threshold level say 'F'. In D-policy queuing system, the server (repairman) starts service (repair) to waiting customer or items when cumulative service or repair times achieved some threshold value say 'D'.

The various applications of *thresholds models* can be made in day-to-day as well as industrial scenarios which motivate us for studying queuing models in different frameworks. For this purpose, we cite an example of the *production systems* wherein an item proceeds through various work station considered as queuing system. The arriving items made at an assembly line where a worker has inactivity between successive jobs. To use the time effectively, production managers can assign secondary tasks to the employee. However, it is important

that the worker returns to complete his or her ancillary activities by applying the concept of various thresholds (N-policy, D-policy or F-policy) for utilizing their time in proper manner.

This study is devoted for queuing system with various thresholds for the past decade. This paper is structured as follows. In section 2, we describe the performance analysis of queuing systems under various thresholds developed by prominent researchers in different frameworks. Classification of threshold with respect to queuing and machining system is discussed in section 3. Section 4 is devoted for analyzing various performance measures for queuing/machining system under threshold. Finally, paper is come to end with conclusions in section 5.

Queuing models under various threshold

Queuing models under various thresholds can be categorized as: (i) Markov modeling (MM) under threshold and (ii) Non-Markov modeling (NMM) under threshold. MM is the most powerful approach which is available to system managers for determining complex queuing systems which provides the results for both time dependent evolution and steady state of the system. In MM, the most idealized assumption is that inputs of items are exponentially-distributed i.e. they possess the memory less property. When this postulation is removed, the resulting Markov process is known as NMM which follow general probability distributions.

The first study on MM under the assumption of N-policy and F-policy was given by Yadin & Naor.[1] MM under various operating policies was suggested by Baker,[2] Medhi & Templetent.[3] Gupta[4] have derived complementary relationships for MM under various thresholds. Ke & Pearn[5] provided the optimal management policies for vacation MM with server breakdown. Additional, batch arrival MM under N-policy was examined by Choudhary & Madan.[6] Further, Choudhary and Paul[7] showed the impact of N-policy on MM for second optional service. MM with phase service under N-policy was investigated by Sharma.[8] Retrial MM with phase service under N-policy was examined by Wu et al.[9] Optimal control of (N, F) policy for MM with unreliable server was studied by Jain et al.[10] They have applied matrix

method for determining the various performance measures of MM. Vacation MM with server breakdown under N-policy was examined by Sharma[11] with the help of hyper exponential distribution. The MM under N-policy with server breakdown was suggested by Sharma[12] in different frameworks. Moreover, NMM under optimal control was investigated by Lee & Srinivasan.[13] Takagi[14] gave NMM under N-policy with set up time. Further, vacation NMM under N-policy was studied by Lee et al.[15] Lee et al.,[16] focused on NMM with single vacation under N-policy. Hur & Paik[17] applied concept of N-policy on NMM with the help of different arrival rates. Wanga et al.[18] provided recursive approach for determining NMM under F-policy. Later on, Jain & Bhahat[19] gave transient analysis of finite NMM retrial queues under F-policy. They have obtained various performance measures of NMM retrial queues. A parametric programming was suggested by Yang & Chang[20] for analyzing the F-policy queue with fuzzy theory. M/G/1 queue with D-policy was given by Artalejo.[21] Also, new fluctuation analysis of D-policy bulk queues with multiple vacations was studied by Agarwal.[22] Lee & Baek[23] analyzed D-policy discrete-time queue with J-optional service. Very recently, a study on NMM under D-Policy was given by Lan & Tang.[24] Repairable system under N-policy and imperfect coverage was investigated by Sharma.[25]

Classification of thresholds in queuing system

Queuing control is one of the most important problems, which can be applicable to many real life situations such as the production/inventory control, telecommunication process control, computer science and so on. This section describes various class of threshold strategies required in queuing system from a social point of view. Various thresholds policies are used in queuing/machining system time to time due to cost effective approach. To increase the cost of MM and NMM, prominent researchers have worked in past years by taking the concepts of N-policy, F-policy and D-policy in their study. When we adopt various thresholds in queuing system, the main objective is to control the system and to maximize social welfare which is defined here as the total expected net benefit of the members of the society, including both customers as well as servers.

Threshold in queuing/machining system plays an important role in making the proper utilization of valuable system resources by switching on the server when the customers/items reach a predetermined level. The provision of threshold in various queuing/machining systems proves to be very appropriate and economic to model some queuing/machining systems in which service/repair does not start until some specified number of arrivals say N are accumulated during idle period. The N-policy states that the service will be started by the server only after the accumulation of N units in the system. Before that the server remains idle in the system or goes for a vacation for during some ancillary work.

Threshold strategies in queuing/machining system can be easily understood with the help of an example of production system wherein the production of product starts when some specified raw material arrives in the machining system. In machining system, when a machine fails which resultant the delay in production. Also, this shows the reduction in expected profit. In machining system, F-policy states that no more failed machine allowed in the system when it reaches to some fixed capacity. As no of failed machine decreases up to some threshold level further failed machines are allowed to join system. In the queuing/machining system under D-policy, upon the completion of each busy period, the server/repairman is switched to the inactive mode. Later on, the server/repairman is switched to active mode when the total service/repair times of all customers waiting in the queue exceed some pre-specified value say 'D'. The pictorial view of N-policy, F-policy and D-policy machining system has been given in Figure 1–3, respectively.
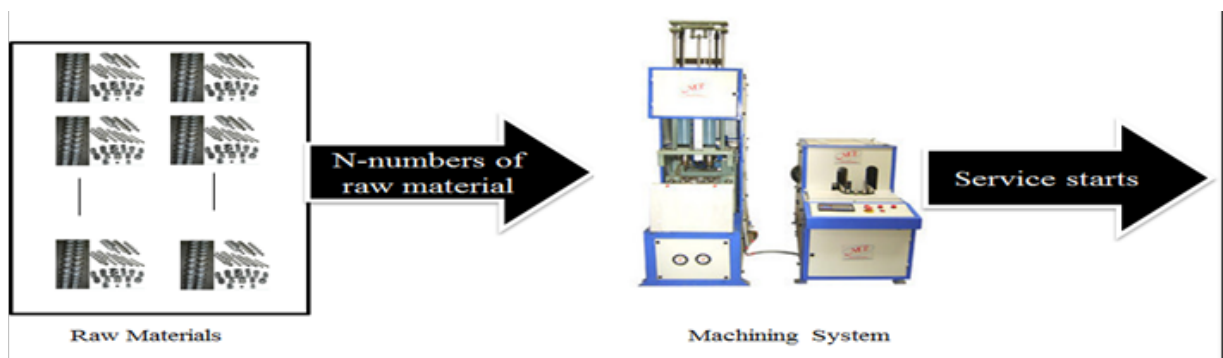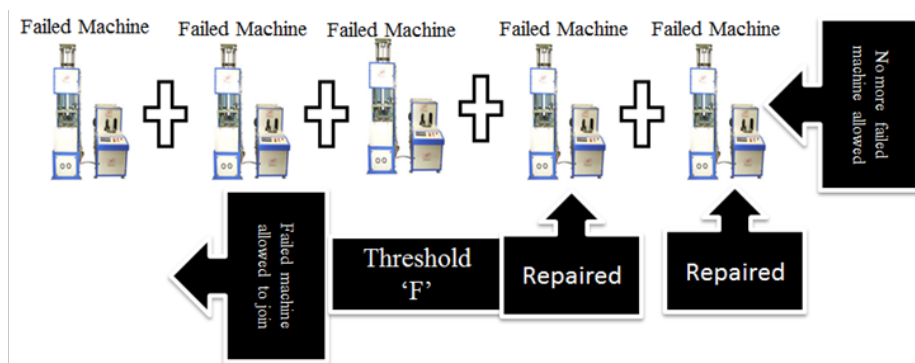

Figure 1 N-policy machining system.

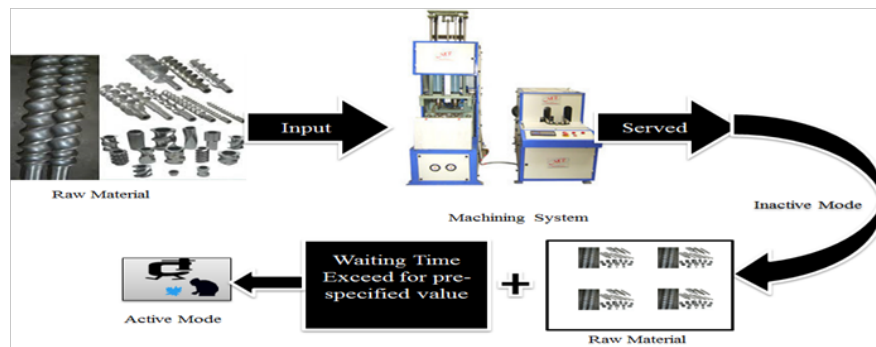**Figure 2** F-policy machining system.

Figure 3 D-policy machining system.

Performance measures

As the queuing/machining systems have become complex over time and their performance analysis has also become complex. Different methodologies have been developed over the years to access the performance of queuing/machining systems. Performance measures of queuing/machining systems are helpful for resolving many real life problems arising day to day life. The research work done provides valuable insight to the system managers and decision makers in forecasting the performance of these systems. The performance measures such as long run probabilities, average system size, average waiting time, throughput, reliability indices, etc. obtained may be helpful to system designers and decision makers in improving the reliability/availability of their systems.

Let 'λ' be the mean arrival rate of the customer/items and 'μ_i' i^{th} $(i=1,2,...,k)$ be the service rate of arriving customer/items (Wang and Yen,[26]). $P_{(0,n)}$ be the probability that there are n $(=0, 1, 2,...,N-1)$ customers in the system when the server is on vacation state. $P_{(i,n)}$ be the probability that there are n $(=1, 2,...)$ customers in the system and the customer in service is in i^{th} $(=1,2,...,k)$ phase when the server is in working state. Let the probability that the next customer to enter service is of type i be q_i $(i=1,2,....,k)$ and $\sum_{i=1}^{k} q_i=1$.

The steady state equations for M/H$_k$/1 queuing system are constructed as follows:

$$P_{(0,0)}=P_{(0,n)}, \quad 1 \leq n \leq N-1 \tag{1}$$

$$\lambda P_{(0,0)}=\sum_{j=1}^{k} \mu_j P_{(j,1)} \tag{2}$$

$$(\lambda+\mu_i)P_{(i,1)}=q_i \sum_{j=1}^{k} \mu_j P_{(j,2)}, \quad 1 \leq i \leq k \tag{3}$$

$$(\lambda+\mu_i)P_{(i,n)}=\lambda P_{(i,n-1)}+q_i \sum_{j=1}^{k} \mu_j P_{(j,n+1)}, \quad 1 \leq i \leq k, \; 2 \leq n \leq N-1 \tag{4}$$

$$(\lambda+\mu_i)P_{(i,N)}=q_i \lambda P_{(0,N-1)}+\lambda P_{(i,N-1)}+q_i \sum_{j=1}^{k} \mu_j P_{(j,N+1)}, \quad 1 \leq i \leq k \tag{5}$$

$$(\lambda+\mu_i)P_{(i,n)}=\lambda P_{(i,n-1)}+q_i \sum_{j=1}^{k} \mu_j P_{(j,n+1)}, \quad 1 \leq i \leq k, \; n \geq N+1 \tag{6}$$

Equations (1–6) can be solved easily using the generating function method to determine the distribution of the number of the customers.

Following these assumption, we provide the various performance measures as given under:

Long run probabilities

Idle period is very important measures in term of long run probabilities. The time span for which the server or repairman is free known as idle period of server or repairman which is given by:

$$P(I)=P_0 \tag{7}$$

Average system size

Average system size can be defined as the number of the customers waiting in the system for their turn. Average system size and queue size can be calculated as

$$E[L_s]=\lambda E[Ws] \tag{8}$$

$$E[L_q]=\lambda E[Wq] \tag{9}$$

Average waiting time

The average waiting time is the time spent by a customer in the system for his service. Using Little's formula, we can calculated as

$$E[Ws]=E[Wq]+\frac{1}{\mu} \tag{10}$$

Throughput

The system throughput is the sum of the service or repair rates that are provided to all customers or units in the queuing/machining system which is obtained as:

$$TP=\mu P_n \tag{11}$$

Reliability indices

Reliability of a system is defined as the probability that the system will perform efficiently throughout the interval (0,t) under operating conditions. The probability that the system is properly functioning at time t(≥ 0) is defined as the availability of the system.

Let 'T' be the random variable which denotes the failure time of the system. Then, reliability and mean time to system failure (MTTF) are given as

$$R(t) = 1 - F(t), \tag{12}$$

$$MTTF = \int_0^t R(t)\, dt \tag{13}$$

where, F(T) is the cumulative distribution function of T.

The performance measures of MM and NMM queues can be helpful to minimize the inconvenience of the users and maximize the reliability aspects of the server. Various performance indices equations (7–13) will be evaluated by employing suitable analytical and/or numerical techniques. To give an insight how a system can be improved to a desired level subject to techno economic constraints, optimal control policies will be suggested for such systems.

Conclusion

This study reviews the work done in the area of threshold queuing/ machining systems. The ideas discussed in various papers have been synthesized. Our study helps to queuing analysts, engineers, system managers for determining the proper use of thresholds with the help of MM and NMM in their study. The study based on thresholds models will be helpful in the quantitative assessment of many realistic systems based on manufacturing system, inventory system, transportation system, telecommunication system and many more. The purpose of this study is to provide sufficient information to queuing models with various thresholds in different frameworks, which may be applicable to many real world congestion situations. A wide range of literature has been covered and proper references have been cited.

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Conflict of interest

The author declares that there is no conflict of interest.

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