

Comparing theoretical and practical solution of the first order first degree ordinary differential equation of population model

Abstract

Population dynamics is the branch of mathematics that studies the size and age composition of populations as dynamical systems, the biological and environmental processes driving them such as birth and death rates and by immigration and emigration. In this paper, we are discussed how to read mathematical models and how to analyze them with the ultimate aim that we can critically judge the assumptions and the contributions of such models whenever we encounter them in your future biological research. Mathematical models are used in all areas of biology. All models in this paper are formulated in ordinary differential equations (ODEs). These will be analyzed by computing steady states. We developed the differential equations by ourselves following a simple graphical procedure, depicting each biological process separately. Experience with an approach for writing models will help us to evaluate models proposed by others.

Keywords: general equation of population growth, logistic equation, logistic, model for given data; solution of logistic model, comparing logistic model with actual data

Volume 1 Issue 1 - 2018

Abdullah Bin Masud, Foyez Ahmed

Department of Computer Science & Engineering, Shanto-Mariam University of Creative Technology, India

Correspondence: Abdullah Bin Masud, Department of Computer Science & Engineering, Shanto-Mariam University of Creative Technology, Dhaka-1230, Bangladesh, India, Email masud05math@gmail.com

Received: January 25, 2018 | **Published:** February 21, 2018

Introduction

In 1798, English economist Thomas Malthus was stated that population would grow at a geometric rate while the food supply grows at an arithmetic rate. The theory has been seen as flawed because of the limited factors observed when he developed the Law. It does not include factors, such as technology, disease, poverty, international conflict and natural disasters.

Malthusian models have the form $P(t) = P_0 e^{kt}$ where P_0 is the initial number of population, k is population growth rate (Malthusian parameter) and t is the time. Sometimes this model is called simple exponential growth model.

General equation of population growth

The rate of change of quantity = the rate of births - The rate of deaths.

Suppose $P(t)$ is the population, α is the per capital births rate and β is the per capital number of deaths population.

$$\begin{aligned} \frac{dP(t)}{dt} &= \alpha P(t) - \beta P(t) \\ &= P(t)(\alpha - \beta) \\ &= P(t)K \quad \text{where } K = \alpha - \beta \end{aligned}$$

This is the first order first degree ordinary differential equation.¹ The solution of (1) is $P(t) = ce^{kt}$. If $t=0, P=P_0 \therefore P_0=C$ and $P=P_0 e^{Kt}$

Birth rate is constant and death rate is linearly increasing

If $\alpha = \alpha_0$ and $\beta = \beta_0 + \beta_1 P(t)$ then we have

$$\frac{dP}{dt} = \alpha_0 P(t) - \{\beta_0 + \beta_1 P(t)\} P(t)$$

$$= (\alpha_0 + \beta_0) P(t) - \beta_1 P^2(t)$$

Birth rate constant and death rate is exponentially increasing

If $\beta = \beta_1 e^{kt}$ and $\alpha = \alpha_0$ then $\frac{dP}{dt} = \alpha_0 P(t) - \beta_1 e^{kt} P(t)$

Birth rate constant and death rate is sine function

If $\beta = \beta_1 \sin t$ and $\alpha = \alpha_0$ then $\frac{dP}{dt} = \alpha_0 P(t) - \beta_1 \sin(t) P(t)$

Death rate constant and birth rate linearly increasing

If $\alpha = \alpha_0 + \alpha_1 P(t)$ and $\beta = \beta$ then $\frac{dP(t)}{dt} = (\alpha_0 + \alpha_1 P(t)) P(t) - \beta P(t)$

Death rate constant and birth rate exponentially increasing

If $\alpha = \alpha_1 e^{kt}$ and $\beta = \beta_0$ then $\frac{dP(t)}{dt} = \alpha_1 e^{kt} P(t) - \beta_0 P(t)$

Death rate and birth rate are linearly increasing

If $\alpha = \alpha_0 + \alpha_1 P(t)$ and $\beta = \beta_0 + \beta_1 P(t)$ then

$$\begin{aligned} \frac{dP(t)}{dt} &= (\alpha_0 + \alpha_1 P(t)) P - (\beta_0 + \beta_1 P) P \\ &= (\alpha_0 - \beta_0) P + (\alpha_1 - \beta_1) P^2 \end{aligned}$$

Logistic equation: In real population growth is not always unlimited but may have an upper limit L where population can no longer

be sustained as time increase. The logistic ODE is

$$\frac{dP}{dt} = KP\left(1 - \frac{P}{L}\right) \dots\dots\dots(1)^2$$

$$P(t+1) - P(t) = KP\left(1 - \frac{P}{L}\right) \Rightarrow \frac{\Delta P}{P} = K\left(1 - \frac{P}{L}\right) \dots\dots\dots(2)$$

Logistic model for given data: Since we have discrete data, then we describe the model using a difference equation. The equation (A) can be written as

The equation says that the ratio of ΔP to P is linear function of P . First of all, let's consider the left hand side (LHS) of equation (2). We calculate the difference of the populations for two consecutive years, and then use those differences against the corresponding function values.³

Year	Bangladesh		India		Pakistan		Canada	
	P(t)	A	P(t)	a	P(t)	a	P(t)	a
1950	2.859358	0.000425	2.982949	0.000264	2.858823	0.00023	2.79945	0.000566
1951	2.860573	0.000402	2.983737	0.00024	2.859481	0.000267	2.801036	0.000579
1952	2.861723	0.000405	2.984562	0.00025	2.860244	0.0003	2.802656	0.00059
1953	2.862882	0.000425	2.985416	0.00025	2.861101	0.000329	2.804309	0.000598
1954	2.864098	0.000455	2.986294	0.00026	2.862042	0.000355	2.805987	0.000601
1955	2.865401	0.000488	2.987191	0.000306	2.863058	0.000379	2.807674	0.000597
1956	2.8668	0.000519	2.988106	0.000311	2.864143	0.0004	2.80935	0.000584
1957	2.868287	0.000542	2.989036	0.000316	2.865289	0.00042	2.810992	0.000563
1958	2.869843	0.000556	2.989982	0.000322	2.866491	0.000437	2.812573	0.000532
1959	2.871439	0.00056	2.990945	0.000328	2.867745	0.000454	2.814068	0.000494
1960	2.873049	0.00056	2.991925	0.000333	2.869047	0.000468	2.815458	0.000454
1961	2.874659	0.00056	2.992923	0.000338	2.87039	0.000481	2.816737	0.000418
1962	2.876269	0.000567	2.993935	0.000342	2.87177	0.000491	2.817913	0.000391
1963	2.877902	0.000584	2.99496	0.000344	2.873181	0.000499	2.819015	0.000377
1964	2.879585	0.000604	2.995992	0.000346	2.874615	0.000505	2.820079	0.000372
1965	2.881326	0.00063	2.997029	0.000347	2.876068	0.000512	2.821129	0.000369
1966	2.883143	0.000644	2.998069	0.000348	2.877539	0.000517	2.82217	0.000363
1967	2.885001	0.000625	2.999113	0.000352	2.879027	0.000521	2.823194	0.000358
1968	2.886806	0.000567	3.000168	0.000358	2.880528	9.55E-09	2.824204	0.000352
1969	2.888444	0.000487	3.001242	0.000366	2.880528	0.001048	2.825197	0.000345
1970	2.889852	0.0004	3.002339	0.000374	2.883547	0.000525	2.826173	0.00034
1971	2.891009	0.000333	3.003461	0.00038	2.885062	0.000528	2.827135	0.000335
1972	2.89197	0.000304	3.004603	0.000383	2.886585	0.000534	2.828082	0.000325
1973	2.89285	0.000326	3.005755	0.000383	2.888127	0.000546	2.829	0.000308
1974	2.893793	0.000382	3.006908	0.000381	2.889702	0.00056	2.829872	0.000288
1975	2.894898	0.000446	3.008055	0.000378	2.891321	0.000573	2.830687	0.000268
1976	2.896189	0.000495	3.009191	0.000376	2.892979	0.000585	2.831444	0.00025
1977	2.897622	0.000526	3.010324	0.000372	2.89467	0.000596	2.832153	0.000236
1978	2.899147	0.000533	3.011443	0.000374	2.896396	0.000607	2.832822	0.000226
1979	2.900694	0.000524	3.01257	0.000375	2.898155	0.000617	2.833463	0.00022
1980	2.902215	0.000512	3.013701	0.000377	2.899945	0.000626	2.834085	0.000213
1981	2.903702	0.000505	3.014838	0.000377	2.901759	0.000631	2.834688	0.000207
1982	2.905169	0.000534	3.015975	0.000375	2.903589	0.000631	2.835275	0.00021
1983	2.90672	0.000466	3.017106	0.000371	2.905421	0.000626	2.835871	0.000223
1984	2.908076	0.000502	3.018225	0.000365	2.90724	0.000617	2.836503	0.000242
1985	2.909535	0.000503	3.019325	0.000358	2.909035	0.000608	2.837191	0.000264
1986	2.911	0.000501	3.020407	0.000352	2.910804	0.000598	2.837941	0.000282

Table Continued..

Year	Bangladesh		India		Pakistan		Canada	
	P(t)	A	P(t)	a	P(t)	a	P(t)	a
1987	2.912459	0.000493	3.02147	0.000346	2.912544	0.000582	2.838741	0.000291
1988	2.913895	0.000478	3.022514	0.000339	2.91424	0.000562	2.839566	0.000287
1989	2.915288	0.000458	3.02354	0.000333	2.915877	0.000538	2.84038	0.000274
1990	2.916624	0.000437	3.024548	0.000327	2.917446	0.000514	2.841157	0.000259
1991	2.917898	0.000418	3.025537	0.000321	2.918945	0.000492	2.841894	0.000247
1992	2.919117	0.000404	3.026507	0.000315	2.92038	0.000475	2.842597	0.000235
1993	2.920298	0.000398	3.02746	0.00031	2.921767	0.000466	2.843265	0.000224
1994	2.921459	0.000395	3.028401	0.000307	2.923128	0.000461	2.843901	0.000213
1995	2.922614	0.000393	3.029329	0.000303	2.924474	0.000458	2.844507	0.000202
1996	2.923763	0.000388	3.030247	0.000299	2.925813	0.000453	2.845083	0.000192
1997	2.924899	0.000381	3.031152	0.000294	2.927139	0.000444	2.84563	0.000186
1998	2.926014	0.00037	3.032042	0.000288	2.928439	0.00043	2.846161	0.000185
1999	2.927098	0.000495	3.032915	0.000281	2.929697	0.000412	2.846688	0.000188
2000	2.928547	0.000205	3.033768	0.000275	2.930905	0.000395	2.847223	0.000191
2001	2.929148	0.000331	3.034601	0.000268	2.932061	0.00038	2.847767	0.000194
2002	2.930118	0.000314	3.035415	0.000262	2.933176	0.000371	2.84832	0.000199
2003	2.931039	0.000352	3.03621	0.000255	2.934264	0.000367	2.848887	0.000206
2004	2.932071	0.000144	3.036985	0.000249	2.935341	0.000356	2.849474	0.000214
2005	2.932492	0.000371	3.037741	0.000243	2.936387	0.000381	2.850083	0.000222
2006	2.933579	0.000152	3.03848	0.000237	2.937506	0.00037	2.850717	0.000229
2007	2.934025	0.000204	3.039199	0.00023	2.938592	0.000371	2.85137	0.000233
2008	2.934624	0.000201	3.0399	0.000223	2.939683	0.000374	2.852033	0.000231
2009	2.935213	0.000205	3.040578	0.000216	2.940781	0.000376	2.852692	0.000226
2010	2.935816	0.000212	3.041235	0.000209	2.941887	0.000378	2.853337	0.00022
2011	2.936438	0.000217	3.04187	0.000202	2.942999	0.00038	2.853965	0.000214
2012	2.937075	0.000219	3.042483	0.000196	2.944117	0.000379	2.854577	0.000209
2013	2.93772	0.000219	3.04308	0.000192	2.945232	0.000375	2.855172	0.000203
2014	2.938363	0.000216	3.043665	0.000189	2.946337	0.00037	2.855752	0.000198
2015	2.938997	0.000213	3.044241	0.000187	2.947427	0.000364	2.856318	0.000193
2016	2.939623	0.00021	3.04481	0.000184	2.948499	0.000357	2.85687	0.000187

Determining the value of K and L: In the Least Square Approximation graph, we know the equation for the line, which is,

Substituting the point P(1950) and P(1951) in (10) we have

$$y = a + bx \dots \dots \dots (3)$$

Variable/Country	India	Pakistan	Canada
	$y = 0.471146818$		
	$- 0.16228x$		
Bangladesh			
	$y = 0.471146818$		
	$- 0.16228x$		
$y=a+bx.$	$y = 0.471146818$	$y = 0.471146818$	
	$- 0.16228x$	$- 0.16228x$	

Table Continued..

P_1	2.859358	2.982949	2.858823	2.79945
P_1	2.860576	2.983737	2.859481	67:13:29
y_1	0.045499	0.033148	0.045835	0:50:01
y_2	0.044285	0.032359	0.045177	0:47:44

Equation (2) can be written as

$$K(1 - P_1 / L) = y_1 \dots\dots\dots(4)$$

$$\text{and } K(1 - P_2 / L) = y_2 \dots\dots\dots(5)$$

$$\text{Solving (3) and (4) we have } L = \frac{P_1 y_2 - P_2 y_1}{y_2 - y_1} \text{ and } K = \frac{y_1}{1 - P_1 / L}$$

Variable/ Country	Bangladesh	India	Pakistan	Canada
L(Caring Capacity)	2.904878989	3.016041509	2.90468156	2.834196893
Exp(Exp(L))	85415102.72	731302266	85107708.4	24562428.95
K (Constant)	2.903479866	3.021126376	2.90316821	2.833056521

Solution of logistic model

Equation (1) is Bernoulli equation,⁴ we have

$$\begin{aligned} \frac{dP}{dt} &= KP(1 - \frac{P}{L}) \\ \Rightarrow \frac{dP}{dt} &= KP - \frac{K}{L}P^2 \\ \Rightarrow \frac{dP}{dt} - KP &= -\frac{K}{L}P^2 \\ \Rightarrow -\frac{1}{P^2} \frac{dP}{dt} + \frac{K}{P} &= \frac{K}{L} \dots\dots\dots(6) \end{aligned}$$

Put $\frac{1}{P} = V$

$$\therefore -\frac{1}{P^2} \frac{dP}{dt} = \frac{dV}{dt}$$

From (13) we have $\frac{dV}{dt} + KV = \frac{K}{L}$

Now this equation is exact. Hence integrating factor

$$\begin{aligned} IF &= e^{\int K dt} \\ &= e^{Kt} \end{aligned}$$

Hence the solution is

$$V \cdot e^{Kt} = \int \frac{K}{L} e^{Kt} dt$$

$$\begin{aligned} \Rightarrow V \cdot e^{Kt} &= \frac{K}{L} \frac{e^{Kt}}{K} + c \\ \Rightarrow \frac{1}{P} e^{Kt} &= \frac{1}{L} e^{Kt} + c \\ \Rightarrow \frac{1}{P} &= \frac{1}{L} + c e^{-Kt} \\ \Rightarrow P &= \frac{L}{1 + L c e^{-Kt}} \dots\dots\dots(7) \end{aligned}$$

If $t \rightarrow \infty$, then $P=L$.

Comparing logistic model with actual data

$$\begin{aligned} \frac{dP}{dt} &= P(K - \frac{PK}{L}) \\ \Rightarrow \frac{dP}{dt} &= KP \left(\frac{L-P}{L} \right) \\ \Rightarrow \frac{1}{K} \frac{L}{P(L-P)} dP &= dt \\ \text{Integrating we have, } \frac{1}{K} \ln \left(\frac{P}{L-P} \right) &= t+c \end{aligned}$$

If $t = 0$ then find the value of c

Variable/ Country	Bangladesh	India	Pakistan	Canada
L(Caring Capacity)	2.904878989	3.016041509	2.90468156	2.834196893
Exp(Exp(L))	85415102.72	731302266	85107708.4	24562428.95
K (Constant)	2.903479866	3.021126376	2.90316821	2.833056521

Putting the values of c in (7), we have

Year	Time	Bangladesh		India		Pakistan		Canada	
		Theoretical Data	Original Data						
1950	0	0.564912	2.859358	0.548991	2.982949	0.565691	2.858823	0.525742	2.79945
1951	1	0.338815	2.860573	2.474081	2.983737	2.367713	2.859481	2.252355	2.801036

Table Continued..

Year	Time	Bangladesh		India		Pakistan		Canada	
		Theoretical Data	Original Data						
1952	2	0.507008	2.861723	2.984176	2.984562	2.868994	2.860244	2.791768	2.802656
1953	3	0.73477	2.862882	3.014472	2.985416	2.902701	2.861101	2.831665	2.804309
1954	4	1.021245	2.864098	3.015965	2.986294	2.904573	2.862042	2.834048	2.805987
1955	5	1.349886	2.865401	3.016038	2.987191	2.904676	2.863058	2.834188	2.807674
1956	6	1.689356	2.8668	3.016041	2.988106	2.904681	2.864143	2.834196	2.80935
1957	7	2.004061	2.868287	3.016042	2.989036	2.904682	2.865289	2.834197	2.810992
1958	8	2.26787	2.869843	3.016042	2.989982	2.904682	2.866491	2.834197	2.812573
1959	9	2.470983	2.871439	3.016042	2.990945	2.904682	2.867745	2.834197	2.814068
1960	10	2.617359	2.873049	3.016042	2.991925	2.904682	2.869047	2.834197	2.815458
1961	11	2.717896	2.874659	3.016042	2.992923	2.904682	2.87039	2.834197	2.816737
1962	12	2.784688	2.876269	3.016042	2.993935	2.904682	2.87177	2.834197	2.817913
1963	13	2.828086	2.877902	3.016042	2.99496	2.904682	2.873181	2.834197	2.819015
1964	14	2.855879	2.879585	3.016042	2.995992	2.904682	2.874615	2.834197	2.820079
1965	15	2.873512	2.881326	3.016042	2.997029	2.904682	2.876068	2.834197	2.821129
1966	16	2.884634	2.883143	3.016042	2.998069	2.904682	2.877539	2.834197	2.82217
1967	17	2.891622	2.885001	3.016042	2.999113	2.904682	2.879027	2.834197	2.823194
1968	18	2.896004	2.886806	3.016042	3.000168	2.904682	2.880528	2.834197	2.824204
1969	19	2.898746	2.888444	3.016042	3.001242	2.904682	2.880528	2.834197	2.825197
1970	20	2.900461	2.889852	3.016042	3.002339	2.904682	2.883547	2.834197	2.826173
1971	21	2.901533	2.891009	3.016042	3.003461	2.904682	2.885062	2.834197	2.827135
1972	22	2.902203	2.89197	3.016042	3.004603	2.904682	2.886585	2.834197	2.828082
1973	23	2.902621	2.89285	3.016042	3.005755	2.904682	2.888127	2.834197	2.829
1974	24	2.902883	2.893793	3.016042	3.006908	2.904682	2.889702	2.834197	2.829872
1975	25	2.903046	2.894898	3.016042	3.008055	2.904682	2.891321	2.834197	2.830687
1976	26	2.903148	2.896189	3.016042	3.009191	2.904682	2.892979	2.834197	2.831444
1977	27	2.903212	2.897622	3.016042	3.010324	2.904682	2.89467	2.834197	2.832153
1978	28	2.903251	2.899147	3.016042	3.011443	2.904682	2.896396	2.834197	2.832822
1979	29	2.903276	2.900694	3.016042	3.01257	2.904682	2.898155	2.834197	2.833463
1980	30	2.903292	2.902215	3.016042	3.013701	2.904682	2.899945	2.834197	2.834085
1981	31	2.903301	2.903702	3.016042	3.014838	2.904682	2.901759	2.834197	2.834688
1982	32	2.903307	2.905169	3.016042	3.015975	2.904682	2.903589	2.834197	2.835275
1983	33	2.903311	2.90672	3.016042	3.017106	2.904682	2.905421	2.834197	2.835871
1984	34	2.903313	2.908076	3.016042	3.018225	2.904682	2.90724	2.834197	2.836503
1985	35	2.903315	2.909535	3.016042	3.019325	2.904682	2.909035	2.834197	2.837191
1986	36	2.903316	2.911	3.016042	3.020407	2.904682	2.910804	2.834197	2.837941
1987	37	2.903316	2.912459	3.016042	3.02147	2.904682	2.912544	2.834197	2.838741
1988	38	2.903317	2.913895	3.016042	3.022514	2.904682	2.91424	2.834197	2.839566
1989	39	2.903317	2.915288	3.016042	3.02354	2.904682	2.915877	2.834197	2.84038
1990	40	2.903317	2.916624	3.016042	3.024548	2.904682	2.917446	2.834197	2.841157
1991	41	2.903317	2.917898	3.016042	3.025537	2.904682	2.918945	2.834197	2.841894

Table Continued..

Year	Time	Bangladesh		India		Pakistan		Canada	
		Theoretical Data	Original Data						
1992	42	2.903317	2.919117	3.016042	3.026507	2.904682	2.92038	2.834197	2.842597
1993	43	2.903317	2.920298	3.016042	3.02746	2.904682	2.921767	2.834197	2.843265
1994	44	2.903317	2.921459	3.016042	3.028401	2.904682	2.923128	2.834197	2.843901
1995	45	2.903317	2.922614	3.016042	3.029329	2.904682	2.924474	2.834197	2.844507
1996	46	2.903317	2.923763	3.016042	3.030247	2.904682	2.925813	2.834197	2.845083
1997	47	2.903317	2.924899	3.016042	3.031152	2.904682	2.927139	2.834197	2.84563
1998	48	2.903317	2.926014	3.016042	3.032042	2.904682	2.928439	2.834197	2.846161
1999	49	2.903317	2.927098	3.016042	3.032915	2.904682	2.929697	2.834197	2.846688
2000	50	2.903317	2.928547	3.016042	3.033768	2.904682	2.930905	2.834197	2.847223
2001	51	2.903317	2.929148	3.016042	3.034601	2.904682	2.932061	2.834197	2.847767
2002	52	2.903317	2.930118	3.016042	3.035415	2.904682	2.933176	2.834197	2.84832
2003	53	2.903317	2.931039	3.016042	3.03621	2.904682	2.934264	2.834197	2.848887
2004	54	2.903317	2.932071	3.016042	3.036985	2.904682	2.935341	2.834197	2.849474
2005	55	2.903317	2.932492	3.016042	3.037741	2.904682	2.936387	2.834197	2.850083
2006	56	2.903317	2.933579	3.016042	3.03848	2.904682	2.937506	2.834197	2.850717
2007	57	2.903317	2.934025	3.016042	3.039199	2.904682	2.938592	2.834197	2.85137
2008	58	2.903317	2.934624	3.016042	3.0399	2.904682	2.939683	2.834197	2.852033
2009	59	2.903317	2.935213	3.016042	3.040578	2.904682	2.940781	2.834197	2.852692
2010	60	2.903317	2.935816	3.016042	3.041235	2.904682	2.941887	2.834197	2.853337
2011	61	2.903317	2.936438	3.016042	3.04187	2.904682	2.942999	2.834197	2.853965
2012	62	2.903317	2.937075	3.016042	3.042483	2.904682	2.944117	2.834197	2.854577
2013	63	2.903317	2.93772	3.016042	3.04308	2.904682	2.945232	2.834197	2.855172
2014	64	2.903317	2.938363	3.016042	3.043665	2.904682	2.946337	2.834197	2.855752
2015	65	2.903317	2.938997	3.016042	3.044241	2.904682	2.947427	2.834197	2.856318
2016	66	2.903317	2.939623	3.016042	3.04481	2.904682	2.948499	2.834197	2.85687

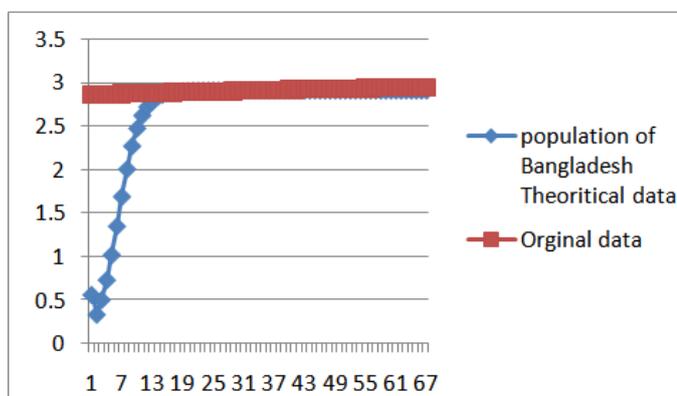


Figure 1 Comparing graph of theoretical data with original data of Bangladesh.

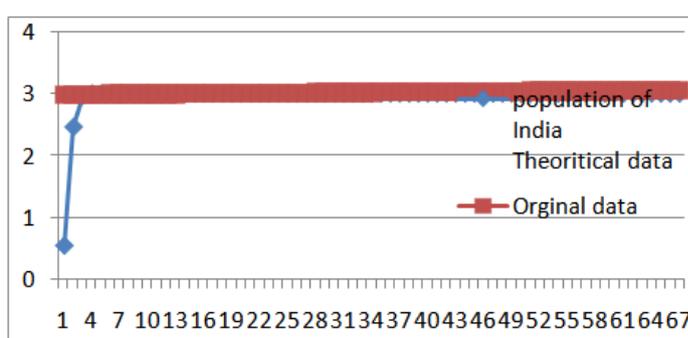


Figure 2 Comparing graph of theoretical data with original data of India.

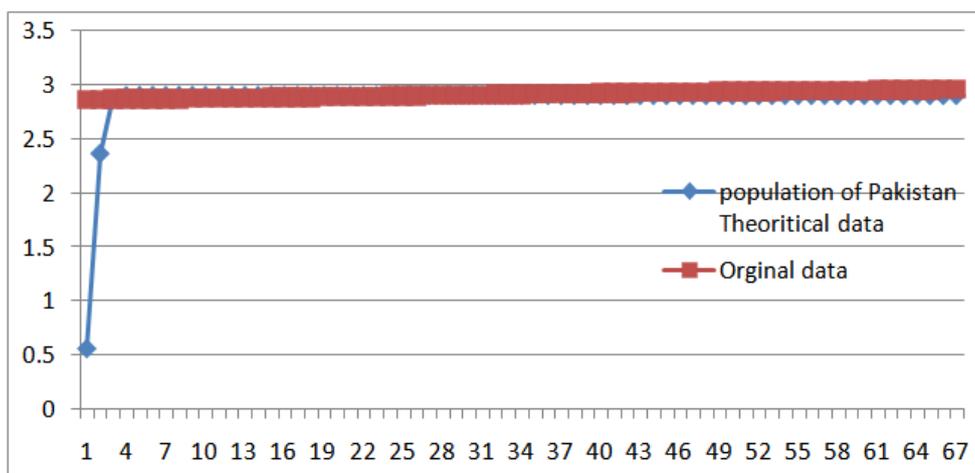


Figure 3 Comparing graph of theoretical data with original data of Pakistan.

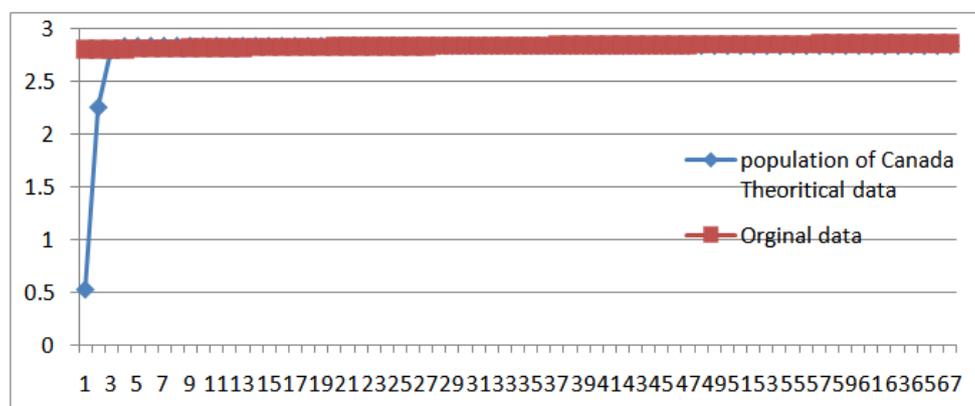


Figure 4 Comparing graph of theoretical data with original data of Canada.

Conclusion

The carrying capacity of Bangladesh is 85415102.72 but at this moment total number of population is 164827718. It is the biggest problem. The government of Bangladesh needs to take necessary step otherwise socio economic system is breakdown. Every country of Subcontinent, the total population of these countries is greater twice of carrying capacity. In Canada, total number of population is greater the carrying capacity.

Acknowledgements

None.

Conflict of interest

The author declares no conflict of interest.

References

1. Dreyer TP. *Modeling with Ordinary Differential Equations*. USA: CRC Press; 1993. 304 p.
2. Kelley W, Peterson A. *Theory of Differential Equations Classical and Qualitative*. Springer; 2004.
3. Mooney DD, Swift RJ. *A Course in Mathematical Modeling*. UK: Cambridge University Press; 1999. 431 p.
4. Zill DD. *A First Course in Differential Equations*. 1993.