Research Article

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Quanta of the magnetic monopole entering the Oersted–Ampere law

Abstract

The paper demonstrates that quantization of the magnetic monopole, much similar to that concerning the magnetic flux in superconductors, can occur also in a non–superconducting case represented by the Oersted–Ampere law. This result can occur on condition the Joule–Lenz law for the action quanta of electrons participating in the effect is taken into account. The quantum result obtained for a monopole can be confirmed with the aid of the uncertainty principle applied to the energy and time period characteristic for the current involved in the Oersted–Ampere phenomenon. Another check is a comparison of the ratio size of the magnetic monopole to the absolute value of the electron charge with the formula obtained by Dirac.

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Introduction

Quantum properties of the physical parameters entering the electrodynamics were examined already on the basis of the old quantum theory. In principle these properties were important first on the mechanical level. Nevertheless discrete properties of the electric charge carried by the electrons, atomic nuclei, and atoms in general, did influence the quantum theory from its very beginning.

Perhaps the most striking and most investigated particle behaviour was that connected with the electron. Its constant and definite electric charge became evident from the very beginning of the atomic theory, and rather soon afterwards a similar interest was attracted by the electron spin. On the other side, however, our knowledge on the magnetostatics and a definite size of the magnetic poles entering the atomic physics, remained much poorer than that obtained in the case of electrostatics. Nevertheless several laws of electrodynamics apply the idea of a definite pole and its use.

An example of such monopole – considered in the present paper–is provided by the Oersted law. In Sec. 2 we demonstrate the quantum aspects of that law and estimate the size of the magnetic pole entering the calculations. In a further Section the quantum properties of the pole are compared with much similar quanta obtained earlier for the magnetic flux. Finally the magnetic pole obtained in Sec. 2 is compared with the pole size estimated on the basis of the uncertainty principle.

Oersted–Ampere law and its quantum behaviour

The discovery of the action of the electric current on a magnetic pole done by Oersted led next Ampere to state the following law (see e.g[1]): a long straight wire carrying the electric current i is acting on a magnetic pole of strength $m^{(p)}$ located at distance r from the wire with a force

$$F = \frac{2im^{(p)}}{cr} \tag{1}$$

where $1/c$ is a proportionality constant having a speed dimension.

With a substitution of the current expression

$$i = \frac{e}{\Delta t} \tag{2}$$

where e is the electron charge and Δt an interval of time, the formula (1) becomes

$$Frc = 2im^{(p)} = 2\frac{e}{\Delta t}m^{(p)}. \tag{3}$$

The product $Fr = \Delta E$ (4)

entering (3) has the dimension of energy and can be considered as an energy amount ΔE.

In a study of the Joule–Lenz law concerning a transfer of energy ΔE from one quantum level to a neighbouring level within the time interval Δt we found that the product of ΔE and Δt satisfies the formula[2,3]

$$\Delta E\Delta t = h \tag{5}$$

The Δt is considered as a shortest interval of time connected with the electron transition between two neighboring quantum levels.

A transformation of (3) into

$$Frc\Delta t = \Delta E\Delta tc = 2em^{(p)} \tag{6}$$

where ΔE is in fact an amount of energy of an arbitrary size, suggests an extension of (5) into

$$\Delta E\Delta t = nh \tag{7}$$

where n is an integer number. This leads to a substitution of (6) by a quantum relation of the kind

$$nhc = 2em^{(p)} \tag{8}$$

which is expected to exist in the case of the law presented in (1). The integer factor n entering (8) is considered to be an unknown number corresponding to a factually unknown size of the magnetic pole entering the right–hand side of (8).

One of the aims of the present paper is to confirm (8) also by examining the magneto–electric relations for the case of the electron motion along an orbit in the hydrogen atom.

1. Electron circulation along an orbit in the hydrogen atom gives the quanta of energy as well as those of the magnetic field.

According to the Bohr approach, the electron orbits in the hydrogen atom are the circles whose radii satisfy the formula[4]

$$r_n = \frac{\hbar^2 n^2}{me^2}. \tag{9}$$

The electron charge e moving along the circle of length $2\pi r_n$ induces the magnetic field of strength B_n. This field corresponds to the frequency of electron circulation equal to

$$\frac{2\pi}{\tau_n} = \frac{eB_n}{mc} \tag{10}$$

where τ_n is the size of the circulation time period.

It is easy to check that (9) and (10) give a correct electron velocity on the orbit.[4] For

$$\frac{2\pi r_n}{\tau_n} = \frac{eB_n}{mc}\frac{n^2\hbar^2}{me^2} = \frac{n^2\hbar^2}{m^2 ec}B_n = \frac{e^2}{n\hbar} \tag{11}$$

is obtained on condition

$$B_n = \frac{m^2 e^3 c}{n^3\hbar^3} \tag{12}$$

In the next step we show that B_n give a correct spectrum of the electron energy in the hydrogen. This is so because the orbital magnetic moment is[5]

$$M_n^{orb} = \frac{e}{2mc}n\hbar = \frac{enh}{4\pi mc} \tag{13}$$

When (13) is interacting with the field B_n in (12) we obtain the correct spectrum of levels of the electron energy in the hydrogen atom:

$$E_n = -M_n^{orb}B_n = -\frac{enh}{4\pi mc}\frac{m^2e^3c}{n^3\hbar^3} = \frac{me^4}{2n^2\hbar^2} \tag{14}$$

The magnetic moments $m^{(p)}$ entering the Oersted–Amper law occur to be equal to the quanta of the magnetic flux in the hydrogen atom.

First we show that the quanta of the magnetic flux Φ_n in the atom calculated for the electron orbits n approach the magnetic moments $m^{(p)}$ in (8). For from (9) and (12) we obtain:

$$\Phi_n = \pi r_n^2 B_n = \pi\left(\frac{\hbar^2 n^3}{me^2}\right)^2 \frac{m^2e^3c}{n^3\hbar^3} = \frac{\pi\hbar nc}{e} = \frac{h}{2}\frac{nc}{e} \tag{15}$$

so

$$\Phi_n = m^{(p)} \tag{16}$$

where the term on the right–hand side of (16) is equal to the magnetic pole introduced in (1); see (8). It can be noted that the ratio of (13) and (15), viz.

$$r_e = \frac{M_n^{orb}}{\Phi_n} = \frac{M_n^{orb}}{m^{(p)}} = \frac{enh}{4\pi mc}\frac{2e}{hnc} = \frac{1}{2\pi}\frac{e^2}{mc^2}, \tag{17}$$

is a constant independent of the quantum number n.

2. Discussion on the result obtained in (17)

The result calculated in (17) has a dimension of a geometrical distance, or length; it is usually identified as being close to the radius of the electron particle. With the factor of $1/6\pi$ instead of $1/2\pi$ it seems that result of (17) has been obtained for the first time by Weyl[6] it is defined sometimes as the radius of the Lorentz electron.[7] In numerous cases[8,9] the expression for the radius of the electron microparticle is simplified into

$$\frac{e^2}{mc^2} \tag{18}$$

With the requirement that an agreement of r_e with the Oersted law should be attained,[10] the electron radius becomes

$$\frac{e^2}{\pi mc^2} \tag{18a}$$

A direct check of the quanta $m^{(p)}$ of the magnetic field obtained in (8) is given below.

This check can be done by taking the well–known formula[11,12]

$$\oint \vec{B}_n d\vec{l}_n = \frac{4\pi}{c} i_n \tag{19}$$

where $d\vec{l}$ is the path element circumventing the electron microparticle, i_n is the electric current along the orbit n. For the left–hand side of (19) we obtain

$$2\pi B_n r_e = 2\pi\frac{m^2e^3c}{n^3\hbar^3}\frac{e^2}{\pi mc^2} = 2\frac{me^5}{cn^3\hbar^3} \tag{20}$$

The right–hand side of (19) is

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$$\frac{4\pi}{c} i_n = \frac{4\pi}{c} \frac{e}{\tau_n} = \frac{4\pi e}{c} \frac{me^4}{2\pi\hbar^3 n^3} = \frac{2me^5}{c\hbar^3 n^3} \quad (20a)$$

3. Proposal of a new formulation of the Oersted–Ampere law.

It looks from (8) that it is more convenient to replace the original Oersted–Ampere law (1) by the formula

$$F = \frac{2i}{cr} \cdot \frac{nhc}{2e} = \frac{i}{r} \frac{nh}{e} \quad (21)$$

or

$$\frac{rF}{i} = \frac{nh}{e} \quad (21a)$$

The equation (21a), when multiplied by $c / 2$, gives in fact on its right–hand side the magnetic flux in (15) expressed by a multiple of the elementary flux, viz.

$$n \frac{hc}{2e} = n \times 2.07 \times 10^{-7}\, gauss\, cm^2 \quad (22)$$

The units on the right–hand side of (22) correspond with the units of the pole

$$m^{(p)} \sim \frac{g^{\frac{1}{2}} cm^{\frac{3}{2}}}{sec} \quad (22a)$$

The formula (22) becomes very similar to that applied in the theory of superconductors.[12–14]

4. Uncertainty principle for energy and time applied in the case of the Ampere–Oersted law.

An insight into the size of the monopole entering the Oersted–Ampere law can be obtained also with the aid of the uncertainty principle for energy and time. The principle is represented by the relation [15]

$$2mc^2 \Delta E \left(\Delta t\right)^2 > \hbar^2 \quad (23)$$

where m is the electron mass and the intervals ΔE and Δt are the considered intervals of energy and time.

Because of (6) the formula (23) can be transformed into

$$\Delta E \Delta t = \frac{2em^{(p)}}{c} > \frac{\hbar^2}{2mc^2 \Delta t} \quad (24)$$

The upper limit of $m^{(p)}$ in (24) is accessible by substituting a minimal acceptable time interval for the electron transition:[3]

$$\left(\Delta t\right)_{min} = \frac{\hbar}{mc^2}. \quad (25)$$

This gives on the basis of (24) the relation

$$\frac{2em^{(p)}}{c} > \frac{\hbar^2}{2mc^2} \frac{mc^2}{\hbar} = \frac{\hbar}{2} = \frac{h}{4\pi} \quad (26)$$

from which we obtain

$$m^{(p)} > \frac{1}{4\pi} \frac{ch}{2e} \quad (27)$$

The limit presented in (27) is smaller only by a factor of $1/4\pi$ than the result obtained for $m^{(p)}$ in (8) on condition the case of n=1 is considered.

5. Extremal values of the physical parameters connected with an elementary electron transition process

By an elementary transition we understand the transition of an electron between two neighbouring quantum energy levels. The study can be accomplished by applying the formula (25) for Δt. First we estimate a maximal value of the electric current intensity i_{max} which can be associated by a single electron transition:

$$i_{max} = \frac{e}{\Delta t_{min}} = \frac{emc^2}{\hbar} \quad (28)$$

This implies

$$i_{max} = \frac{4.8 \times 10^{-10} \times 9.1 \times 10^{-28} \times \left(3 \times 10^{10}\right)^2}{1.6 \times 10^{-27}} CGS\, units \cong 82 amper \quad (28a)$$

by taking into account that $1\,A = 3 \times 10^9\ CGS\, units$

Since the resistance associated with the electron transition between two neighbouring quantum levels in an energy emission process is approximately a constant number [2]

$$R = \frac{h}{e^2}, \quad (29)$$

we can estimate a maximal potential difference between two neighbouring quantum states as equal to

$$V_{max} = Ri_{max} = \frac{h}{e^2} \frac{emc^2}{\hbar} = \frac{2\pi mc^2}{e}. \quad (30)$$

In effect the corresponding energy difference between two neighbouring electron states becomes limited by a maximal value

$$\Delta E_{max} = eV_{max} = 2\pi mc^2 \quad (31)$$

It is easy to calculate the corresponding minimal capacitance associated with the elementary electron transition. With q equal to the electron charge e this is [1]

$$C = C_{min} = \left(\frac{q}{V}\right)_{min} = \frac{q_{min}}{V_{max}} = \frac{e.e}{2\pi mc^2} = \frac{e^2}{2\pi mc^2} = \frac{1}{2}r_e \qquad (32)$$

where r_e is the radius of the electron microparticle given in (18a).

Summary

The paper is analyzing the size of the magnetic monopole entering the Oersted–Ampere law of electrodynamics. With an application of the quantum character of the Joule–Lenz law obtained for the product of the intervals of energy (ΔE) and time (Δt), it is found that the magnetic monopole $m^{(p)}$ becomes equal to a multiple of the magnetic flux characteristic for the first electron orbit in the hydrogen atom. Precisely the same kind of the magnetic flux is observed since a long time in superconductors.

The result for $m^{(p)}$ can be approached also by applying the uncertainty principle to the intervals of energy and time specific for the Oersted–Ampere law.

A very special check is a comparison of the result obtained in (8) with that given by the Dirac ratio between $m^{(p)}$ and e (see [16-20]):

$$\frac{m^{(p)}}{e} = \frac{1}{2}\frac{\hbar c}{e^2}n \cong \frac{1}{2}137n \qquad (33)$$

This gives in place of (8) the formula

$$n\hbar c = 2em^{(p)} \qquad (34)$$

Therefore, according to Dirac, the left-hand side of (8) should be smaller by the factor of 2π.

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Conflict of interest

Author declares there is no conflict of interest.

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