Research Article

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Three dimensional mathematical modeling of plunge pool scour

Abstract

Existing empirical plunge pool scour predictive methods vary significantly in their scour depth predictions when applied to field cases. Physically–based methods are complex and require detailed data for their calibration. This fact motivated the development of a novel three dimensional mathematical modeling approach that is simple and yet efficient. The mathematical modeling is based on an energy transfer theory that links the energy available in the attacking flow jet issuing from a dam outlet to the work consumed in lifting the bed material out of the plunge pool scour hole. This yields an equation expressing the dependence of the plunge pool scour depth on flow jet parameters such as: jet velocity, discharge, angle of impingement and falling height; pool parameter such as the downstream pool water depth; and bed material parameters such as: side slope, porosity and specific gravity. The developed approach has an added value of predicting the volume of the scoured material. An equation is developed to predict the maximum particle size of the disintegrated rock beds due to scour by falling jets. The developed approach predictions showed excellent agreement with theoretical, experimental, and field data; in particular at twelve well–known dams all over the world.

Keywords: plunge pool scour holes, free–falling jets, high head dams, three–dimensional mathematical modeling, scour depth, volume of scour, energy–transfer theory

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Introduction

Plunge pools are very economical means for discharging high energy flows downstream of high head dams. However, the high energy discharge usually falls down from a considerable height and with high velocity, penetrates the plunge pool with a significant velocity and often scours the bed material even when it is a hard rock. The resulting scour could be severe to the point of endangering the safety of the dam structure, and the downstream valley or river channel. Therefore, in order to safeguard against scour it is extremely of paramount importance to predict with reliable and sufficient accuracy the long term or equilibrium scour depth downstream of dams at plunge pools. Like many other scour phenomena, existing plunge pool scour predictive equations are often two–dimensional and vary significantly in their scour depth predictions when applied to field cases by orders of magnitude. Being two–dimensional, they only predict the depth of scour while not being able to determine the volume of scoured material which could be very important to know a priori before dam construction. In addition, recent physically–based geomechanical methods such as Anandale[1] & Bollaert[2] are complex and require detailed data for their calibration. This fact motivated the development made herein in which a novel three dimensional mathematical modeling approach is applied to predict accurately and in a simple manner the scour depth and volume of scoured material at a plunge pool. The model is simple but yet efficient. It is also physically based through relating the energy transfer from the attacking flow jet (erosive capacity) to the plunge pool resistance capacity.

Previous work

Prediction methods of plunge pool scour are basically divided mainly into five methods which are:

1. Empirical formulas,
2. Physical model tests,
3. Theoretically–based formulas,
4. Geomechanical methods,
5. Computational fluid dynamics simulations.

The empirical formulas are obtained based on dimensional analysis and laboratory experiments and most of them have the general form, Gioia et al:[3]

$$D_s + h_2 = K\, q^{e_q}\, h^{e_h}\, g^{e_g}\, d^{e_d} \left(\frac{\rho}{\rho_s - \rho} \right)^{e_\rho} \qquad (1)$$

where D_s is the equilibrium scour depth at the plunge pool scour hole measured from the original bed level to the deepest point of the scour hole , h_2 is the downstream water depth, K is a coefficient, q is the flow discharge per unit width, h is the total head on the dam usually taken as the difference between the upstream and downstream water levels, g is the gravitational acceleration, d is a bed material characteristic diameter, ρ is the fluid density (usually water), ρ_s is the density of the bed material grains and e_q, e_h, e_g, e_d and e_ρ are exponents whose values are often determined experimentally, see for example Table 1.

The inclusion of the unit discharge q in Eq. (1) assumes that the scour phenomenon is treated in a two–dimensional fashion neglecting three–dimensionalities especially estimation of the volume of the scoured bed material. The inclusion of the downstream water depth on the left side indicates an inverse relation with the scour depth which is not always true. For large values of the downstream water depth, negative or very low scour depths may result which is unrealistic. In addition, most equations in this group lack dimensional homogeneity. The difference in the exponents' values reflects difference in the range of experimental conditions which may limit their use.

Table 1 Comparison of the exponents in scour depth prediction equations due to falling jets

Researcher(s) and Year	eq	eh	eg	ed	eρ
Veronese[25]	0.54	0.225	0	–0.42	0
Kotoulas[40]	0.7	0.35	–0.35	–0.4	0
Bormann et al.[39]	0.6	0.5	–0.30	–0.4	0.8
Gioia et al.[3]*	0.67	0.67	–0.33	–0.67	1
Bombardelli et al.[23]+	0.4	0.4	–0.20	–0.4	0.6
The Present Study (High Heads)	0.5	0.25	–0.25	–0.5 /0.5	0.25

*theoretical–cylindrical

+ theoretical–axisymmetric

In physical model tests, a scale model is built often using Froude number similarity. Häusler[4] & Water Power[5] reported that initial model tests for high head structures predicted a scour depth of 30 m below the original rock surface for the high outlet spillway of the Kariba Dam in Zimbabwe. By 1979 the scour depth was 85 m below the original rock surface and Häusler[4] predicted this would reach 100 m. For four dams in British Colombia Canada, Monfette[6] stated that in only one of the four study cases the results of downstream scour from small–scale model tests were comparable to prototype observations. Whittaker[7] stated that the model bed material type and size must be chosen carefully to allow scaling and reported details about the difficulties involved with grain size effects. Hafez[8] discussed in detail the scale effects inherited in flume experiments in which Froude scale modeling is used.

An example of theory–based approach is the one proposed by Hoffmans.[9] He applied Newton's law of motion to a mass of fluid particles and presented an equation based on the solution of eight equations with eight unknowns. Hoffmans[9] equation is given as:

$$D_s + h_2 = C_{2v}\sqrt{\frac{qV\sin\beta}{g}} \qquad (2)$$

Where V is the mean velocity of the jet, β is angle of inclination of the jet relative to the horizontal downstream water surface, and C_{2v} is usually taken as 2.9 but it can be in terms of several soil bed and jet characteristics. Formulas that have $D_s + h_2$ on the left hand side would indicate, in case of zero discharge, a negative scour depth value which is physically impossible.

Among the geomechanical methods are the Erodibility Index (EI) method by Annandale[1] and the Comprehensive Scour Model (CSM) by Bollaert.[2] Bollaert[2] stated that despite the experimental validation of EI method, no direct dynamic parameters were incorporated and no physical background of rock break–up was evident in the model. Further, Monfette[6] applied the Erodibility Index (EI) method to four hydro dam sites in BC, Canada, and found its performance disappointing and did not outclass the conventional methods. It was found that the EI method was characterized by a tendency for underestimation and a standard error of estimate of 16.15 m. Bollaert[2] stated that for cases where previous scour–hole development has been observed and described, the CSM model will be useful to predict future scouring. The CSM requires calibration, for example as in Bollaert[10] & Lesleighter et al.,[11] which limit its use as a predictive method. Examples of Computational fluid dynamics simulations are those by Neyshabouri et al.[12], Epely Chauvin et al.[13] & Castillo.[14] The modeling process is often done in three steps: hydrodynamic modeling of the jet, sediment transport modeling and bed erosion modeling. These models, though they are very useful in predicting the hydrodynamic aspects of the falling jet and the transient analysis of scour–hole development, require in addition to calibration too much detailed data sets and are too complex to be used by the design engineers. Based on all of the above, a three–dimensional mathematical approach was developed herein that was sought to be physically based, three–dimensional, simple, accurate, and general enough to cover wide range of scour cases at plunge pools.

The present approach

The work/energy transfer theory, which was used successfully by Hafez[8] for predicting the equilibrium scour depth downstream of low and high head hydraulic structures due to horizontal turbulent wall jets, is reformulated herein for predicting plunge pool scour. The same theoretical principles were also used successfully for predicting bridge pier scour, Hafez.[15] The principles of work/energy transfer theory, for local scour phenomena, state that the energy in the attacking flow jet is transferred to the work done in disintegrating and removing the volume of the scoured bed material out of the scour hole. The scope here does not include the time development of the profile of the scour hole. The mechanics of energy exchange between the flow jet and the bed particles are complex and beyond the scope of this work.

The basic assumptions, their explanations and validations are as follows:

1. Erosion or scour of the bed surface downstream of the dam has taken place and equilibrium conditions have been reached. This means that initially the surface bed material is ready for erosion due to the energy available in the attacking jet. For pre–excavated plunge pools where most of the energy is dissipated, scour is not possible. Anandale's[1] Erodibility Index method can be used to check the threshold of erosion.
2. During of the early stages of the scour process of rock beds, the jet flow energy is consumed in disintegration, fracturing and dislodging of the rock bed. After that, the jet flow energy acts in moving these rock pieces from the scour hole. This is validated from Gerodetti[16] who stated that the actual development of a scour hole depends on two related steps: disintegration and/or entrainment of base material and evacuation of the material from the scour hole.
3. The jet flow is falling from a height, H. Its vertical component is penetrating the pool depth while keeping most of its momentum throughout the pool depth till it reaches the river bed or ground level and starts to scour the bed surface down to a depth of D_s. This is true if the pool water depth is relatively small compared to the falling jet height or kinetic energy (total head>h_2).

4. The vertical component of the jet is responsible for the plunge pool scour provided it can penetrate deep enough inside the plunge pool and hits hard the bed surface while the horizontal component of the jet is responsible for transporting the scoured material out of the scour hole to the downstream channel. This is validated by Whittaker[7] who indicated that the ability of the flow to carry entrained material beyond the mound depends on the horizontal velocity components of the flow within the scour hole and so the angle of impingement of the jet is important.
5. When the attacking falling flow jet hits the original pool bed, it will erode the surface layer. Often the energy would be sufficient to erode and transport the next surface layer away. This erosion and transport process will continue as long as there is sufficient energy to transport the eroded material outside the formed scour hole. When the energy in the attacking flow jet is incapable to transport the deepest layer in the scour hole, scour ceases and equilibrium conditions are reached. This means that at equilibrium conditions, the energy in the attacking flow jet is equal to the work done to lift the scoured material outside of the scour hole.
6. The scour hole shape is assumed to be as an inverted cone with its height equal to the scour depth, D_s. Originally it is hypothesized that the cone base should have an elliptical shape. However, as an approximation, a simplification is made herein considering the cone has a circular base, Figure 1. The cone circular base radius is equal to $D_s/\tan\phi$ where ϕ is the average side slope angle of the scour hole (ϕ can be assumed as the angle of repose of the bed material). Mason[17] observed that the impressive plunge pool scour hole of Kariba dam on Zambezie River had volume grossly compared to a cone of 60°C to the horizontal direction which was also evident in Noret et al.[18] The conical shape of the scour hole is also evident at the four dams in B.C. Canada, Monfette.[6]
7. The scour depth increases with increase in the downstream pool water depth (tail water depth), h_2. This is true under certain conditions such as that the pool water depth is relatively small compared to the falling jet height or its kinetic energy. When the pool water depth is significantly large compared with the jet falling head or its kinetic energy, the pool water would have a cushioning effect. The cushioning effect would make the scour depth decrease with increase in the pool water depth. Lencastre[19] & Martins[20] stated that scour increases with increasing tail water depth to a critical value, and then decreases as tail water increases beyond this value.
8. The jet velocity issuing from the dam outlet or from the flip bucket is constant along the jet trajectory path. The jet energy losses, because their determination is difficult for example due to friction and aeration, are neglected and their inclusion would result in lesser scour depth. Most previous methods neglected energy losses. Neglecting of energy dissipation will compensate for other opposite factors which are not accounted for such as energy coming from instantaneous turbulent velocity fluctuations discussed in Hafez.[15]

The flow jet force, F_j, is expressed as from basic fluid mechanics, Roberson,[21] when applied to a mixture jet as $F_j = \rho_m Q_m V_m$ where ρ_m is the jet–mixture density, Q_m is the jet–mixture discharge and V_m is the jet–mixture impingement velocity. The quantities ρ_m, Q_m and V_m could be for the water–air–sediment mixture as the jet may contain water, air and suspended sediment in it, i.e.

$$F_j = \rho_a Q_a V_a + \rho_w Q_w V_w + \rho_s Q_s V_s$$

Where subscripts a, w and s refer to air, water and sediment, respectively. For simplicity the subscripts will be dropped from here on, i.e., $F_j = \rho\, Q\, V$, with the understanding that these quantities could represent either complete mixture, or both of water and air only, or water only. The vertical component of this jet is travelling a total vertical distance downward equal to $(H + h_2 + D_s)$ where H is the jet falling head, h_2 is the plunge pool depth and D_s is the maximum scour hole depth, see Figure 1 & Figure 2. The work done by gravity, W_g, on this vertical jet is equal to the vertical force times the vertical travelled distance, i.e.

$$W_g = \rho Q\left(V_1 \sin\beta_1 H \pm V_2 \sin\beta_2 h_2 + V_3 \sin\beta_3 D_s\right) \tag{3}$$

where V_1 is the velocity of the jet in the air before entering the plunge pool, V_2 is the velocity of the jet inside the plunge pool and V_3 is the velocity of the jet inside the scour hole (all velocities are in an average sense), β_1, β_2 and β_3 are the inclination angles of the jet inside the air, the plunge pool and the scour hole, respectively.

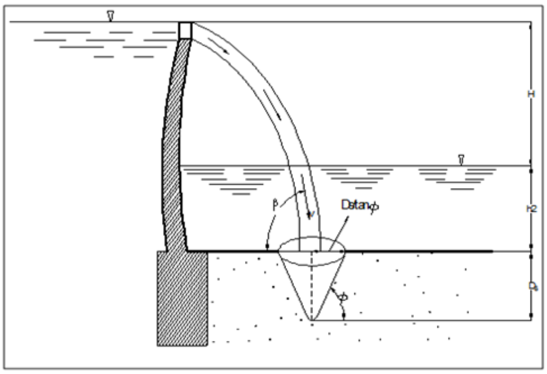

Figure 1 Schematic representation of plunge-pool scour at high-outlet jets.

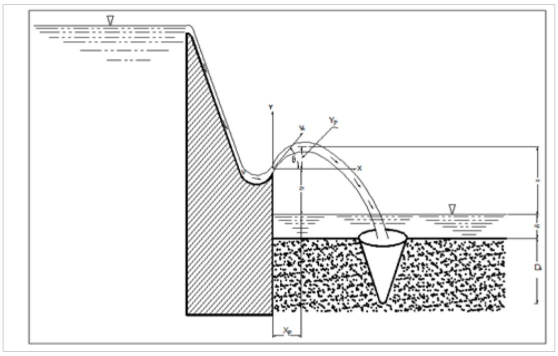

Figure 2 Schematic representation of plunge-pool scour at ski-jump jets.

A plus sign (+) in front of h_2 is used for the case of increase of the scour depth with the increase in tail water depth while a negative sign (–) is used for the case of decrease of the scour depth with increase in tail water depth. It indicates the resistance by the pool water mattress to the falling jet, i.e., the cushioning effect. This condition applies if h_2 > total head (h).

For simplicity and for the lack of actual data, it will be assumed that all the three velocity components are equal to the velocity of the impingement jet and that the β' s are equal to the jet inclination to the water surface. Therefore, one symbol for V and β will be used in Eq. (3), i.e.:

$$W_g = \rho Q V \sin\beta\left(H \pm h_2 + D_s\right) \quad (4)$$

The volume of the scoured bed material in the scour hole is equal to the volume of a cone. The submerged weight of the volume of the scoured material in the conical scour hole is

$$(\gamma_s - \gamma)(1-p)\frac{1}{3}\pi\left(\frac{D_s}{\tan\varphi}\right)^2 D_s$$

where γ_s is the specific weight of the bed material, γ is the specific weight of the pool water, p is the porosity of the bed material and ϕ is the slope angle of the conic scour hole. This weight force of the scoured material is assumed to act at the center of gravity of the scour hole which is $D_s/3$ below the original river bed for cone–shaped volumes. The work done in moving this material out of the scour hole, W_o, is equal to the submerged weight force times $D_s/3$ and is given as:

$$W_o = \left(\gamma_s - \gamma\right)\left(1-p\right)\frac{1}{3}\pi\left(\frac{D_s}{\tan\varphi}\right)D_s\frac{D_s}{3}\rho g\left(S_G - 1\right)\left(1-p\right)\frac{1}{9}\pi\frac{D_s{}^4}{\left(\tan\varphi\right)} \quad (5)$$

Where S_G is the specific gravity of the bed material.

The work/energy transfer theory, Hafez[8], states that the work done by gravity on the jet flow is converted, or transferred to a work done in moving the bed material out of the scour hole (Basic mechanics: the change in potential energy is equal to the work done). This results in equality of equations (4) and (5), i.e., $\lambda W_g = W_o$ where λ is an energy transfer efficiency coefficient accounting for the various energy loses. λ may be taken as unity for simplification. Equalizing and solving for D_s yield:

$$D_s{}^4 = \frac{9\lambda\left(\tan\varphi\right)^2}{\pi g\left(S_G - 1\right)\left(1-p\right)} Q V \sin\beta\left(H \pm h_2 + D_s\right) \quad (6)$$

When the falling jet penetrates deeply into the plunge pool at high h/h_2 (i.e. $h/h_2 > 1$) the plus sign in front of the h_2 term is used in Eq. (6) which becomes:

$$D_s{}^4 = \frac{9\lambda\left(\tan\varphi\right)^2}{\pi g\left(S_G - 1\right)\left(1-p\right)} Q V \sin\beta\left(H + h_2 + D_s\right) \quad (7a)$$

For the case of relatively low h/h_2 (i.e. $h/h_2 < 1$) Eq. (6) becomes:

$$D_s{}^4 = \frac{9\lambda\left(\tan\varphi\right)^2}{\pi g\left(S_G - 1\right)\left(1-p\right)} Q V \sin\beta\left(H - h_2 + D_s\right) \quad (7b)$$

It should be noted that Eq. (7a) or (7b) is dimensionally correct (homogeneous) and therefore they can be used under any system of units (SI or English unit systems). They can be solved by trial and error approach in which D_s on the right hand side of the equation is set to zero in the first trial and then iteration continues till the difference of D_s in the two sides of the equation is less than a specified accuracy.

Using $(S_G - 1) = (\rho_s - \rho)/\rho$, $Q = q B$ where q is the unit width discharge, B is the lateral flow width, and $V = q/b_o$ where b_o is the jet flow width in Eq. (6) yields:

$$D_s{}^4 = \frac{9\lambda\left(\tan\varphi\right)^2}{\pi g\left(1-p\right)}\frac{\rho}{\rho_s - \rho} q^2 \frac{B}{b_o}\sin\beta\left(H + h_2 + D_s\right) \quad (8)$$

When the jet falling head, H, is relatively large compared to the sum of the pool depth, h_2, and the scour depth D_s, h_2+D_s can be neglected on the right hand side of Eq. (8). This situation is when the rock bed is very resistive to erosion resulting in small D_s. In that case, D_s on the left hand side varies to the fourth power. Equation (8) then indicates that the plunge pool scour hole depth in this case $D_s \propto (\tan\phi)^{1/2} q^{1/2} H^{1/4} g^{(-1/4)} (\rho/\rho_s - \rho)^{1/4}$. If ϕ is assumed equal to the angle of repose of the bed material, then according to Simon,[22] $\tan\phi \propto (1/d)$ for bed material grains<2.4 mm and $\tan\phi \propto d$ for bed material grains≥2.4 mm where d is the representative or mean bed sediment diameter. This means that $D_s \propto d^{(-1/2)}$ for $d < 2.4$ mm and $D_s \propto d^{(1/2)}$ for $d > 2.4$ mm.

Table 1 shows comparison of the exponents found in the literature with those according to Eq. (8) for the case at high heads ($H>>(h_2+D_s)$). It is clear from Table 1 that, the present study derived exponents are within the range of the previous experimental and theoretical investigations which supports the validity of Eq. (8) and its source equation; Eq. (6). The theoretically derived equations by Gioia[3] & Bombardelli23 have the discharge and head exponents as equal and their negative values are equal to the sediment size exponent. Such trend is not supported with the other experimental findings. It should be noted that for rock scouring to large pieces or blocks, their sizes or characteristic lengths cannot be considered as influencing the scour mechanism as is done by most researchers, because they are the result of the scour process not the cause of the scour. Akhmedov[24] comments that fractured rocks disintegrate within the scour hole due to flow action. The existence of sediment size exponent is not necessary in the present approach as ϕ can be interpreted merely as the side slope angle. It was done here for comparison purposes.

Application to field data

The developed scour equation is tested herein via calculating the plunge pool scour depth at twelve well–known dams all over the world which are: Kariba Dam, Cabora–Bassa Dam, and Gibe III Dam in Africa, four dams in Canada, British Colombia (Peace Canyon Dam, Miles Dam, Portage Mountain Project, Revelstoke Dam), Kondopoga Dam in USSR, Chucas Dam in Costa Rica, Tucurui Dam in Brazil, Picote Dam in Portogal and Wivehoe dam in Australia. These dams were mainly chosen because of the existence of complete data set. Table 2 shows the input scour data for these dams while Table 3 shows the results of applying the formulas of Veronese,[25] Martins[26] & Hoffmans[9] in addition to the present study equations, Eq. (7a) and Eq. (7b) to the aforementioned dams. Veronese[25] and Martins[26] equations represent the empirical approach and Hoffmans[9] equation represents the theoretical approach. Veronese[25] equation is still recommended by USBR. The volume of the scour hole is unknown in addition to the scour hole depth. For a conic volume, two parameters determine this volume; namely the cone circular–base–radius and height. Alternatively the side slope angle of the cone and cone height determine the same volume. For the Kariba dam case where the jet impingement angle was 60°C the measured side slope angle was also 60°C. It might be the only side slope angle to be explicitly reported in the literature and therefore this values was used herein for scour depth prediction reducing the problem to predicting only the scour hole depth. Guided by the close proximity to the angle of repose of most river bed materials, a value of the side slope angle of 30°C is assumed for the other cases with less jet impingement angle than that at the Kariba dam. There are exceptional cases where a pre–excavated plunge pool existed at which the side slope angle is known already. Hoffmans[9] assumed the upstream scour angle in the equilibrium phase as close to the angle of repose and that the downstream scour angle is equal to the upstream one. Hoffmans[9] analysis was two–dimensional, i.e., the scour hole has an isosceles triangular vertical section. The three–dimensional extension of this triangle would be a cone. The side slope angle depends on all the variables affecting scour such as the jet falling angle, falling height, velocity, jet diameter; and bed resistance to erosion characteristics. This makes determining the side slope angle just as challenging as determining the scour depth itself. However, the correlation between the side slope angle at one hand and the jet falling angle and angle of repose on the other hand should be the strongest among all the variables. Therefore, to close the plunge pool scour problem, one of the two unknown variables has to be prescribed. Indeed, assuming the side slope angle is much more practical and was assumed to be in the proximity of the jet falling angle and angle of repose. A value of 30 °C for the side slope angle at naturally formed plunge pools was found, as will be seen later, to be sufficient for scour depth predictions. Six, five and four iterations were found sufficient when calculating the scour depths by Eq. (7a) or (7b) for an accuracy of 0.001 m, 0.01m, and 0.1 m, respectively.

Table 2 Input data for plunge pool scour calculations at various dams

Dam	Discharge Q (m^3/s)	Jet head, H (m)	Total Head, h (m)	Impact Velocity (m/s)	Angle β (degrees)	Tailwater depth, h_2 (m)	Unit Discharge, q m^2/s	Angle ϕ (degrees)	Observed scour depth (m)
Kariba	8400	82.52	82.52	40.84	60	40.5	155.6	60	70.0–75.0
Cabora Bassa	13100	19.2	100.8	40.3	42	40	275	30	35
PCN803456	2577	5.73	36.58	26.79	24	4.27	36.23	30	14.33
SEV9712	2039	24.55	58.52	33.89	36	9.14	57.6	30	18.29
PMD96	3455	112.01	161.54	56.3	51	7.92	83.61	30	35.97
REV86	1416	49.07	127.71	50.07	40	5.18	30.66	30	21.34
Gibe_III	6500	134.5	204	42	54	23	77.38	20$	31.8
Kondogopa	70	11.4	11.4	14.95	30	1.45	NA	30	4.8
Chucas	5400	33.89	33.89	25.79	45	16.51	82.32	26.6$	20
Tucurui	50000	40	40	28.28	45	10	130.43	30	48
Picote	7000	45	45	29.71	30	36	50.4	30	19
Wivenhoe*	3750	10	30	25	30	12	62.5	17$	13
Wivenhoe**	7500	10	30	25	30	12	125	17$	15

*Physical model with Q =3750 m^3/s.

**Physical model with Q = 7500 m^3/s.

$ Angle of pre–excavated plunge pool.

Table 3 Predicted versus observed plunge pool scour depths at various dams

Dam Name	Veronese 1937 (m)	Martins 1975 (m)	Hoffmans 2009 (m)	Present approach Eq. (7a) (m)	Present approach Eq. (7 b)	Observed scour depth
Kariba	37.78	7.69	28.19	74.73	63.94	70 – 75
Cbassa–pr	71.37	29.19	39.73	37.7	32.17	35
PCN803456	25.41	14.26	14.13	14.14	12.28	14.33
SEV9712	33.22	16.51	22.21	18.83	16.65	18.29
PMD96	57.18	27.58	48.06	34.24	33.26	35.97
REV86	30.74	13.81	23.9	21.22	20.39	21.34
Gibe_III	42.81	11.69	24.61	31.5	29.26	31.8
Kondogopa	NA	NA	NA	4.83	4.6	4.8
Chucas	28.93	13.58	19.36	23.87	20.12	20

Dam Name	Veronese 1937 (m)	Martins 1975 (m)	Hoffmans 2009 (m)	Present approach Eq. (7a) (m)	Present approach Eq. (7 b)	Observed scour depth
Tucurui	50.47	30.32	37.29	49.18	46.02	48
Picote	1.16	–12.94	–10.67	28.64	20.65	19
Wivenhoe*	26.1	13.2	13.88	12.82	11.34	13
Wivenhoe**	43.39	26.19	24.6	15.53	13.87	15

*Physical model with Q =3750 m^3/s.

** Physical model with Q = 7500 m^3/s.

Verification case: Kariba dam, Zimbabwe

Whittaker[7] provided valuable data for scour downstream of Kariba Dam in Zimbabwe. The Kariba Dam has a high level outlet spillway. The relevant parameters were: the discharge was 8400m^3/s, the unit width–discharge was 155.6m^2/s, the total head was 82.52m, the jet issuing velocity was 40.84m/s and the downstream water depth was 40.5m. The jet impingement angle is 60 °C based on Noret et al.[18] and Hoffmans[9]. The porosity is assumed as zero for the rocky bed and the specific gravity as 2.65. Noret et al.[18] showed the scour hole is bounded between level 381m and level 306m (between the years 1981 and 2001) with a resulting scour depth of 75m. However, recently Duarte[27] reported depth of scour of 70.0m below the original river bed.

The case of Kariba dam is chosen as a verification case because of:

a. The actual scour hole had a conic shape which is in line with assumption 6 of the present approach;

b. It is one of the rare occasions where the side slope angle of the scour hole was reported and it its value was given as 60 °C; and

c. This case is very unique as it had the largest observed scour hole with depth ranging from 70.0m to 75.0m below the original riverbed.

Noret et al.,[18] show a jet diffusion angle inside the scour hole of 69 °C which indicates a case of high angle of the flow jet. Therefore, it is assumed that the slope angle of the scour–hole is 60 °C. For verification at Kariba Dam of the present approach equations (4) and (5), we consider that the scour depth was 75m and used the relevant data in Table 2, to calculate the work done by gravity, W_g, in Eq. (4) (with the plus sign for h_2) which becomes 5.88×10^{10}N.m. The work done to lift the scoured material out of the scour hole, W_o, as given by Eq. (5) becomes 5.96×10^{10}N.m. The ratio between these two values is 0.99 which is close to unity. If a value of the side angle slightly more than 60°C was chosen, say 61°C, W_o would be 5.49×10^{10}N.m. which satisfies conservation of energy as the energy input to a system must be much more than the output work if no generation of energy occurs. Regardless of that, the close proximity between the two values indicates a complete balance between the energy added and the work done, i.e. that the energy supplied by the attacking flow jet is almost equal to the work done in lifting the scoured material out of the scour hole. This statement is the heart of the energy transfer theory developed herein. To test the predicting capability of the present approach, the measured value of 60°C will be assumed for the conic scour hole side angle and focus is given only to predicting the scour–hole depth. This leaves out any uncertainty in the scour–hole shape and side angle. It limits the prediction to predicting only the scour–hole depth as seen below. Equations (7a) and (7b) predict a scour depth, D_s, equal to 75.34 m and 64.82m respectively. The ratio of h/h_2 = 82.52/40.5>1; therefore Eq. (7a) is recommended. Thus good agreement is there with the 70.0m or 75.0m reported measured scour depths. Using the same data, all other formulas predicted very low scour depth. Hoffmans[9] used different data set (15 m for h_2, 9400m^3/s for the discharge, and 90m for the head) and used Eq. (2) to predict D_s+h_2 = 80m which results in a better predicted scour depth of 65m. Noret et al.,[18] reported that it was estimated that about 150000m^3 of rock have been removed under level of 350m between 1962 and 1981. The calculated scour volume from the present approach with a cone of depth of 75.34m and side angle of 60 °C is 149274m^3.

Cabora–bassa Dam, Mozambique

Whittaker[7] provided valuable data for scour downstream of Cabora–Bassa Dam on the Zambezi River in Mozambique. Such data is utilized herein for scour prediction. The Cabora–Bassa Dam has a middle–level outlet consisting of eight sluices, with the outlet section of each being 6 by 7.8m^2. The maximum discharge at reservoir level of 326m a.s.l. (above sea level) through these 8 sluices was 13100m^3/s (the corresponding unit–width discharge was 275m^2/s), the total head was 100.8m, and the downstream water level was at 225.10m a.s.l. The lip of the spillway sluices was located at an elevation of 244.3m a.s.l. The riverbed was very irregular and had its elevations varying from 170m to 180m a.s.l. The jet exit velocity was 40.3m/s issuing at an angle of 32.5°C with the horizontal direction. The depth of water downstream of the dam was 40m. The difference between level of the lip of spillway sluices and the downstream water level, which is needed in Eq. (7a or 7b) for H was 19.2m. In model tests performed at scale of 1:75, Ramos,[28] the modeled scour depth for all the eight sluices discharging was D_s+h_2 = 75m. In February 1982, D_s+h_2 was measured in the field to be approximately 68m. With h_2 = 40m, the scour depth below the original river bed was 28m at the field scale and 35m at the model scale. The 1982 measured scour depth of 28m might not be the ultimate or equilibrium scour depth. Therefore, to be in the safe side the measured scour depth from the model tests, which was 35m, was assumed herein to be the ultimate scour depth and was used for comparison with predictions.

The jet impingement angle β, is calculated according to Whittaker,[7] from the following equation which is based on the kinematic theory of free jets as

$$\tan\beta = \frac{1}{\cos\theta}\sqrt{\sin^2\theta + Z_1/Z_0} \tag{9}$$

where θ is the jet exit angle, Z_o is the distance between the upstream reservoir level and the center line of the spillway sluices and Z_1 is the distance between the center line of spillway sluices and the downstream water level. When θ = 32.5°C, Z_o = 78.2m, and

Z_1 = 22.6m are used in Eq. (9), it yields a jet impingement angle of approximately 42°C. As the downstream river bed is rocky, porosity is assumed as zero and the specific gravity as 2.65. The side angle of the scour hole is assumed to be 30°C guided by the physical model and prototype scour–hole profile. Equations (7a) and (7b) predict a scour depth, D_s equal to 39.26 m and 34.4 m, respectively. The ratio of h/ h_2 = 100.8/40 > 1; therefore Eq. (7a) is recommended. The predictions are in close match to the 35 m scour depth from the model tests. Veronese[25] prediction is almost double the measured value, while Martins[26] prediction of 29.2 m is low. Very good prediction was obtained by Hoffmans[9] equation, Eq. (2), which predicted scour depth of 39.73 m; almost equal to that by Eq. (7a). Most scour formulae in the literature are for plunging free falling or ski–jump jets which should not be used as such for middle level pressure outlet jets like the Cabora–Bassa dam case.

Peace Canyon Dam, BC, Canada

Monfette[6] investigated the plunge pool performance at four of British Colombia (Canada) dam sites. The sites of Peace Canyon Dam, Seven Miles Dam, Portage Mountain Project and Revelstoke Dam were described in terms of spillway design, historical spillway outflows, plunge pool geology, and scour–hole development. The imperial or English system of units was used in Monfette;[6] however the standard SI units are used herein. For these dams, Monfette[6] found no successful predictions of the plunge pool scour depth using scale model tests, the empirical equations and Anandale's[1] EI method. The Peace Canyon Project consists of 344.4 m long and 60.96 m high concrete gravity structure built across steep canyon walls. The Peace Canyon spillway is a gated six–bay overflow spillway with flip buckets at the toe. The bucket lip angle (θ) is reported as 20°. Bedrock exists in the riverbed downstream of the spillway. The spillway operation commenced in late of October 1979. The following spills of 1981, 1983, and 1984 were all of a shorter duration (maximum of two weeks in 1983) and of a lower magnitude per day than the 1979/1980 spill. The scour–hole depth downstream of the spillway flip buckets has formed for the most part during the 1979/1980 spillway operation. Maximum scour of about 13.72 m to 15.24 m extended some 54.86 m to 85.3 m downstream of the flip buckets of bays 3 and 4. Downstream of bays 5 and 6, the plunge pool had a maximum scour depth of approximately 9.14 m. These scour depths for the range of the existing operating flows can be considered as equilibrium scour depths because in 1996 the maximum scour depth was 15.24 m downstream of bay 3 and 9.14 m downstream of bays 5 and 6, i.e., no change in scour depth occurred after 16 years. However, there have been changes in the observed volume of the scour hole in 1980 with a volume of 9004 m^3 compared to 1996 with a volume of 16452 m^3 signaling tendency toward equilibrium in both depth and volume of scour. As part of a diving inspection, six large rock blocks "the size of a Volkswagen buses" were identified down the base of the slope and at the deepest part of the plunge pool. The jet impingement angle (β) is calculated as approximately as 24° according to Eq. (9). For ski–jump jets the jet head, H, needed in applying Eq. (7a/b) is different from the total head, h, which is needed for exit velocity calculations. The total head, h, is the difference between the reservoir elevation and the jet outlet elevation while H for ski–jump jets is the difference between the maximum height at which the jet travels in the air and the plunge pool water surface, see Figure 2. For ski–jump jets, H is evaluated as follows. From the kinematic theory of free jets, the profile of the jet as reported in Whittaker et al.[7] is given as:

$$y = x \tan\theta - \frac{gx^2}{2V_o^2 \cos^2\theta} \tag{10}$$

where y and x are the vertical and horizontal coordinates of the jet (the origin is taken at the jet outlet point as in Figure 2)),V_o is the jet exit velocity for ski–jump jets and θ is the jet exit angle. The velocity of the issuing jet, V_o, could be calculated from the well–known equation:

$$V_o = \sqrt{2gh} \tag{11}$$

where h is the total head and approximately h ≈ Z_o herein. The maximum height of the jet, Y_p, is obtained by setting dy/dx = 0 in Eq. (10). Also Y_p occurs at X_p which is then given via setting dy/dx = 0 in Eq. (10) as:

$$X_p = \frac{V_0^2 \cos^2\theta(\tan\theta)}{g} \tag{12}$$

Substituting X_p from Eq. (12) into Eq. (10), resulting in that Y_p is given by:

$$Y_p = X_p\left(\tan\theta - \frac{gX_p}{2V_0^2\cos 2\theta}\right) = \frac{X_p \tan\theta}{2} \tag{13}$$

The total head, h, on Peace canyon Dam in April 1980 was 36.58 m which according to Eq. (11) yields jet exit velocity, V_o, of 26.79 m/s. Upon substitution of V_o, θ and g values in Eq. (12) X_p becomes 23.5 m and hence Y_p from Eq. (13) becomes 4.28 m. The jet head, H, equals then to $Y_p + Z_1$ according to Fig. 2 where Z_1 is the height of the jet outlet above the plunge pool water level. With Z_1 = 1.45 m, H becomes 5.73 m. Peace Canyon dam has two outlet jet discharges in April 1980; one is 1557.4m^3/s from bays 3 and 4 and the second is 1019.4m^3/s from bays 5 and 6. The combined discharge is therefore 2576.8 m^3/s with corresponding downstream water depth of 4.27 m. In calculating the unit width discharge for use in the other foumulas, the average unit discharge value between the unit discharge in bays 3 and 4 (43.66 m^2/s) and that of bays 5 and 6 (28.8 m^2/s) is used. The average unit discharge is then 36.2m^2/s. Note then the advantage of treating the scour problem in three–dimensional fashion by the present approach. It allows the addition of the two aforementioned flows which sounds physically correct while the two–dimensional treatment requires using average unit discharge. The side angle of the scour hole was assumed to be 30° which is slightly higher than the impingement angle of 24° due to jet deflection in the pool water column. The ratio of h/ h_2 = 36.58 m /4.27 m = 8.6 > 1; therefore Eq. (7a) is recommended. With the above data, the calculated plunge pool scour depth from Eq. (7a) was 14.14 m while it was 12.28 m by Eq. (7b). Nearly close values are predicted by Martins[26] Hoffamns[9] which along with the prediction of Eq. (7a) are in excellent agreement with the observed scour depth. Veronese[25] Eq. (2), highly over–predicts the scour depth. The calculated volume of the scour hole is 15030 m^3 according to the scour depth prediction by Eq. (7a) whereas the measured total volume scoured below elevation 451.1 m since the beginning of spillway operation in 1979 had reached 16452.1 m^3. The close match between the calculated and observed scour depth and eroded volume supports the assumption of a conical shaped scour hole by the present approach. Calculating the volume of the scoured material is an added advantage of the present approach which is not offered by other methods.

Seven Mile Dam, BC, Canada

The Seven Mile Project consists of a concrete gravity dam with a crest length of 349.9 m and a maximum height of 80.0 m above the foundation. Gravity blocks are adjacent to both abutments followed by a four–unit power intake section on the right and a five–bay spillway arrangement on the left. The Seven Mile Dam was operated as a run–of–the–river plant (no storage). The Seven Mile spillway is a gated five–bay overflow spillway with flip buckets at the toe. The bucket lip angle (θ) is reported as 30°. The Seven Mile spillway became operational in early November 1979 and has been operated every year since. The spring freshet of 1997 caused the largest spill event at Seven Mile Dam since completion of the dam. Successive surveys of the plunge pool have shown the formation of a scour hole gradually increasing in size, although at a decreasing rate. In 1997 a maximum scour depth from 16.76 m to 18.29 m at bays 1 and 2 was reported under a discharge of 2038.8m^3/s with a corresponding unit discharge of 57.6 m^2/s, total head of 58.52 m and tail water depth of 9.14 m. The ratio of h/ h_2 = 58.52 m /9.14 m = 6.4 > 1; therefore Eq. (7a) is recommended. The outlet exit velocity is calculated as 33.89 m/s according to Eq. (11), X_p as 50.68 m according to Eq. (12) and Y_p as 14.63 m from Eq. (13). With Z_1 equals to 9.92 m, the jet head, H, becomes 24.55 m. The jet impingement angle (β) was calculated as 36° according to Eq. (9) while the side angle of the scour hole was assumed to be 30°. With these values used in Eq. (7a), the predicted scour depth is 18.83 m which is in excellent agreement with the measured scour depth of 18.29 m. Equation (7b) predicted a lower scour depth of 16.65 m. Hoffmans[9] equation, Eq. (2), predicted scour depth of 22.22 m. Over–prediction of the scour depth resulted from Veronese[25] equation and under–prediction from Martins[26] equation. The reported observed volume of scoured material was given for a depth of scour equals 2/3 of the equilibrium scour depth and this volume was 14725 m^3. The two–third volume of the scoured material according to Eq. (7a) is 16455 m^3.

Portage Mountain Project, BC, Canada

The Portage Mountain Project includes the W.A.C. Bennett embankment dam, the Williston Lake, the G.M. Shrum generating Station, and a long spillway chute on the right abutment. The W.A.C. Bennett embankment dam is one of the world's largest earth fill structures with 2042.16m stretch across the head of the Peace River Canyon and a maximum height of 182.88 m. The Portage Mountain spillway consists of an approach channel, head–works including gated overflow bays and gated sluices, a long discharge channel, and a steep chute with a downstream flip bucket. Overall spillway length from the ogee crest to the nearest bucket lip is 731.52 m. The bucket has a 30° flip angle while the jet impingement angle (β) was calculated as 51° according to Eq. (9). The flood at W.A.C. Bennett Dam in 1996 caused the largest spill event with reference to the volume of water released. The 1996 scour–hole configuration was essentially the same as in 1972, with the invert roughly 1.22m deeper. The scour–hole depth was 35.97m in 1996 corresponding to a discharge of 3454.7 m^3/s and unit discharge of 83.6 m^3/s/m, total head of 161.54 m and tail water depth of 7.92m. The ratio of h/ h_2 = 161.54 m /7.92 m = 20.4 > 1; therefore Eq. (7a) is recommended.

The outlet exit velocity was calculated as 56.3m according to Eq. (11), X_p as 139.9 m according to Eq. (12) and Y_p as 40.38m from Eq. (13). With Z_1 equals to 71.62 m, the jet head, H, becomes 112.01 m. The side angle of the scour hole is assumed to be 30°. With these values used in Eq. (7a), the predicted scour depth was 34.24 m while Eq. (7b) predicted 33.26m. Hoffmans[9] equation, Eq. (2), predicted scour depth of 48.06 m. As in the last case, Over–prediction of the scour depth results from Veronese (1937) equation and under–prediction from Martins[26] equation.

Revel stoke dam, BC, Canada

The Revel stoke Project consists of a concrete gravity dam within the river canyon with a long spillway chute on the right edge, an embankment dam on the right bank terrace, and a downstream four–unit powerhouse. The concrete dam has a maximum height of 175.26m and a total length of 472.44 m. The Revel stoke spillway includes head works comprising two gated overflow bays and two intermediate level outlets, a long chute with a steep initial portion, and a terminal horizontal ski–jump structure which differentiates this case from the last three B.C. dam cases. The Overall spillway length from the ogee crest to the downstream end of the ski–jump is 405.38m. In 1986, a discharge 1415.8 m^3/s with a unit width discharge 30.66 m^2/s caused a maximum scour depth of 21.34 m. Because the bucket lip angle is 0°, the jet head H is the difference between the bucket lip elevation of 493.78 m and the tail water level of 444.7m which becomes 49.07 m. The total head of 127.71 m resulted in exit velocity of 50.07 m/s according to Eq. (11). The tail water depth was 5.18 m and the calculated jet impingement angle was 40° according to Eq. (9). The ratio of h/ h_2 = 127.71 m /5.18 m = 24.7 > 1; therefore Eq. (7a) is recommended. For an assumed scour–hole side slope of 30°, the predicted scour depth by Eq. (7a) was 21.22 m which is almost identical to the 21.34 m measured scour depth. For an assumed scour hole side slope of 35°, the predicted scour depth by Eq. (7a) was 23.56 m which still gives a good prediction. Eq. (7b) predicted scour depth of 20.39m. Hoffmans[9] equation, Eq. (2) , predicted well a scour depth of 23.9 m. Veronese (1937) equation exhibited over–prediction of 30.7 m while Martins[26] equation highly under–predicted the scour depth as 13.8 m.

Gibe III dam, Ethiopia

The information and data cited herein about this project is from de Azevedo et al.[29] Gibe III dam is a hydropower project located on the Omo River in Ethiopia. This dam is a gravity structure, about 250 m high, the world's highest using Roller Compacted Concrete (RCC) technology. The spillway is located on the dam crest and controls the discharge of the floods (up to18000 m^3/s) through seven large radial gates (12 m × 17.5 m). The key elevations are: 896 m a.s.l. dam crest level, 892 a.s.l. full supply level (FSL) and 875 a.s.l. spillway sill level. The spillway chute is about 75 m long ending in flip buckets with the lip at elevation 800 m a.s.l. Sidewalls divide the chute into seven independent bays arranged in a slightly convergent plan.

A pre–excavated plunge pool controls the scour of the jets in the riverbed and protects the outdoor power house, located downstream. The pre–excavated plunge pool is about 300 m long and less than 100 m wide. The excavation was designed using a flood with a return period of 100 years (Q = 6500 m^3/s). The pool makes use of the river stretch between the dam and the power house, which is located on the left bank about 300 m downstream of the dam toe. The bottom of the pool is unlined but the slopes are bolted and, above the water level, also lined with concrete. The depth and steepness of the slopes have been established considering the geotechnical characteristics of the rock and the energy of the water jets to be dissipated. The lowest elevation of the pool is about 640 m a.s.l., nearly 50 m below the water level corresponding to the 100–year return period flood (the downstream tail water level is 688.0 a.s.l.). The bed level just downstream of the dam and upstream of the plunge pool is 665 m a.s.l. The spillway and plunge pool design were based on numerical analysis and optimized by means of a physical model (scale 1:60) built at the LACTEC laboratory in Curitiba, Brazil. The spillway model, built in Plexiglas, was designed to accommodate a variable number of chutes (from 3 to 9) and several alternatives for the flip buckets (varying the lip angle in the range of –5 to 30°). Erosion in the plunge pool was studied using a movable bed. The granular material was selected adopting a geometrical similarity to the fractured rock of the river bed. Plunge pool behavior was also analyzed using a fixed bed measuring the pressures of the water jets through electronic transducers. The measured flow velocity reaches maximum values of 40–45 m/s, near the end of the chute.

The plunge pool design is based on the outputs of numerical analysis carried out using several independent approaches and then verified and refined using the physical model. The first analysis carried out was based on empirical equations such as the modified Veronese formula (adapted by Yildiz et al.[30]) and Mason's[31] formula. The excavation level of the plunge pool was preliminarily set at 640 m a.s.l, slightly above the value obtained with the modified Veronese equation (636.6 m a.s.l.). The findings of the movable bed model showed that the pre–excavated plunge pool is effective. After the occurrence of the design flood ($Q = 6500$ m^3 /s, return period = 100 years) the scour is quite small, a few meters only (from level of 636.6 m to level 633.2 m), and the eroded material remains within the plunge pool so operation of the powerhouse will not be affected by erosion in the plunge pool.

The plunge pool scour–hole depth will be calculated here assuming non–existence of the pre–excavated plunge pool and that the downstream river bed level is flat at 665 m a.s.l., i.e. conditions before formation of any scour. The parameters needed to calculate the scour depth are as follows. $Z_o = 892$ m – 800 m = 92 m, $Z_1 = 800$ m – 688 m = 112 m, θ is assumed as 30° (the worst case scenario), then β is found from Eq. (9) as 54.4°, with total head h of 90 m (892 m–802 m), the flip bucket exit velocity Vo becomes 42 m/s, X_p becomes 77.9 m according to Eq. (12), then Y_p becomes 22.5 m according to Eq. (13), then $H = Y_p + Z1 = 134.5$ m. The tail water depth h_2 is 23 m (688 m–665 m). The ratio of $h/h_2 = 90$ m /23 m = 3.9 > 1; therefore Eq. (7a) is recommended. Using these data, Eq. (7a) and eq. (7b) yielded scour depth of 31.5 m and 29.26 m, respectively. These predictions by both equations are enveloping the physical model scour depth of 31.8 m. The pre–excavated plunge pool had a trapezoidal vertical section which is quite different that the conical shape which is assumed in the present approach. The space available for the plunge pool was limited by the narrow width of the valley and by the powerhouse located just downstream, de Azevedo et al.[29] These factors in addition to the scale factor in physical models explain the differences between the predicted and calculated scour depths. The performance of the other three equations was not good.

Kondopoga dam, USSR

Asadollahi et al.,[32] reported data for Kondopoga dam in USSR from Cunha and Lancastre where the discharge is 70 m^3/s, the fall height is 11.4 m, the jet velocity at impact is 14.95 m/s, the water cushion (pool water depth) is 1.45 m and the resulting measured scour depth is 4.8 m. It is assumed that the scour hole has an angle of 30° and that the jet impingement angle is 30°, or 45°, or 60° (as no exact information was found). The ratio of $h/h_2 = 11.4$ m /1.45 m = 7.9 > 1; therefore Eq. (7a) is recommended. With these data Eq. (7a) predicted scour depths of 4.83 m, 5.3 m and 5.6 m for the jet impingement angles of 30°, 45° and 60°, respectively. For the wide range of values assumed for the jet angle, all the three predictions are excellent. For jet impingement angle of 30° Eq. (7b) predicted 4.6 m. As there is no information about the unit discharge, the other three equations could not be tested.

Chucas Dam, Costa Rica

Capuozzo et al.,[33] reported data for the case of Chucas Dam, Costa Rica. The Chucás hydroelectric project, owned by ENEL Costa Rica, is located about 40 km west of San José, the capital. The project, dams the Tárcoles River by a 63 m height dam, then the water is transported about 400 m downstream by a 6.5 m penstock, up to a powerhouse where 2 Francis type units generate a total output of 50 MW. For flood control, the dam is equipped with four radial gates, with dimensions 15 m × 12.4 m, capable of discharging 5400 m^3/s under design conditions and up to 8100 m^3 /s, as a verification flood. These large discharges are handled through a ski–jump spillway chute discharging on the rock bed downstream of the dam. A critical consideration in this kind of spillway was the plunge pool, which ineffably will form because rock scour under the free falling jet. A pre–excavated plunge pool with depth of 20.0 m was seen necessary by Capuozzo et al.,[33] to avoid future uncontrolled scouring.

A physical model study, on a scale 1:65, was tested at the hydraulic laboratory of INA, Argentina. The model includes a good deal of the approaching reservoir, the complete dam and spillway, and a large section downstream, covering the zone of the plunge pool in addition to a zone where the powerhouse discharges back to the river. The model tests showed that the pre–excavated pool is very effective in the dissipation of energy, for the operational discharge of 5400 m^3/s, but for the verification flood high residual energy is observed. The relevant data is that the discharge is 5400 m^3/s, the total head is 33.89 m, the tail water depth is 16.51m, the unit discharge is 82.32m^2/s, and the flip bucket take–off angle is 40°. The ratio total head /$h_2 = 2.1$, so Eq. (7a) or eq. (7b) could be recommended. The impingement jet angle is 45° based on the model jet trajectory. With θ = 40°, $Z_o = 29$ m (300 m– 271 m) and $Z_1 = 3.49$ m (271 m– 267.51 m), the calculated impingement jet angle according to Eq. (9) is 43.64° which is close to the physical model value. The side slope angle of the pre–excavated scour hole had slope of 1V:0.5 H which corresponds to an angle of 26.6°. With these data and assuming non–existence of the pre–excavated plunge pool, Eq. (7a) and (7b) predicted a plunge pool scour depth of 23.87 m and 20.12 m, respectively which compare very well to the 20.0 m from the physical model. Hoffmans[9] equation also predicted well a scour depth of 19.36 m. Again, Veronese[25] equation over–predicted while Martins[26] equation under–predicted the sour depth.

Tucurui Dam, Brazil

Bollaert[34] reported about the Tucurui Dam spillway in Brazil where the design discharge is 110000 m^3/s under a gross head of 60 to70 m. The spillway is characterized by an ogee type gate controlled structure topped by 23 radial gates (20.75m high × 20m wide), a compact flip bucket with issuing angle of 32° and a 50 m deep plunge pool. Hydraulics laboratory model tests resulted in the forecast of a satisfactory scouring behavior for a pre–excavated plunge pool at an elevation of – 40 m a.s.l. The bed level just downstream of the dam is at level 3.0 m a.s.l. Thus the depth of the pre–excavated scour hole can be assumed as 43.0 m. Scour formation in the downstream plunge pool has been described by a series of bathymetric surveys since 1984. These showed that, as predicted by the laboratory tests, the maximum observed scour depth was of only 5 m. It was assumed that this erosion is related to removal of partially detached rock blocks during initial spillage. These blocks were fractured and detached by blasting during dam construction. Hence, it was considered that the pre–excavated plunge pool behaves like expected during dam construction. For a recorded period of 17 years, incorporating 6 flood events of more than a 31000 m^3/s and a maximum value of a 43400 m^3/s, no significant scour formation could be observed. However, the 5 m of scour below the level of the pre–excavated plunge pool is taken into account yielding the maximum scour depth to 48.0 m. Bollaert[2] CSM model has been calibrated based on the assumption that, for flood events of up to 50000 m^3/s, no significant scour forms at the plunge pool bottom. This discharge can be considered as the critical discharge for scour. The upstream water level is 74 m and with a lip elevation of the flip bucket of 30 m yields Z_o = 44 m. The downstream water level is 10 m yielding Z_1 = 20 m. The impingement angle is calculated from Eq. (9) as 45.3°. The jet profile in Bollaert et al.,[35] shows an angle of nearly 45° which is almost identical. In addition, the jet profile has its maximum level at 50 m which for a tail water level of 10 m yields a jet head equals to 40 m. This head results in jet impact velocity of 28 m/s according to Eq. (11). Assuming flat bed at level zero representing conditions without the pre–excavated plunge pool and for a discharge of 50000 m^3/s, head of 40 m, velocity of 28.28 m/s, tail water depth of 10.0 m, and jet impingement angle of 45°, scour side angle of 30°, $h/h_2 = 4$, Eq. (7a) yields scour depth of 49.18 m which is almost close to the model result. Eq. (7b) yields scour depth of 46.02 m which is also good. Veronese[25] equation performed fairly well with 50.47 m predicted depth of scour.

Picote Dam, Portugal

Cunha and Lancastre and Asadollahi et al.,[32] reported data for Picote dam in Portugal where the discharge is 7000 m^3/s, the fall height is 45 m, the jet velocity at impact is 29.71 m/s, the water cushion (pool water depth) is 36 m and the resulting measured scour depth is 19 m. With a crest length of 139 m, the unit width discharge becomes 50.4 m^2/s. It is assumed that the scour hole has an angle of 30° and that the jet impingement angle is 30° (no information was found). With these data Eq. (7a) predicts a scour depth of 28.64 m. The ratio of the falling height to the pool water depth is 1.25 which is not high enough signaling the existence of a cushioning effect by the deep water pool. In that case the negative sign (–) can be used as in Eq.(7b) which then yields predicted scour depths of 20.65 m, 22.9 m and 24.4 m for the jet impingement angles of 30°, 45° and 60°, respectively. The calculated volume of scoured material is 27865 m^3, 37727 m^3, and 45637 m^3 respectively. Heng et al.,[35] reported a volume of scoured material equal to 36900 m^3. Regardless of the value assumed for the jet angle, all the three predictions by Eq. (7b) are excellent and the cushioning effect could be successfully accounted for. Martins[25] & Hoffman[9] equations predicted unrealistic negative scour depths while Veronese[26] equation predicted very low scour depth.

Wivenhoe Dam, Australia

The data and information in this section about Wivenhoe Dam, Austrailia, are according to Bollaert et al.[36] Wivenhoe Dam was constructed in the early 1980s, on the Brisban River. The earth–fill dam incorporates a gated ski–jump spillway on the left abutment, which discharges into a pre–excavated unlined plunge pool and outlet channel. The spillway has five radial gates each 12 m wide and 16 m high. The pre–excavated plunge pool comprised benches stepping down in increments generally of 3 m, to the lowest invert level of El. (elevation) of 17.0 m Australian Height Datum (AHD). Major floods occurred at the site in January 2011 that necessitated the spillway to discharge 7500 m^3/s which was twice as the design flood for the plunge pool. The head differential during this event rarely exceeded 30 m, the velocities exiting the flip bucket were less than 25 m/s and the jet angle of impact is about 30°. When the tail water subsided four days after passing the peak discharge, the top of an enormous rock mound was revealed. A pile of rocks was approximately 11 m high, covered nearly the full width of the channel. Boulders of up to 15 m × 10 m × 3 m, weighing over 1000 tones were observed in the pile of eroded rocks in the spillway channel. Material was removed from the downstream extent of the plunge pool to extend its length by more than 40 m and its base down to 2 m below design, Lesleighter et al.[37]

The ratio $h/h_2 = 33/12 > 1$. Eq. (7a) was applied assuming non–existence of the pre–excavated plunge pool with the original bed level assumed at about El. (elevation) 30 m which with tail water levels of El. 42 m yields tail water depth of about 12 m. The resulting scour–hole side slope angle, ϕ, is assumed as 20° which is close to the pre–excavated plunge pool slope ($\tan^{-1} 3/10 \approx 17°$). The head on the jet from Bollaert et al.[36] CFD jet trajectory simulation is nearly 10 m (El. 50 m – El. 40 m). With V_o as 25 m/s, θ as 30°, X_p is 31.9 m from Eq. (12), Y_p is 9.2 m from Eq. (13) and Z_1 is about 1 m yielding H as 10.2 m. which is close to the value from CFD results of 10 m. For the plunge pool design flood of about 3750 m^3/s (50% of the 2011 peak flood) and using the rest of the aforementioned variables Eq. (7a) predicted a scour depth of 12.82 m. This scour depth is to be compared to physical model scour depth of 13.0 m. Hoffman[9] & Martins[25] equations provided excellent predictions while Veronese[26] double–predicted the measured value. With peak flood of 7500 m^3/s and the rest of the aforementioned variables, Eq. (7a) predicted a scour depth of 15.53 m. The pool bed–level from the bathymetric survey after the flood was at El. 15 m. Thus it can be considered that a scour depth of 15 m (El. 30 m –El. 15 m) would occur in reality in the absence of the pre–excavated plunge pool. Equation (7b) predicted scour depths of 11.34m and 13.87 m for the flows of 3750 m^3/s and 7500 m^3/s, respectively. The author believes that the lateral shift or extension of the downstream face of the pre–excavated plunge pool at the peak flood is due to the fact that with almost doubling of the peak flood discharge in comparison to the design discharge the jet range and the point of impact of the jet shifted in the downstream direction causing the core of the jet to attack heavily the downstream face. This is a risk of having a predetermined or pre–excavated plunge pool where the jet impact could change its position compared to the design conditions. The present approach which resulted in Eq. (7 a/b) could be considered as the scour–hole version of the energy transfer theory where the whole scour hole is treated as a one unit. Now the particle version of the energy transfer theory is considered where attention is given to one single bed particle. Since lifting of bed particles against gravity is the phenomenon herein the vertical component of the velocity is thought to be the cause for this lifting through giving its kinetic energy to the bed particles. It is assumed that a particle lying on the bed surface of the scour hole at its deepest level will have a projected area to the falling flow jet as A_p and a volume as V_p. For simplicity it is assumed that the deepest level of the scour hole is horizontal. The work done by the flow energy of the falling jet on this particle is W_{in} which is given as:

$$W_{in} = \lambda \rho A_p \left(\{V \sin \beta\}^2 H + \{V_{av} \sin \beta\}^2 (h_2 + D_s) \right) \tag{14}$$

Where λ is a momentum transfer coefficient, $V_{av} = (V+V_b)/2$ is the average of the jet velocity at impact with water, V, and the jet velocity at impact with the particle surface, V_b. The energy needed to lift this particle vertically out of its original position at the deepest point of the scour hole to the bed original position, i.e. a distance equal to the scour depth D_s is given as:

$$W_o = (\gamma_s - \gamma) V_p D_s \tag{15}$$

Equating Eq. (14) and Eq. (15) and considering that $h_p = V_p/A_p$ where h_p is the bed particle height yields:

$$h_p = \frac{\lambda \left(\{V \sin \beta\}^2 H + V_{av} \{\sin \beta\}^2 (h_2 + D_s) \right)}{g (S_G - 1) D_s} \tag{16}$$

The velocity at the particle bed surface, V_b is calculated according to Bohrer et al.[38] For the Wivenhoe dam, the total pool depth is taken as 29.1 m (D_s+h_2 = 17.1 + 12 = 29.1 m) while the jet diameter at impact is about 10 m according to Bollaert et al.[33] CFD results. Bohrer et al.,[36] curve of % jet– impact on the bed velocity shows for pool depth to jet diameter ratio (which is about 3 at V) a value of 50%. For a jet impingement velocity of 25 m/s, the bed velocity V_b becomes then 12.5 m/s and consequently the average velocity V_{av} becomes 18.8 m/s. Now substituting in Eq. (16) a momentum transfer efficiency coefficient of 0.75 according to Hafez[15] V= 25 m/s, β = 30°, H = 10m, V_{av} = 18.8 m/s, h_2 = 12 m, and the previously calculated value of D_s of 17.1 m yields h_p = 11.2 m. Field observations showed a pile of rocks with a height approximately ranging from 10 to 11 m high moved out of the scour hole. With boulders of 15 m × 11 m × 3 m observed in the rock pile, it is more likely that these boulders were cut from the rock bed by the flow jet to a height of 3 m, then turned up vertically 90° where the height became 11 m and the horizontal dimensions became 15 m × 3 m in order to minimize the resistance to the upward vertical movement. This close match between prediction and field data supports the particle version of the energy transfer theory. Equation (16) can be used to predict the maximum particle size of the disintegrated rock beds due to scour by falling jets at plunge pools' bottom which is needed in conducting model tests in order to scale the model bed material.

Discussion

It should be noted that the proposed approach can make use of past methods by utilizing the Erodibility Index method by Anandale[1] and Comprehensive Scour Model by Bollaert[2] for checking the threshold of erosion and obtaining relevant falling jet parameters from computational hydrodynamic simulations. In other words, integration between the proposed approach herein and some existing methods seems very possible and quite useful. Though not addressed clearly in past research, the scour hole average side slope angle, ϕ, should depend primarily on the jet impingement angle β in addition to other important factors such as the bed material composition, jet velocity and falling head. For naturally formed scour holes, the scour hole average side slope angle, ϕ, was assumed 30° for eight cases for which the angle for jet impingement was in the range (24° to 51°). For the severe case of Kariba Dam where the jet impingement angle was as high as 60°, a value of 60° was assumed. Further research is needed in this area to find the exact functional relationship for ϕ. The present approach is quite useful in providing the necessary information for designing of pre–excavated scour holes. The predicted scour depth from the present approach assuming non–existence of the pre–excavated scour hole is the key to this step. It is noted that for well designed –excavated scour holes the jet velocity structures should diminish (i.e. close to zero or very minimal) at the pool bottom and hence no more scour should occur. As velocity is a key component in influencing formation of the scour hole, a near zero velocity would predict no further scour and ensure stability of the scour hole. The velocity structure inside the plunge pool and near the pool bottom is best obtained from CFD models. If a relationship exists between the length of the scour hole and its width at plunge pools, then it would be possible to use an elliptical conical shape for the scour–hole model which would be more general. This is also another area for further research. The velocity used in the present approach and all other existing methods is the mean flow velocity, i.e. neglecting explicitly the effect of the turbulent velocity fluctuations. The discharge Q can be written as $Q = A_j V$, where A_j is the cross sectional area of the flow jet. Eq. (4) for the work done by the attacking flow jet can then be written as:

$$W_g = \rho A_j V^2 \sin \beta (H \pm h_2 + D_s) = \rho A_j (V_m + v)^2 \sin \beta (H \pm h_2 + D_s) \tag{17}$$

Where V_m is the time average mean velocity and v is the instantaneous turbulent velocity fluctuation component. Taking the time average of this equation with a time scale that is longer than the time scale of the instantaneous turbulent velocity fluctuation yet smaller than the scour time scale yields:

$$W_g = \rho A_j (V_m^2 + v^2) \sin \beta (H \pm h_2 + D_s) \tag{18}$$

Where the quantities in Eq. (18) are understood to be time averaged quantities. Eq. (18) suggests that an additional energy or work done by the turbulent velocity fluctuations (mean turbulent kinetic energy) should be accounted for. This may compensate for neglecting the energy frictional losses, or jet aeration, or energy dissipation and other losses as these two factors have opposite effects as was stated in the assumption section. The success of the energy/work transfer theory in predicting scour at plunge pools due to falling jets in addition to scour due to turbulent wall jets, Hafez,[8] and scour at bridge piers, Hafez[15] suggests the universality and generality of this theoretical approach and that it can provide a general theory for local scour at hydraulic structures. The general form of the energy/work transfer theory can be stated as:

$$\left(\textit{Flow jet Force}\right) \times \left(\textit{force travelling distance}\right) = \left(\gamma_s - \gamma\right) \times \left(\textit{Volume of the scour hole}\right) \times \left(Y_{C.G} + h_r\right) \tag{19}$$

Where $Y_{C.G}$ is the distance of the center of mass of the volume of the scour hole below the original ground level and h_r is the ridge height. Here it is considered that for formation of scour hole the bed particles must be lifted to a distance equal to the sum of the maximum scour depth and the ridge height. No data existed about the ridge height; therefore, its influence was not considered. Every local scour phenomenon has its own representation of the variables in Eq. (19). However, all local scour phenomena can be expressed well as by Eq. (19). As observed from Table 3, the close match between the present approach and Hoffman[9] predictions of scour depth suggests some sort of connection as both methods are theory–based. For H >> ($h_2 + D_s$), using $H = V^2/2g$, Eq. (8) can be written as

$$D_s^{\,4} = \frac{9\,(\tan\varphi)^2}{\pi\ \ (1-p)}\ \frac{\rho}{\rho_s-\rho}\,\frac{B}{b_o}\,q^2\,\frac{V^2}{2g^2}\sin\beta \tag{20}$$

Now taking the fourth root of Eq. (20) yields

$$D = \sqrt[4]{\frac{9\,(\tan\varphi)}{2\pi\ \ (1-p)}\ \frac{\rho}{\rho_s-\rho}\,\frac{B}{b_o}}\ \sqrt{\frac{q\,V\sqrt{\sin\beta}}{g}} \tag{21}$$

Comparing Eq. (2) and Eq. (21) suggests remarkable similarity between the two equations where the scour depth varies in both equations with the square root of q V/g. Eq. (21) suggests weaker dependence in the present approach by the scour depth on the jet impingement angle than by Hoffmans[9] equation. It is interesting to note that Hoffmans [9] equation is based on force balance while the present approach is based on energy balance. The two equations provided by the present approach; namely Eq. (7a) and Eq. (7b) performed quite well with Eq. (7a) performing better. Equation (7b) considers the cushioning effect by the pool water mattress and therefore its scour depth predictions are lower than that by Eq. (7a). On the other hand Eq. (7a) neglects the cushioning effect and assumes that all the energy coming from the falling flow jet contributes to the plunge pool scour. It seems plausible that an average value of the three equations; Eq. (7a), Eq. (7b) and Eq. (2) by Hoffmans[9] would be a very good estimate to plunge pool scour depth. The current approach has the advantage that it is physically based which allows its mathematical structure to accommodate several additional important factors such as jet aeration, ridge height and sediment–laden jets. These factors cannot be easily reflected upon in most existing methods.[39,40]

Conclusion

The proposed three dimensional mathematical models for predicting plunge pool scour–hole depth and volume of scoured material due to falling jets proved to be very accurate and reliable. Not only it provides predictions for the scour depth as typical in other methods but also it predicts the volume of the scoured material and maximum size of fractured bed material. It showed excellent agreement with theoretical, experimental, and field data. In particular, outstanding performance resulted in when comparing to plunge pool scour at well–known twelve dams all over the world such as Cabora–Bassa Dam, Kariba Dam, and Gibe III Dam in Africa and four dams in British Colombia, Canada, which are: Peace Canyon Dam, Miles Dam, Portage Mountain Project, and Revel stoke Dam. Then, Kondopoga Dam in USSR, Chucas Dam in Costa Rica, Tucurui Dam, in Brazil, Picote Dam in Portogal and Wivenhoe dam in Australia were studied. The close matching of the predicted and measured scour depth at these dams and in volume of the scoured material (when data for scour volume existed) supports the validity of the energy/work transfer theory and its underlying assumptions such as considering the whole scour hole as circular conical body. A particle version of the theory seems promising in predicting particle sizes of disintegrated rock beds due to scour. Elliptical conic shape of the scour hole at plunge pools, determination of the side slope of the scour hole and scour–hole length to width ratio are recommended for further studies.

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Conflict of interest

The author declares that there is none of conflicts.

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