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Rayleigh waves in thermo elastic medium with double porosity

Abstract

The present paper deals with the propagation of Rayleigh waves in isotropic homogeneous thermoelastic half–space with double porosity whose surface is subjected to stress–free, thermally insulated/isothermal boundary conditions. The compact secular equations for thermoelastic solid half–space with voids are deduced as special cases from the present analysis. In order to illustrate the analytical developments, the secular equations have been solved numerically. The computer simulated results for copper materials in respect of determinant of Rayleigh wave secular equation, Rayleigh wave velocity and attenuation coefficient have been presented graphically for different values of phase velocity.

Keywords: rayleigh waves, double porosity, thermoelastic, secular equation

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Introduction

Porous media theories play an important role in many branches of engineering including material science, the petroleum industry, chemical engineering, biomechanics and other such fields of engineering. Biot[1] proposed a general theory of three–dimensional deformation of fluid saturated porous salts. Biot[1] theory is based on the assumption of compressible constituents and till recently, some of his results have been taken as standard references and basis for subsequent analysis in acoustic, geophysics and other such fields. One important generalization of Biot's[1] theory of poroelasticity that has been studied extensively started with the works by Barenblatt et al.,[2] where the double porosity model was first proposed to express the fluid flow in hydrocarbon reservoirs and aquifers. The double porosity model represents a new possibility for the study of important problems concerning the civil engineering. It is well–known that, under super– saturation conditions due to water of other fluid effects, the so called neutral pressures generate unbearable stress states on the solid matrix and on the fracture faces, with severe (sometimes disastrous) instability effects like landslides, rock fall or soil fluidization (typical phenomenon connected with propagation of seismic waves). In such a context it seems possible, acting suitably on the boundary pressure state, to regulate the internal pressures in order to deactivate the noxious effects related to neutral pressures; finally, a further but connected positive effect could be lightening of the solid matrix/fluid system.

Wilson and Aifanits[3] presented the theory of consolidation with the double porosity. Khaled et al.,[4] employed a finite element method to consider the numerical solutions of the differential equation of the theory of consolidation with double porosity developed by Aifantis.[3] Wilson et al.,[5] discussed the propagation of acoustics waves in a fluid saturated porous medium. Beskos et al.,[6] presented the theory of consolidation with double porosity–II and obtained the analytical solutions to two boundary value problems. Aifantis[7–10] introduced a multi–porous system and studied the mechanics of diffusion in solids. Khalili et al.,[11] presented a fully coupled constitutive model for thermo–hydro –mechanical analysis in elastic media with double porosity structure. Straughan[12] studied the stability and uniqueness in double porous elastic media. Svanadze[13–17] investigated some problems on elastic solids, viscoelastic solids and thermoelastic solids with double porosity. Rayleigh waves are always generated when a free surface exists in a continuous body. Rayleigh firstly introduced them as solution of the free vibration problem for an elastic half–space (on waves propagated along the plane surface of an elastic solid). Rayleigh wave play an important role in the study of earthquakes, seismology, geophysics and geodynamics. During earthquake, Rayleigh waves play more drastic role than other seismic waves because these waves are responsible for destruction of buildings, plants and loss of human lives etc. Geophysical and thermal problems consist of the study of propagation of progressive elastic and thermoelastic waves and hence the effect of voids on the surface waves propagating in the thermoelastic media has got its due importance where the situation so demands. The cooling and heating of the medium also results in the expansion and contraction of the voids along with the core material which contributes towards thermal stress and vibration developments in solids. In coating or casting applications, the voids that are not detected and removed, can result in defects that compromise the adhesion, electric properties, surface finish and durability of the product.

Rayleigh L,[18] investigated the propagation of waves along the plane surface of an elastic solid. Lockett[19] studied the effect of thermal properties on Rayleigh wave's velocity. Propagation of Rayleigh waves along with isothermal and insulated boundaries discussed by Chadwick et al.,[20] Kumar et al.,[21,22] presented the problem of Rayleigh waves in an isotropic generalized thermoelastic with diffusive half–space medium. Sharma et al.,[23] presented the problem of Rayleigh waves in rotating thermoelastic with voids. Kumar et al.[24] discussed the problem of Rayleigh waves in isotropic micro stretch thermoelastic diffusion solid half–space .Kumar and Gupta[25] discussed the problem of Rayleigh waves in generalized thermoelastic medium with mass diffusion. Abd–Alla et al.,[26–33] investigated the propagation of Rayleigh waves in different theories. Singh et al.[34] examined the propagation of the Rayleigh wave in an initially stressed transversely isotropic dual phase lag magneto–thermoelastic half space. Kumar et al.,[35] studied the propagation of Rayleigh waves in generalized thermoelastic medium with mass diffusion. Biswas et al.,[36] investigated the Rayleigh surface wave propagation in orthotropic thermoelastic solids under three–phase lag model. Singh et al.,[37] examined the propagation of Rayleigh wave in two–temperature dual–phase–lag thermo elasticity. Biswas et al.,[38] studied the effect of phase–lags on Rayleigh wave propagation in initially stresses magneto–thermoelastic orthotropic medium. Hussien et al.,[39] investigated the effect of rotation on Rayleigh waves in a fiber–reinforced solid anisotropic magneto–thermo–viscoelastic media. In the present paper, we investigate the propagation of Rayleigh waves in homogeneous isotropic elastic material with double porosity structure .Secular equations are derived mathematically for the boundary conditions. The values of determinant of Rayleigh wave secular equation, Rayleigh wave velocity and attenuation coefficient with respect to wave number are computed numerically and depicted graphically.

Basic equations

Following Iesan et al.,[40] the constitutive relations and field equations for homogeneous elastic material with double porosity structure without body forces, extrinsic equilibrated body forces and without heat sources can be written as:

Constitutive Relations

$$t_{ij} = \lambda e_{rr}\delta_{ij} + 2\mu e_{ij} + b\delta 6\varphi + d\delta_{ij}\psi - \beta\delta_{ij}T, \quad (1)$$

$$\sigma_i = \alpha\varphi_{,i} + b_1\psi_{,i}, \quad (2)$$

$$\tau_i = b_1\varphi_{,i} + \gamma\psi_{,i}, \quad (3)$$

Equation of motion

$$\mu\nabla^2\vec{u} + (\lambda+\mu)\nabla\nabla\cdot\vec{u} + b\nabla\varphi + d\nabla\psi - \beta\nabla T = \rho\frac{\partial^2 u}{\partial t^2} \quad (4)$$

Equilibrated Stress Equations of motion

$$\alpha\nabla^2\varphi + b_1\nabla^2\psi - b\nabla\cdot\vec{u} - \alpha_1\varphi - \alpha_3\psi + \gamma_1 T = \kappa_1\frac{\partial^2\varphi}{\partial t^2}, \quad (5)$$

$$b_1\nabla^2\varphi + \gamma\nabla^2\psi - d\nabla\cdot\vec{u} - \alpha_3\varphi - \alpha_2\psi + \gamma_2 T = \kappa_2\frac{\partial^2\psi}{\partial t^2}, \quad (6)$$

Equation of Heat conduction

$$K^*\nabla^2 T - \beta T_0\nabla.\dot{\vec{u}} - \gamma_1 T_0\dot{\varphi} - \gamma_2 T_0\dot{\psi} - \rho C^*\dot{T} = 0 \quad (7)$$

where $\vec{u}$ is the displacement vector ; t_{ij} is the stress tensor; κ_1 and κ_2 are coefficients of equilibrated inertia; φ and ψ are the volume fraction fields corresponding to pores and fissures respectively ; σ_i is the equilibrated stress corresponding to pores; τ_1 is the equilibrated stress corresponding to fissures; K^* is the coefficient of thermal conductivity; C^* is the specific heat at constant strain, ρ is the mass density; $\beta = (3\lambda + 2\mu)\alpha_t$; α_t is the linear thermal expansion; λ and μ are Lame's constants and $b, d, b_1, \gamma, \gamma_1, \gamma_2$ are constitutive coefficients; δ_{ij} is the Kronecker's delta; T is the temperature change measured form the absolute temperature $T_0\ (T_0 \neq 0)$; a superposed dot represents differentiation with respect to time variable t .

$$\nabla = \hat{i}\frac{\partial}{\partial x_1} + \hat{j}\frac{\partial}{\partial x_2} + \hat{k}\frac{\partial}{\partial x_3}, \quad \nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}$$

Are the gradient and Laplacian operators, respectively.

Formulation of the problem

We consider homogeneous isotropic thermoelastic with double porous half space .We take the origin of the coordinate system (x_1, x_2, x_3) at any point plane on the horizontal surface and x_1 – axis in the direction of the wave propagation and x_3 – axis pointing vertically downward to the half–space so that all particles on line parallel to x_2 – axis are equally displaced. Therefore, all the field quantities will be independent of x_2 – coordinate.

For the two–dimensional problem, we take

$$u_1 = u_1(x_1, x_3, t), u_2 = 0, u_3 = u_3(x_1, x_3, t), \varphi = \varphi(x_1, x_3, t), \psi = \psi(x_1, x_3, t), T = T(x_1, x_3, t) \quad (8)$$

We define the following non–dimensional quantities:

$$x_1' = \frac{\omega_1}{c_1}x_1,\ x_3' = \frac{\omega_1}{c_1}x_3, u_1' = \frac{\omega_1}{c_1}u_1,\ u_3' = \frac{\omega_1}{c_1}u_3, t_{ij}' = \frac{t_{ij}}{\beta T_0}, T' = \frac{T}{T_0},$$

$$\varphi' = \frac{\kappa_1\omega_1^2}{\alpha_1}\varphi,\ \psi' = \frac{\kappa_1\omega_1^2}{\alpha_1}, t' = \omega_1 t, \sigma_1' = \left(\frac{c_1}{\alpha\omega_1}\right)\sigma_1, \tau_1' = \left(\frac{c_1}{\alpha\omega_1}\right)\tau_1 \quad (9)$$

Where $c_1^2 = \dfrac{\lambda + 2\mu}{\rho}, \omega_1 = \dfrac{\rho C^* c_1^2}{K^*}$

Here ω and c_1 are the constants having the dimension of frequency and velocity in the medium respectively.

Using (8) in Eqs. (4)–(7) and with the aid of (9), after suppressing the primes, we obtain

$$\left(\frac{\lambda+\mu}{\rho c_1^2}\right)\frac{\partial e}{\partial x_1} + \frac{\mu}{\rho c_1^2}\nabla^2 u_1 + a_1\frac{\partial\varphi}{\partial x_1} + a_2\frac{\partial\psi}{\partial x_1} - a_3\frac{\partial T}{\partial x_1} = \frac{\partial^2 u_1}{\partial t^2} \quad (10)$$

$$\left(\frac{\lambda+\mu}{\rho c_1^2}\right)\frac{\partial e}{\partial x_3} + \frac{\mu}{\rho c_1^2}\nabla^2 u_3 + a_1\frac{\partial\varphi}{\partial x_3} + a_2\frac{\partial\psi}{\partial x_3} - a_3\frac{\partial T}{\partial x_3} = \frac{\partial^2 u_3}{\partial t^2} \quad (11)$$

$$a_4\nabla^2\phi + a_5\nabla^2\psi - a_6 e - a_7\varphi - a_8\psi - a_9 T = \frac{\partial^2\varphi}{\partial t^2} \tag{12}$$

$$a_{10}\nabla^2\phi + a_{11}\nabla^2\psi - a_{12}e - a_{13}\varphi - a_{14}\psi - a_{15}T = \frac{\partial^2\psi}{\partial t^2} - a_{16}\frac{\partial e}{\partial t} - a_{17}\frac{\partial\varphi}{\partial t} - a_{18}\frac{\partial\psi}{\partial t} + a_{19}\nabla^2 T - \frac{\partial T}{\partial t} = 0 \tag{13}$$

Where $a_1 = \frac{b\alpha_1}{\rho c_1^2\kappa_1^2\omega_1^2}, a_2 = \frac{d\alpha_1}{\rho c_1^2\kappa_1^2\omega_1^2}, a_3 = \frac{-\beta T_0}{\rho c_1^2}, a_4 = \frac{\alpha}{\kappa_1 c_1^2}, a_5 = \frac{b_1}{\kappa_1 c_1^2}, a_6 = \frac{b}{\alpha_1}, a_7 = \frac{\alpha_1}{\kappa_1\omega_1^2}, a_8 = \frac{\alpha_3}{\kappa_1\omega_1^2},$

$a_9 = \frac{\gamma_1 T_0}{\alpha_1}, a_{10} = \frac{b_1}{\kappa_2 c_1^2}, a_{11} = \frac{\gamma}{\kappa_2 c_1^2}, a_{12} = \frac{d\kappa_1}{\kappa_2\alpha_1}, a_{13} = \frac{\alpha_3}{\kappa_2\omega_1^2}, a_{14} = \frac{\alpha_2}{\kappa_2\omega_1^2}, a_{15} = \frac{\gamma_2 T_0\kappa_1}{\alpha_1\kappa_2}, a_{16} = \frac{\beta}{\rho C^*},$

$a_{17} = \frac{\gamma_1\alpha_1}{\rho C^*\kappa_1\omega_1^2}, a_{18} = \frac{\gamma_2\alpha_1}{\rho C^*\kappa_1\omega_1^2}, a_{19} = \frac{K^*\omega}{\rho c_1^2 C^*}$

Here $\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}$, $e = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}$

The displacement components u_1 and u_3 are related by potential functions φ_1 and ψ_1 as

$$u_1 = \frac{\partial\varphi_1}{\partial x_1} - \frac{\partial\psi_1}{\partial x_3}, \quad u_3 = \frac{\partial\varphi_1}{\partial x_3} + \frac{\partial\psi_1}{\partial x_1} \tag{15}$$

Making use of (15) in equations (10)–(14), we obtain

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right)\varphi_1 + a_1\varphi + a_2\psi - a_3 T = 0 \tag{16}$$

$$-a_6\nabla^2\varphi_1 + \left(a_4\nabla^2 - a_7 - \frac{\partial^2}{\partial t^2}\right)\varphi + \left(a_5\nabla^2 - a_8\right)\psi + a_9 T = 0 \tag{17}$$

$$-a_{12}\nabla^2\varphi_1 + \left(a_{10}\nabla^2 - a_{13}\right)\varphi + \left(a_{11}\nabla^2 - a_{14} - \frac{\partial^2}{\partial t^2}\right)\psi + a_{15}T = 0 \tag{18}$$

$$-a_{16}\frac{\partial}{\partial t}(\nabla^2\varphi_1) - a_{17}\frac{\partial\varphi}{\partial t} - a_{18}\frac{\partial\psi}{\partial t} + \left(a_{19}\nabla^2 - \frac{\partial}{\partial t}\right)T = 0 \tag{19}$$

and

$$\left(a_{20}\nabla^2 - \frac{\partial^2}{\partial t^2}\right)\psi_1 = 0 \tag{20}$$

Here

$$a_{20} = \frac{\mu}{\rho c_1^2}$$

Solution of the problem

We assume the solution of the form

$$(\varphi_1, \varphi, \psi, T, \psi_1) = (\varphi_1^*, \varphi^*, \psi^*, T^*, \psi_1^*)\,e^{i\xi(x_1 - ct)} \tag{21}$$

Where ξ is the wave number, $\omega = \xi c$ is the angular frequency and c is the phase velocity of the wave.

Making use of (21) in Eqs.(16)–(20) ,we obtain four homogeneous equations in four unknowns and these equations have non–trivial solutions if the determinant of the coefficient $\varphi_1^*, \varphi^*, \psi^*$ and T^* vanishes, which yield to the following characteristics equation:

$$E_1\frac{d^8}{dz^8} + E_2\frac{d^6}{dz^6} + E_3\frac{d^4}{dz^4} + E_4\frac{d^2}{dz^2} + E_5 = 0 \tag{22}$$

Where E_1, E_2, E_3, E_4 and E_5 are given in the appendix and

$$\left(\frac{d^2}{dx_3^2} - \zeta_5^2\right) = 0 \tag{23}$$

Where

$$\zeta_5^2 = \xi^2 + \frac{\xi^2 c^2}{a_{20}}$$

Since we are interested in surface waves only, it is essential that the motion is confined to the free surface $x_3 = 0$ of the half–space. Therefore, to satisfy the radiation conditions, $\varphi_1, \varphi, \psi, T, \psi_1 \to 0$ as $x_3 \to \infty$ are given by

$$\left(\varphi_1, \varphi, \psi, T\right) = \sum_{i=1}^{4}(1, r_i, s_i, h_i)B_i e^{-m_i x_3} e^{i\xi(x_1 - ct)} \tag{24}$$

and from (21), we get

$$\psi_1 = B_5 e^{-m_5 x_3} e^{i\xi(x_1 - ct)} \tag{25}$$

where $m_i (i = 1,2,3,4)$ are roots of the equation (22) and m_5 is root of equation (23) . $B_i\left(i = 1,2,3,4,5\right)$ are arbitrary constants in equation(21) and (22) .

Here the coupling constants are

$$r_i = -\frac{D_{1i}}{D_{0i}}, s_i = \frac{D_{2i}}{D_{0i}}, h_i = -\frac{D_{3i}}{D_{0i}}; i = 1,2,3,4$$

$D_{0i}, D_{1i}, D_{2i}, D_{3i}$ are given in the Appendix.

Boundary conditions

The boundary conditions at the free surface $x_3 = 0$. Mathematically these can be written as

$$t_{33} = 0 \quad (26)$$

$$t_{31} = 0 \quad (27)$$

$$\sigma_3 = 0 \quad (28)$$

$$\tau_3 = 0 \quad (29)$$

$$\frac{\partial T}{\partial x_3} = 0 \quad (30)$$

Derivation of the seqular equation

Making use of (21) and (22) in the boundary conditions (26)–(30) and with the aid of (1)–(3), we obtain a system of five simultaneous homogeneous linear equations

$$\sum_{j=1}^{5} Q_{ij} B_j = 0 \qquad \text{for } i=1,2,3,4,5 \quad (31)$$

where

$$Q_{1j} = \begin{cases} p_1 m_j^2 - \xi^2 p_2 + p_3 r_j + p_4 s_j - h_j, & \text{for } j = 1,2,3,4 \\ (p_1 - p_2) i\xi m_j, & \text{for } j = 5 \end{cases} \quad (32)$$

$$Q_{2j} = \begin{cases} 2i\xi m_j, & \text{for } j = 1,2,3,4 \\ (m_j^2 + \xi^2), & \text{for } j = 5 \end{cases} \quad (33)$$

$$Q_{3j} = \begin{cases} -m_j (p_5 r_j + p_6 s_j), & \text{for } j = 1,2,3,4 \\ 0, & \text{for } j = 5 \end{cases} \quad (34)$$

$$Q_{4j} = \begin{cases} -m_j (p_6 r_j + p_7 s_j), & \text{for } j = 1,2,3,4 \\ 0, & \text{for } j = 5 \end{cases} \quad (35)$$

$$Q_{5j} = \begin{cases} h_j, & \text{for } j = 1,2,3,4 \\ 0, & \text{for } j = 5 \end{cases} \quad (36)$$

where

$$p_1 = \frac{\lambda + 2\mu}{\beta T_0},\ p_2 = \frac{\lambda}{\beta T_0}, p_3 = \frac{b\alpha_1}{\beta T_0 \kappa_1 \omega_1^2},\ p_4 = \frac{d\alpha_1}{\beta T_0 \kappa_1 \omega_1^2},\ p_5 = \frac{\alpha_1}{\kappa_1 \omega_1^2},\ p_6 = \frac{b_1 \alpha_1}{\alpha \kappa_1 \omega_1^2}, p_7 = \frac{\gamma \alpha_1}{\alpha \kappa_1 \omega_1^2}$$

The system of Eqs.(31) has a non–trivial solution if the determinant of the unknowns $B_j\ (j = 1,2,3,4)$ vanishes i.e.

$$\left| Q_{ij} \right|_{5\times5} = 0 \quad (37)$$

Particular case

If $b_1 = \gamma = \alpha_3 = \alpha_2 = d = \gamma_2 \to 0$, Eq. (37) yields the expressions for thermoelastic material with voids.

Numerical results and discussion

The material chosen for the purpose of numerical computation is copper, whose physical data is given by Sherief et al.,[41] as,

$$\lambda = 7.76\times10^{10}\, Nm^{-2}, C^* = 3.831\times10^3\, m^2 s^{-2} K^{-1}, \mu = 3.86\times10^{10}\, Nm^{-2}, K^* = 3.86\times10^3\, Ns^{-1}K^{-1},$$

$$\text{ù} = 1\times10^{11}\, s^{-1}, T_0 = 0.293\times10^3\, K, \alpha_t = 1.78\times10^{-5}\, K^{-1}, t = 0.1s, \rho = 8.954\times10^3\, Kgm^{-3}$$

Following Khalili, [42] the double porous parameters are taken as,

$$\alpha_2 = 2.4\times10^{10}\, Nm^{-2}, \alpha_3 = 2.5\times10^{10}\, Nm^{-2}, \gamma = 1.1\times10^{-5}\, N, \alpha = 1.3\times10^{-5}\, N, \gamma_1 = 0.16\times10^5\, Nm^{-2},$$

$$b_1 = 0.12\times10^{-5}\, N, d = 0.1\times10^{10}\, Nm^{-2}, \gamma_2 = 0.219\times10^5\, Nm^{-2}, \kappa_1 = 0.1456\times10^{-12}\, Nm^{-2}s^2,$$

$$b = 0.9\times10^{10}\, Nm^{-2}, \alpha_1 = 2.3\times10^{10}\, Nm^{-2}, \kappa_2 = 0.1546\times10^{-12}\, Nm^{-2}s^2$$

Figure1–3 depicts the variation of determinant of Rayleigh wave secular equation, Rayleigh wave velocity and Attenuation coefficient w.r.t ξ for different values of c. In all these figs. solid line, small dashes lines and big dashes line correspond to the value of $c = 0.1, 0.12 \text{ and } 0.13$ respectively. From Figure 1, it is noticed that determinant of Rayleigh wave secular equation is equal to zero for the region $0 \le \xi < 0.01$,then it slightly increases and decreases for the region $0.01 \le \xi < 0.022$, again becomes almost zero for the region $0.022 \le \xi < 0.043$ and then increase for the remaining region as ξ increases. It is also evident from the fig. that magnitude of the determinant of Rayleigh wave secular equation increases with the increase in the value of c. Figure 2 shows that Rayleigh wave velocity initially increases and decreases with very small magnitude for the region, become almost stationary near the boundary surface for the region and then increases sharply with ads .It is obvious that magnitude values of Rayleigh wave velocity also increase as value of increases. It is found from Figure 3 that value of attenuation coefficient initially decreases and increases for the region, become almost stationary near the boundary surface for the region and then start to increase sharply as. Also, it is clear that the attenuation coefficient increases monotonically with the increase in the value of.

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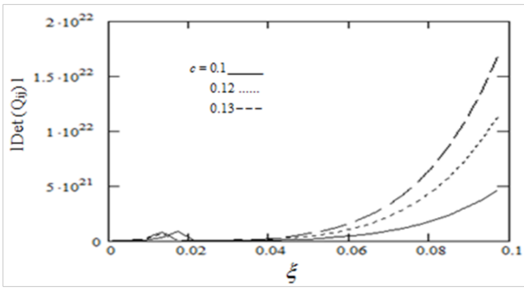

Figure 1 Determinant of Rayleigh waves secular equation with varies values of c with respect to ξ.

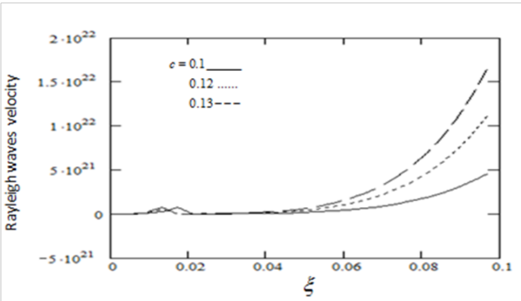

Figure 2 Rayleigh waves velocity with varies values of c with respect to ξ.

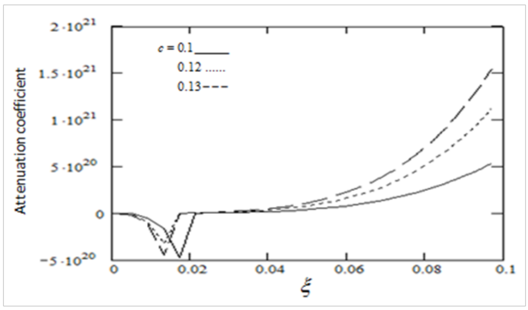

Figure 3Attenuation coefficient with varies values of c with respect to ξ.

Conclusion

In this work, a problem of propagation of Rayleigh waves in thermoelastic material with double porosity structure has been investigated. Secular equations are derived mathematically for the boundary conditions. The values of determinant of Rayleigh wave secular equation, Rayleigh wave velocity and attenuation coefficient with respect to wave number are computed numerically and depicted graphically.

From the theoretical and numerical discussion we can draw the following concluding remarks:

1. It is observed that porosity has a significant effect on the propagation of Rayleigh waves.
2. It is found that values of determinant of Rayleigh wave secular equation, Rayleigh wave velocity and Attenuation coefficient, all have similar trend of variation for all the values of phase velocity.
3. The magnitude of the determinant of Rayleigh wave secular equation, Rayleigh wave velocity and Attenuation coefficient increase with the increase in the value of phase velocity for higher values of wave number.

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Conflict of interest

The author declares there is no conflict of interest.

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