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Elastic instability analysis of biaxially compressed flat rectangular isotropic all-round clamped (CCCC) plates

Abstract

The Galerkin's method was used to carry out the elastic instability analysis of biaxially compressed flat rectangular isotropic all-round clamped plates. The biaxial critical buckling load equation was obtained by substituting the plate deflection equation (obtained via the polynomial series) into the Galerkin's functional. Throughout the analysis, the aspect ratios (defined as the ratio of length, "b" of the plate on the y axis to the length, "a" of plate on the x- axis) was considered to range from 1 to 2. A linear relationship was obtained for the buckling load on the y axis in terms of that on the x-axis. Results for the critical buckling load were obtained for the various aspect ratios (1 to 2) and "k" (relationship constant between forces on the Y- axis and forces on the X-axis) values (0.1 to 1). A maximum buckling load coefficient of 108.0006 was obtained for a square plate at a "k" value of 0, while the least buckling load was 40.50021, obtained for a rectangular plate of aspect ratio equal to 2 and a "k" value of 1.0. At k equal to zero and for all aspect ratios, the results of the present study showed a maximum percentage difference of 0.69389 with those given by Ibearugbulem et al, which shows that the results for the buckling analysis of biaxially loaded CCCC plates presented in this paper for the given aspect ratios and "k" values are very accurate.

Keywords: elastic instability analysis, thin plates, biaxial forces, galerkin's method, boundary conditions

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David Ogbonna Onwuka, Stanley Emeka Iwuoha
Department of Civil Engineering, Federal University of Technology Owerri, Nigeria

Correspondence: Stanley Emeka Iwuoha, Department of Civil Engineering, Federal University of Technology Owerri, Nigeria, Email iwuohastanley1@gmail.com

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Abbreviations: A, coefficient of deflection of the plate; a, length of the plate; b, width of the plate; W, deflection equation of the plate; H, shape function of the plate; D, flexural rigidity of the plate; α - aspect ratio=b/a; N_x, load applied in the x-direction; N_y, load applied in the y-direction; N_{xcr}, critical buckling load in the x-direction; h, thickness of the plate; X, primary axis of the plate; Y, secondary axis of the plate; C, clamped support; R, non-dimensional parameter equal to x/a; Q, non-dimensional parameter equal to y/b; F, buckling load coefficient; k, constant, relating ny and nx; Nxi, the critical buckling load coefficients at k=0.i ; w'^R, w'^Q –first derivative of the deflection equation with respect to r and q respectively

Introduction

Thin rectangular plates are used in the construction of thin walled structures for the transmission of both in-plane and lateral loads. The aeronautic and marine industries make particular use of such materials as thin plates. Thin plates had been defined by Szilard[1] as one whose ratio of its basic dimension to its thickness falls within the range 8…10≤ a/h ≤ 80…100. Due to the importance and wide application of this structural material, several researches had been carried out with the aim of maximizing its potentials for wider structural applications. Areas of research of plate analysis include the vibration of plates, bending of plates and the buckling of plates. Buckling is the phenomenon in which a material under the action of in-plane compressive loads, begins to move from the state of stable equilibrium to a state of unstable equilibrium at a critical value of the compressive loads even when transverse loads are not applied. According to Ventsel & Krauthermmer[2] failure of thin plate elements may be attributed to an elastic instability and not to the lack of their strength" Therefore, determination of the critical buckling loads of a plate, is essential to safe design of a plate for the intended use of the plate for any Engineering purpose within the safe load. Several works on the buckling analysis of plates had been done in the past. Makhtar et al.,[3] used a first order shear deformation theory to carry out the thermal buckling analysis of simply supported functionally graded plate and showed that when the plate aspect ratio, $\frac{a}{b}$ is decreased, the critical temperature reduces and the plate becomes thinner. Chajes[4] showed that the buckling load of a plate simply supported all round and uniformly compressed in one direction is given by Equation 1

$$N_x = \frac{D\pi^2}{b^2}\left(\frac{mb}{a}+\frac{n^2 a}{mb}\right)^2 \quad (1)$$

While for a plate fully clamped on all sides and uniaxially loaded, the buckling load is given by Equation 2.

$$N_x = \frac{10.67\pi^2 D}{a^2} \quad (2)$$

Singh & Chakrabarti[5] developed an efficient C°FE model based on higher zigzag theory for the buckling analysis of a uniaxially loaded simply supported cross ply square plate. Jayashankarbabu et al.[6] used the finite element method to obtain the elastic buckling load factor for square plates of different boundary conditions (such as, SCSC, CCCC, SSSS) containing square and circular cutouts, subjected to uniaxial compression, with the loads applied at the simply supported and at the clamped edges. Yao et al.,[7] proposed a new method which do not require the global stiffness matrix of the system but, reduces the system matrix order and improves the computational efficiency for analyzing plates which are simply supported on all edges. Ezeh et al.,[8] proposed shape functions based on the characteristic orthogonal polynomial and used them in the Galerkin's indirect variational principle for carrying out the elastic buckling analysis of a thin plate clamped at all edges, and subjected to axial load in the x-direction. Ventsel & Krauthermmer,[2] Iyengar[9] & Chajes,[4] individually, demonstrated that for a biaxially loaded square SSSS plates subject to uniform pressure on both sides, the critical load Ncr, is given by Equation 3.

$$N_{cr} = \frac{2\pi^2 D}{a^2} \qquad \text{Eqn 3}$$

Ibearugbulem et al.,[10] derived a polynomial shape function and used it in the Ritz method to carry out the buckling analysis of plates with boundary conditions (such as the SSSS, CCCC, CSSS, CCSS, CSCS, and the CCCS).From available literature, it will be discovered that works on the buckling analysis of plates had revolved mostly around uniaxially loaded, and square simply supported biaxially loaded plates subject to uniform pressure. To the best of our knowledge, there is extreme dearth of literature on the buckling analysis of all-round clamped thin isotropic square and rectangular plates subject to biaxial loading. The objective of this work, is to fill the gap in literature, by providing solutions to the buckling analysis of all-round clamped thin isotropic rectangular plates subject to biaxial loading (with unequal forces in the both axes) by using the polynomial shape function proposed by Ibearugbulem[11] in the Galerkin's work method to derive the equation for buckling of plates.

Material and method

The method of solution is detailed as presented in the following stages below.

Formulation of the equation of buckling of biaxially compressed thin rectangular isotropic plates

Consider a fully clamped flat isotropic plate under the action of biaxial compressive in-plane loads as shown in Figure 1. Let the thickness of the plate in the z- direction be far smaller than both the length and width of the plate in the x-and y-directions.

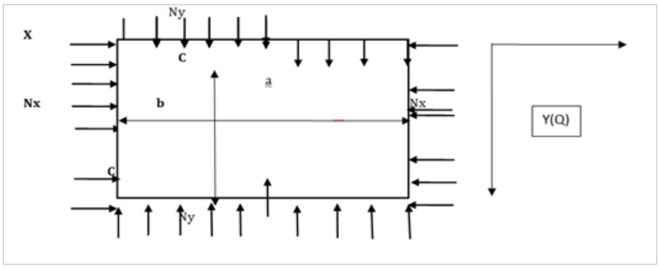

Figure 1 A CCCC Plate Under-going Biaxial Compression.

The overall governing differential equations for plates, is given by Ibearugbulem et al.[10] as, Equation 4

$$q - N_x\left(\frac{d^2w}{dx^2}\right) - 2N_{xy}\left(\frac{\partial^2 w}{\partial x\partial y}\right) - N_y\left(\frac{d^2w}{dy^2}\right) + m\lambda^2 w = D\left[\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2\partial y^2} + \frac{\partial^4 w}{\partial y^4}\right]$$

Where W = AH (5)

For biaxial buckling $q = Nxy = m?^2\ w = 0$, hence, Ventsel and Krauthammer (2001) gave the buckling equation as; Equation (6)

$$-N_x\left(\frac{\partial^2 w}{\partial x^2}\right) - N_y\left(\frac{\partial^2 w}{\partial y^2}\right) = D\left[\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2\partial y^2} + \frac{\partial^4 w}{\partial y^4}\right]$$

The Equation (6), is the equation of forces acting on the biaxially loaded plate. These forces (both internal and external) acting on the plate, together have the tendency to cause the plate to be deformed. If "w" is the average deformation caused on the plate by the forces, then the work done by the forces on the plates is as given by Equation (7)

$$-N_x\left(\frac{\partial^2 w}{\partial x^2}\right)w - N_y\left(\frac{\partial^2 w}{\partial y^2}\right)w = D\left[\frac{\partial^2 w}{\partial x^4}.w + 2w.\frac{\partial^4 w}{\partial x^2\partial y^2} + \frac{\partial^4 w}{\partial y^4}.w\right]$$

The Equation (7) being Galerkin's expression for the biaxial buckling of plates at any arbitrary point, was obtained by multiplying Equation (5) by the average deformation "w" of the plate.

The entire work done on the plate, obtained by integrating Equation (7) completely along the x-and y-axes, is given by Equation (8).

$$-N_x\iint\left(\frac{\partial^2 w}{\partial x^2}\right).w\partial x\partial y - N_y\iint\left(\frac{\partial^2 w}{\partial y^2}\right).w\partial x\partial y = D\iint\left[\frac{\partial^4 w}{\partial x^4}.w + 2w\frac{\partial^4 w}{\partial x^2\partial y^2} + \frac{\partial^4 w}{\partial y^4}.w\right]\partial x\partial y$$

For rapid solution of the plate problem, the Cartesian coordinates, are expressed in terms of non-dimensional parameters as;

$$R = \frac{x}{a}\ and Q = \frac{y}{b} \qquad (9)$$

Let the aspect ratio, $\propto$ be given by the Equation (10)

$$\propto = \frac{b}{a}$$

Substituting Equations (5), (9) and (10) into equation (8), and simplifying gives Equation (11)

$$-\frac{N_x}{a^2}\iint\left(\frac{\partial H}{\partial R}\right)^2\partial R\partial Q - \frac{N_y}{a^2\propto^2}\iint\left(\frac{\partial H}{\partial Q}\right)^2\partial R\partial Q$$

$$= \frac{D}{a^4}\iint\left[\left(\frac{\partial^4 H}{\partial R^4}\right)H + \frac{2H}{\propto^2}\left(\frac{\partial^4 H}{\partial R^2\partial Q^2}\right) + \frac{H}{\propto^4}\left(\frac{\partial^4 H}{\partial Q^4}\right)\right]\partial R\partial Q$$

Let the forces in the x- and y- axes of the plates be related by Equation 12.

$$N_y = KN_x$$

Substituting Equation (12) into (11), and multiplying the resulting equation by a^2, yields Equation (13)

$$-N_x\iint\left(\frac{\partial H}{\partial R}\right)^2\partial R\partial Q - \frac{KN_x}{\propto^2}\iint\left(\frac{\partial H}{\partial Q}\right)^2\partial R\partial Q = \frac{D}{a^2}\iint\left[\left(\frac{\partial^4 H}{\partial R^4}\right)H + \frac{2H}{\propto^2}\left(\frac{\partial^4 H}{\partial R^2\partial Q^2}\right) + \frac{H}{\propto^4}\left(\frac{\partial^4 H}{dQ^4}\right)\right]\partial R\partial Q$$

Making N_x the subject of the Equation (13), gives the general equation of buckling of a biaxially compressed thin rectangular isotropic plate as Equation (14)

$$N_x = -\frac{D/a^2\iint\left[\left(\frac{\partial^4 H}{\partial R^4}\right)H + \frac{2H}{\propto^2}\left(\frac{\partial^4 H}{\partial R^2\partial Q^2}\right) + \frac{H}{\propto^4}\left(\frac{\partial^4 H}{\partial Q^4}\right)\right]\partial R\partial Q}{\iint\left[\left(\frac{\partial H}{\partial R}\right)^2 + \frac{K}{\propto^2}\left(\frac{\partial H}{\partial Q}\right)^2\right]\partial R\partial Q}$$

Taylor-McLaurin's series formulated deflection function of the CCCC isotropic rectangular plate

Since, for an all-round clamped plate, the deflections and rotations, are zeros at all edges of the plate, the boundary conditions of the SSSS plate is as follows:

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$$w(R=0) = w'^{R}(R=0) = 0 \quad (15)$$

$$w(R=1) = w'^{R}(R=1) = 0 \quad (16)$$

$$w(Q=0) = w'^{Q}(Q=0) = 0 \quad (17)$$

$$w(Q=1) = w'^{Q}(Q=1) = 0 \quad (18)$$

They w'^{R} *and* w'^{Q} are the first derivatives of the deflection function, w, in the R-and Q-directions respectively.

Ibearugbulem[11] assumed the shape function, w, to be continuous and differentiable. He expanded it in Taylor-Mclaurin series and truncated the infinite polynomial series at m=n=4 and got the general polynomial deflection equation of rectangular plates as follows.

$$w = \sum_{m=0}^{4}\sum_{n=0}^{4} a_m b_n R^m . Q^n \quad (19)$$

Where a_m *and* b_n are constants and R *and* Q are as already defined earlier.

Substituting the first and second boundary conditions (i.e Equations (15) and (16)) into Equation (19) and solving the resulting simultaneous equations, yields,

$$a_0 = a_1 = 0,\ a_3 = -2a_4 \text{ and } a_2 = a_4 . \quad (20)$$

In the same way, substituting Equations (17) and Equations (18) into Equation (19), yields,

$$b_0 = b_1 = 0,\ b_3 = -2b_4 \text{ and } b_2 = b_4 . \quad (21)$$

Substituting Equations (20) and (21) into Equation (19), gives the particular deflection equation of an all-round clamped isotropic thin rectangular plate as Equation (22)

$$w = a_4 b_4 \left(R^2 - 2R^3 + R^4\right)\left(Q^2 - 2Q^3 + Q^4\right)$$

Where,

$$A = a_4 b_4$$

$$H = \left(R^2 - 2R^3 + R^4\right)\left(Q^2 - 2Q^3 + Q^4\right)$$

Determination of the critical biaxial buckling loads for CCCC Plates

Differentiating Equation (24) with respect to the R-and Q-axes, gave the following results;

$$\left(\frac{\partial H}{\partial R}\right)^2 = \left(4R^2 - 24R^3 + 52R^4 - 48R^5 + 16R^6\right) * \left(Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8\right)$$

$$\left(\frac{\partial^4 H}{\partial R^4}\right) H = 24\left(Q^2 - 2Q^3 + Q^4\right)$$

$$\left(\frac{\partial^4 H}{\partial Q^4}\right) H = 24\left(R^2 - 2R^3 + R^4\right)$$

$$\left(\frac{\partial^4 H}{\partial R^2 \partial Q^2}\right) H = \left(2R^2 - 16R^3 + 38R^4 - 35R^5 + 12R^6\right)\left(2Q^2 - 16Q^3 + 38Q^4 - 35Q^5 + 12Q^6\right)$$

Integrating the derivatives of Equations (25) to (2.9) with respect to R and Q from 0-1, yields the results given as Equations (30) to (34):

$$\int_0^1\int_0^1 \left(\frac{\partial H}{\partial R}\right)^2 \partial R \partial Q = \left[\frac{4}{3} - \frac{24}{4} + \frac{52}{5} - \frac{48}{6} + \frac{16}{7}\right] * \left[\frac{1}{5} - \frac{4}{6} + \frac{6}{7} - \frac{4}{8} + \frac{1}{9}\right] = 3.023431587 * 10^{-5}$$

$$\int_0^1\int_0^1 \left(\frac{\partial H}{\partial Q}\right)^2 \partial R \partial Q = \left[\frac{1}{5} - \frac{4}{6} + \frac{6}{7} - \frac{4}{8} + \frac{1}{9}\right]\left[\frac{4}{3} - \frac{24}{4} + \frac{52}{5} - \frac{48}{6} + \frac{16}{7}\right] = 3.023431587 * ?10?^{(-5)}$$

$$\int_0^1\int_0^1 \left(\frac{\partial^4 H}{\partial R^4}\right) H \partial R \partial Q = \left[24\left(\frac{1}{3} - \frac{2}{4} + \frac{1}{5}\right)\left(\frac{1}{5} - \frac{4}{6} + \frac{6}{7} - \frac{4}{8} + \frac{1}{9}\right)\right] = 1.269841257 * ?10?^{(-3)}$$

$$\int_0^1\int_0^1 \left(\frac{\partial^4 H}{\partial Q^4}\right) H \partial R \partial Q = \left[24\left(\frac{1}{5} - \frac{4}{6} + \frac{6}{7} - \frac{4}{8} + \frac{1}{9}\right)\left(\frac{1}{3} - \frac{2}{4} + \frac{1}{5}\right)\right] = 1.269841257 * ?10?^{(-3)}$$

$$\int_0^1\int_0^1 \left(\frac{\partial^4 H}{\partial R^2 \partial Q^2}\right) H \partial R \partial Q = \left[\frac{2}{3} - \frac{16}{4} + \frac{38}{5} - \frac{36}{6} + \frac{12}{7}\right]^2 = 3.628117911 * ?10?^{(-4)}$$

Substituting the numerical values obtained from Equations (30) – (34) into Equation (14), gave

$$N_x = -\frac{\frac{D}{a^2}\left[1.26984125*10^{-3} + \frac{2}{\propto^2}\left(3.628117911*10^{-4}\right) + \frac{1}{\propto^4}\left(1.269841257*10^{-3}\right)\right]}{\left[3.023431587*10^{-5} + \frac{k}{\propto^2}\left(3.02341587*10^{-5}\right)\right]}$$

The Equation (35) is the expression for the critical buckling load of a biaxially loaded CCCC plate. While the Equation (36)

$$F = -\frac{\left[1.26984125*10^{-3} + \frac{2}{\propto^2}\left(3.628117911*10^{-4}\right) + \frac{1}{\propto^4}\left(1.269841257*10^{-3}\right)\right]}{\left[3.023431587*10^{-5} + \frac{k}{\propto^2}\left(3.02341587*10^{-5}\right)\right]}$$

Is the expression for the biaxial coefficient of an all-round CCCC plate.

Results and discussion

The values of the plate aspect ratios (1-2) and the constant "k" varying from 0-1, at intervals of 0.1, were substituted into Equation (36). The results of the critical buckling load coefficients for a biaxially compressed all-round clamped (CCCC) isotropic thin rectangular plate were obtained and presented in Table 1.

The results presented in Table 1 were plotted as shown on Figure 2. From Figure 2 and Table 1, it could be observed that, as the aspect ratios increase from one to two, the buckling load coefficients of the CCCC plates reduces. This is because as the aspect ratio increases, the plate begins to behave as a slender column and hence loses its ability to resist load in the y- direction, which causes it to buckle faster; hence, the reduction in the buckling load. It is also observed that as the forces in the y-direction increased (i.e. as "k" increases), the buckling coefficient reduces, this is because as the loads in the y-direction of the plate, increased (with loads applied on the x-axis), the plate becomes weaker and less resistant to applied loads and hence, it buckles faster.

Table 1 Critical Buckling Load Coefficients for biaxially loaded CCCC Plates

Aspect Ratio	CRITICAL BUCKLING LOAD COEFFICIENTS (Nxi)										
	N_x	N_{x1}	N_{x2}	N_{x3}	N_{x4}	N_{x5}	N_{x6}	N_{x7}	N_{x8}	N_{x9}	N_{x10}
1	108.0006	98.18233	90.00047	83.07735	77.14326	72.00037	67.50035	63.52974	60.00031	56.8424	54.00028
1.1	90.52175	83.61169	77.68178	72.53729	68.03187	64.0534	60.51454	57.34624	54.49319	51.91057	49.56168
1.2	78.92171	73.79692	69.29711	65.31452	61.76481	58.58106	55.70944	53.10619	50.73538	48.5672	46.57674
1.3	70.90692	66.94564	63.40354	60.21743	57.33622	54.71812	52.32869	50.1392	48.12558	46.26745	44.54747
1.4	65.17818	62.01419	59.14316	56.52621	54.13103	51.93058	49.90204	48.02603	46.28595	44.66756	43.15853
1.5	60.96328	58.3691	55.98669	53.79113	51.76128	49.87905	48.1289	46.49742	44.97291	43.5452	42.20535
1.6	57.78399	55.61166	53.59675	51.72273	49.97534	48.34216	46.81235	45.37639	44.0259	42.75347	41.55253
1.7	55.33346	53.48284	51.75201	50.12969	48.60599	47.17218	45.82055	44.54421	43.33705	42.19359	41.10892
1.8	53.4086	51.80954	50.30345	48.88245	47.53952	46.26841	45.06351	43.91976	42.83264	41.79803	40.81223
1.9	51.87128	50.47313	49.14838	47.89139	46.69709	45.56091	44.4787	43.44671	42.46152	41.52003	40.61938
2	50.62526	49.3905	48.21454	47.09327	46.02297	45.00023	44.02197	43.08533	42.18772	41.32675	40.50021

Where $N_{(xi)}$ $(i=1,2,3\ldots10)$ are the critical buckling load coefficients at $k = 0.i$.

(where k=Constant, relating N_y and N_{xi}: and $N_{(xi)}$ =The Critical buckling load coefficients at various fractions of k)

Table 2 Comparison of the Biaxial Buckling Load Coefficients Obtained in this work for CCCC Rectangular Plates under Uniform Unilateral Pressure (i.e. at K= 0), with those of Ibearugbulem et al.[8]

Biaxial buckling load coefficient for CCCC plate			
Aspect Ratios	**Ibearugbulem et al.[10]**	**Present Study**	**Percentage Difference**
1	108.667	108.001	0.61298
1.1	91.082	90.5218	0.61511
1.2	79.415	78.9217	0.62117
1.3	71.3565	70.9069	0.63005
1.4	65.5979	65.1782	0.63984
1.5	61.3621	60.9633	0.64995
1.6	58.167	57.784	0.65847
1.7	55.706	55.3335	0.66876
1.8	53.773	53.4086	0.67766
1.9	52.229	51.8713	0.68491
2	50.979	50.6253	0.69389

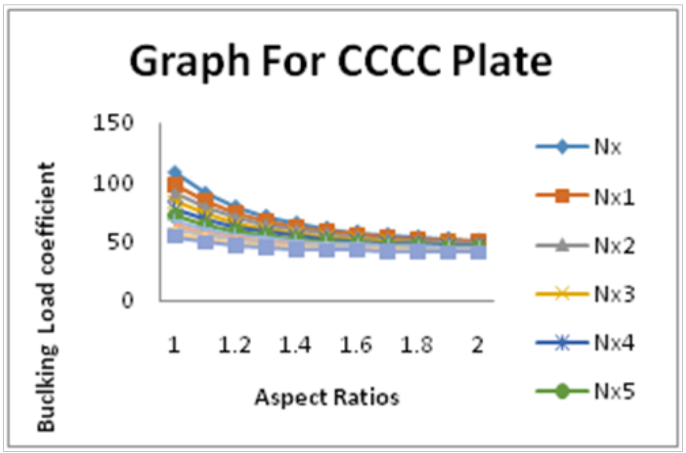

Figure 2 Graph of Critical Buckling Load Coefficients Versus Aspect Ratios of a Biaxially Loaded all round clamped Plate.

The results of the present study were compared with that obtained by Ibearugbulem[11] at k=0 (uniaxial buckling only, of CCCC plates) as presented on Table 2. From the Table 2, it is seen that, the results of this present work agrees very closely with established results of uniaxially loaded CCCC plates subject to uniform pressure along the x-axis. (i.e. at k=0), for different aspect ratios. This therefore, validates the results of the critical buckling load coefficients for the other k values (presented on Table1) for which there are no existing results in literature.[12]

Conclusion

From the study, the following conclusions have been, drawn: The equation for the determination of the critical buckling loads for a biaxially loaded all- round clamped thin rectangular isotropic plate, for all aspect ratios and k-values, has been derived in this work. The critical buckling load coefficients for a biaxially loaded thin rectangular isotropic plates, have been determined for different aspect ratios and k-values. Given that the results obtained from this work agrees with the results of Ibearugbulem[11] at k=0, it therefore follows that, the results obtained in this work for other k values (for which there are no other existing results to compare with, in literature), are also correct.

The polynomial shape function used in this work (based on the Tailor-Mclaurin series) can be said to have accurately defined the plate's deformed shape, given the high accuracy of the buckling load coefficients obtained. Given the difficulty in the use of the trigonometric shape functions to determine the critical buckling load coefficients of an all-round clamped thin rectangular isotropic plate, the use of the equations and tables developed in this work is recommended for a quick and easy analysis of the buckling loads of such plates, for other aspect ratios which are not covered in this work.

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None.

Conflict of interest

The author declares no conflict of interest.

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