Review Article
Open Access

A one-step extended block hybrid formulae for solving orbital problems

Abstract

A One-step Extended Block Hybrid Formulae (OEBHF) for solving Orbital problems is presented. The processes compute the solutions of Orbital Problems in a block by block fashion by some discrete schemes obtained from the associated continuous schemes and its first derivatives which are combined and implemented as a set of block formulae. The analysis of the method was examined, it was found to be zero stable, consistence, convergent and A-stable. The order, error constants and the region of absolute stability was also investigated, MATLAB package was used to plot the region of stability within which the method is stable. Numerical results revealed this method to be efficient and very accurate, and especially suitable for orbital problems.

Keywords: A-atable, extended block hybrid formulae, one-step, orbital problems, zero stability

Volume 3 Issue 1 - 2019

Momoh Adelegan Lukman,[1] Omole Ezekiel Olaoluwa[2]

[1]Department of Mathematical Sciences, Federal University of Technology, Akure, Ondo State, Nigeria

[2]Department of Mathematics & Statistics, Joseph Ayo Babalola University, Ikeji Arakeji, Osun State, Nigeria

Correspondence: Omole Ezekiel Olaoluwa, Department of Mathematics & Statistics, Joseph Ayo Babalola University, Ikeji Arakeji, Osun State, Nigeria, Email eoomale@jabu.edu.ng

Received: February 11, 2019 | **Published:** April 29, 2019

Introduction

In modern work of engineering, physics, applied mathematics and science, second order equations arise very frequently. To date, these equations have been extensively studied and books have been composed along the mathematical methods available for such equations. In recent times, the integration of Ordinary Differential Equations (ODEs) is investigated using some kind of block methods. This research discusses the formation of One-step Extended Block Hybrid Formulae (OEBHF) for solving ordinary differential equations which is in turn possesses Orbital problems in nature. This kind of problem often arise frequently in the area of science and engineering especially mechanical system, control theory and celestial mechanics. The equation that defines this type of the problem is

$$y''=f(x,y,y'),\quad y(a)=y_0,\quad y'(a)=\eta_0,\quad x\in[a,b] \tag{1}$$

Many scholars have been developed numerical methods for solving equation of the form (1) using different techniques See.[1–17] These techniques have been introduced in literatures such as[1,4,5–12] and others. Most real-life problems that arise in various fields of study be it engineering or science are modeled as mathematically before they are solved. These models often lead to differential equations. Numerous problems such as chemical kinetics, orbital dynamics, Orbital problem, circuit and control theory and Newton's second law applications involve second-order ODEs as discussed by.[13,15,17] Ordinary differential equations (ODEs) are commonly used for mathematical modeling in many diverse fields such as engineering, operation research, industrial mathematics, behavioral sciences, artificial intelligence, management and sociology.[7,9,10] This mathematical modeling is the art of translating problem from an application area into tractable mathematical formulations whose theoretical and numerical analysis provides insight, answers and guidance useful for the originating application.[13,14] This type of problem can be formulated either in terms of first-order or higher order ODEs. Here, the ordinary differential equations are being formulated to an Orbital Problem as studies by.[13,15–17] This research work is motivated by the need to address some setbacks associating with existing method, the new method is capable of solving orbital problems efficiently and accurately. The results is also in conformity.

This research is organized as follows: in the coming section, we carried out the derivation of the method, where we considered the newly developed method through the approach of interpolation and collocation. The details of the analysis of the method were discussed in Section three. In fourth section, some numerical problems were solved, the analytical solution and exact solution in terms of accuracy and performance of the developed method were compared. Finally, the conclusion was drawn in the fifth section.

Theoretical formulation of the (OEBHF)

We considered power series of the form

$$y(x)=\sum_{j=0}^{r+s-1} w_j x^j \tag{2}$$

as an approximate solution to (1) where a_j's are parameters to be determined and $k=1$ which is the step length. The first and second derivatives of (2) are

$$y'(x)=\sum_{j=1}^{r+s-1} j w_j x^{j-1}$$

$$A=[w_0,w_1,w_2,w_3,w_4,w_5,w_6,w_7]^T$$
$$x''+x=\varpi\cos(\psi t),\ x(0)=1,\ x'(0)=0$$
$$h=\tfrac{1}{n}$$

$$y'(x)=\sum_{j=1}^{r+s-1} j w_j x^{j-1} \tag{3}$$

$$y''(x)=\sum_{j=2}^{r+s-1} j(j-1) w_j x^{j-2} \tag{4}$$

Equating equations (4) and (2) yields

$$\sum_{j=2}^{r+s-1} j(j-3)(j-2)(j-1) w_j x^{j-4}=f(x,y,y') \tag{5}$$

Here, a step-length of $k=1$ with constant step size of "a". Interpolating (3) at $x=x_{n+a},x_{n+b}$ and collocating (4) at

$x = x_n, x_{n+a}, x_{n+b}, x_{n+c}, x_{n+d}, x_{n+1}$ gives system of non-linear equations of the form

$$AX = B \quad (6)$$

Where $A = [w_0, w_1, w_2, w_3, w_4, w_5, w_6, w_7]^T$

$$B = \left[y_{n+a}, y_{n+b}, f_n, f_{n+a}, f_{n+b}, f_{n+c}, f_{n+d}, f_{n+1}\right]^T$$

$$X = \begin{bmatrix} (xn+a)^0 & (xn+a)^1 & (xn+a)^2 & (xn+a)^3 & (xn+a)^4 & (xn+a)^5 & (xn+a)^6 & (xn+a)^7 \\ (xn+b)^0 & (xn+b)^1 & (xn+b)^2 & (xn+b)^3 & (xn+b)^4 & (xn+b)^5 & (xn+b)^6 & (xn+b)^7 \\ 0 & 0 & 2(xn)^0 & 6(xn)^1 & 12(xn)^2 & 20(xn)^3 & 30(xn)^4 & 42(xn)^5 \\ 0 & 0 & 2(xn+a)^0 & 6(xn+a)^1 & 12(xn+a)^2 & 20(xn+a)^3 & 30(xn+a)^4 & 42(xn+a)^5 \\ 0 & 0 & 2(xn+b)^0 & 6(xn+b)^1 & 12(xn+b)^2 & 20(xn+b)^3 & 30(xn+b)^4 & 42(xn+b)^5 \\ 0 & 0 & 2(xn+c)^0 & 6(xn+c)^1 & 12(xn+c)^2 & 20(xn+c)^3 & 30(xn+c)^4 & 42(xn+c)^5 \\ 0 & 0 & 2(xn+d)^0 & 6(xn+d)^1 & 12(xn+d)^2 & 20(xn+d)^3 & 30(xn+d)^4 & 42(xn+d)^5 \\ 0 & 0 & 2(xn+1)^0 & 6(xn+1)^1 & 12(xn+1)^2 & 20(xn+1)^3 & 30(xn+1)^4 & 42(xn+1)^5 \end{bmatrix}$$

Solving (6) for the $w_i's$ using Gaussian elimination method or Crammer's rule and substitute the values into (2), gives a continuous hybrid computation method of the form;

$$y(x) = \alpha(q)_a y_{n+a} + \alpha(q)_b y_{n+b} + \mathrm{h}^2\left(\sum_{i=0}^{k} \sigma_i(q) f_{n+i} + \sigma_k(q) f_{n+k}\right), i = 0, a, b, c, d, \quad k = 1 \quad (7)$$

The coefficient of $y_{n+i}, i = a, b, c, d, 1$ and $f_{n+i}, i = 0(a)1$

$$\left.\begin{aligned}
\alpha(q)_a &= (2-5q) \\
\alpha(q)_b &= (5q-1) \\
\sigma(q)_0 &= \left(\frac{-h^2}{6300000}\right)\left((5q-1)(5q-2)(93750q^5 - 372500q^4 + 574875q^3 - 422400q^2 + 138070q - 9366)\right) \\
\sigma(q)_a &= \left(\frac{h^2}{176400}\right)\left((5q-1)(5q-2)(93750q^5 - 346250q^4 + 469875q^3 - 259125q^2 + 16935q + 30891)\right) \\
\sigma(q)_b &= \left(\frac{81h^2}{2450000}\right)\left((5q-1)(5q-2)(31250q^5 - 95000q^4 + 95375q^3 - 28800q^2 - 3910q - 42)\right) \\
\sigma(q)_c &= \left(\frac{-h^2}{25200}\right)\left((5q-1)(5q-2)(18750q^5 - 58750q^4 + 61425q^3 - 19695q^2 - 2731q - 63)\right) \\
\sigma(q)_d &= \left(\frac{-h^2}{252000}\right)\left((5q-1)(5q-2)(93750q^5 - 267500q^4 + 249375q^3 - 69600q^2 - 9210q + 42)\right) \\
\sigma(q)_1 &= \left(\frac{h^2}{2100000}\right)\left((5q-1)(5q-2)(93750q^5 - 241250q^4 + 207375q^3 - 54025q^2 - 7005q + 119)\right)
\end{aligned}\right\}$$

where

$$q = \frac{x - x_n}{h}, \quad \frac{dq}{dx} = \frac{1}{h}, \quad y_{n+i} = y(x_n + ih), \quad f_{n+i} = f(x_n + ih), \quad y'(x_n + ih) \quad (8)$$

Citation: Lukman MA, Olaoluwa OE. A one-step extended block hybrid formulae for solving orbital problems. *MOJ App Bio Biomech.* 2019;3(1):37–43. DOI: 10.15406/mojabb.2019.03.00101

Solving (7) for the independent solution at the grid and off grip points give the continuous block method.

$$y(x)=\sum_{i=0}^{k}\frac{(ih)^n}{n!}(y_n)^n+h^2\left(\sum_{i=0}^{k}\tau_i(q)f_{n+i}+\tau_k(q)f_{n+k}\right), i=0,a,b,c,d,\ k=1 \quad (9)$$

For simplicity, we set $a=\frac{1}{5},\ b=\frac{2}{5},\ c=\frac{3}{5},\ d=\frac{4}{5}$

The coefficient of f_{n+i} and f_{n+k} give;

$$\left.\begin{aligned}
\tau(q)_0 &= \frac{-1}{1260000}\left(3281250q^6-12862500q^5+20146875q^4-15942500q^3+6562500q^2-1260000q+83326\right)\\
\tau(q)_a &= \frac{1}{352800}\left(3281250q^6-12075000q^5+17128125q^4-11375000q^3+3150000q^2-85899)\right)\\
\tau(q)_b &= \frac{81}{490000}\left(1093750q^6-3412500q^5+3871875q^4-1872500q^3+315000q^2-1438)\right)\\
\tau(q)_c &= \frac{-1}{25200}\left(3281250q^6-10500000q^5+12271875q^4-6125000q^3+1050000q^2-4517)\right)\\
\tau(q)_d &= \frac{-1}{10080}\left(656250q^6-1942500q^5+2086875q^4-962500q^3+157500q^2-762)\right)\\
\tau(q)_1 &= \frac{1}{1420000}\left(3281250q^6-8925000q^5+8990625q^4-3955000q^3+630000q^2-3159)\right)
\end{aligned}\right\} \quad (10)$$

Evaluating (9) at $q=0(a)1$ give a discrete block formula of the form

$$A^0Y_m^{(i)}=\sum_{i=0}^{k}h^ie_iy_n^{(i)}+h^2d_if(y_n)+h^2b_if(y_m), i=0,1 \quad (11)$$

$$y_m=\begin{bmatrix}y_{n+a}\\y_{n+b}\\y_{n+c}\\y_{n+d}\\y_{n+1}\end{bmatrix}, f(y_m)=\begin{bmatrix}f_{n+a}\\f_{n+b}\\f_{n+c}\\f_{n+d}\\f_{n+1}\end{bmatrix}, y_n^i=\begin{bmatrix}y_{n+1}^{(i)}\\y_{n+d}^{(i)}\\y_{n+c}^{(i)}\\y_{n+b}^{(i)}\\y_n^{(i)}\end{bmatrix}, f(y_n)=\begin{bmatrix}f_{n-1}\\f_{n-d}\\f_{n-c}\\f_{n-b}\\f_n\end{bmatrix}, A^0=6*6$$

When $i=0$

$$e_0=\begin{bmatrix}0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\end{bmatrix}, e_1=\begin{bmatrix}0&0&0&0&a\\0&0&0&0&b\\0&0&0&0&c\\0&0&0&0&d\\0&0&0&0&1\end{bmatrix}, d_0=\begin{bmatrix}0&0&0&0&\frac{32297}{3150000}\\0&0&0&0&\frac{1849}{78750}\\0&0&0&0&\frac{771}{21875}\\0&0&0&0&\frac{1312}{7875}\\0&0&0&0&\frac{59}{1008}\end{bmatrix}, b_0=\begin{bmatrix}\frac{8039}{588000}&\frac{61641}{12250000}&\frac{-5147}{126000}&\frac{-207}{14000}&\frac{2921}{2100000}\\\frac{382}{6125}&\frac{2997}{30625}&\frac{-604}{7875}&\frac{-157}{5250}&\frac{38}{13125}\\\frac{24363}{196000}&\frac{6561}{612500}&\frac{291}{14000}&\frac{-9}{700}&\frac{1269}{700000}\\\frac{688}{3675}&\frac{-14256}{153125}&\frac{1312}{7875}&\frac{32}{2625}&\frac{16}{65625}\\\frac{1175}{4704}&\frac{-81}{392}&\frac{325}{1008}&\frac{25}{336}&\frac{1}{672}\end{bmatrix}$$

When $i=1$

$$e_1=\begin{bmatrix}0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\end{bmatrix}, d_1=\begin{bmatrix}0&0&0&0&\frac{12673}{180000}\\0&0&0&0&\frac{343}{5625}\\0&0&0&0&\frac{1161}{20000}\\0&0&0&0&\frac{326}{5625}\\0&0&0&0&\frac{17}{288}\end{bmatrix}, b_1=\begin{bmatrix}\frac{1191}{7200}&\frac{4617}{10000}&\frac{-271}{720}&\frac{-971}{7200}&\frac{757}{60000}\\\frac{133}{450}&\frac{-81}{625}&\frac{7}{45}&\frac{4}{225}&\frac{-1}{3750}\\\frac{1761}{5600}&\frac{-41553}{70000}&\frac{57}{80}&\frac{93}{800}&\frac{-153}{20000}\\\frac{496}{1575}&\frac{-1944}{4375}&\frac{32}{45}&\frac{38}{225}&\frac{-16}{1875}\\\frac{625}{2016}&\frac{-81}{112}&\frac{125}{144}&\frac{125}{288}&\frac{5}{96}\end{bmatrix}$$

Convergence analysis of (OEBHF)

Order and error Constants of the (OEBHF)

According to[1,4,6,9] the order of the new method in Equation (11) is obtained by using the Taylor series and it is found that the developed method has uniformly order six, with an error constants vector of:

$$C_8=\left[-2.7937\times10^{-9},\ -1.4550\times10^{-7},\ -1.0836\times10^{-7},\ -8.0571\times10^{-8},\ -5.1471\times10^{-8}\right]^T$$

Consistency

Definition 3.1: The hybrid block method (7) is said to be consistent if it has an order more than or equal to one i.e. $P\geq1$. Therefore, the method is consistent.[1,4,6]

Zero Stability

Definition 3.2: The hybrid block method (7) said to be zero stable if the first characteristic polynomial $\pi(r)$ having roots such that $|r_z|\leq1$ *and if* $|r_z|=1$, then the multiplicity of r_z must not greater than two.[6] In order to find the zero-stability of hybrid block method (7), we only consider the first characteristic polynomial of the method according to definition (3.2) as follows

$$\Pi(r)=\left|r\begin{bmatrix}1&0&0&0&1\\0&1&0&0&1\\0&0&1&0&1\\0&0&0&1&1\\0&0&0&1&1\end{bmatrix}-\begin{bmatrix}0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\end{bmatrix}\right|=r^4(r-1)$$

Which implies $r=0,0,0,0,1$. Hence the method is zero-stable since $|r_z|\leq1$.

Convergence

Theorem (3.1): Consistency and zero stability are sufficient condition for linear multistep method to be convergent. Since the method (7) is consistent and zero stable, it implies the method is convergent for all point.[6]

Regions of absolute stability (RAS)

The absolute stability region of the new method is found according to[1,4,6,8,12,16] and is shown as

Numerical implementation of the (OEBHF)

In this section, the efficiency and the performance the new One-Step Extended Block Hybrid Formulae (OEBHF) is tested on three test problems. We present

A problem by franco and palacios

We consider the almost periodic problem studied by Simos (2003)

$$z''+z=\varpi e^{i\psi t}\quad z(0)=1,\ z'(0)=i,\ z\in C$$

With equivalent from

$$x''+x=\varpi\cos(\psi t),\ x(0)=1,\ x'(0)=0$$

$$y''+y=\varpi\sin(\psi t),\ y(0)=0,\ y'(0)=1$$

where $\varpi=0.001$ and $\psi=0.001$.

Analytical solution of this problem is given as

$$z(t)=x(t)+iy(t),\quad x,\ y\in\Re$$

$$x(t)=\frac{1-\varpi-\psi^2}{1-\psi^2}\cos(t)+\frac{\varpi}{1-\psi^2}\cos(\psi t),$$

$$y(t)=\frac{1-\varpi\psi-\psi^2}{1-\psi^2}\sin(t)+\frac{\varpi}{1-\psi^2}\sin(\psi t),$$

A problem by stiefel and bettis

The second almost periodic orbital problem earlier studied by Stiefel and Bettis (1969) and later by Simos (1998, 2003), Vigo-Anguiar and Simos (2001)).

$$z''+z=e^{it}\quad z(0)=1,\ z'(0)=0.9995i,\ z\in C$$

With equivalent from

$$x''+x=0.001\cos(t)\quad x(0)=1,\ x'(0)=0$$

$$y''+y=0.001\sin(t)\quad y(0)=0,\ y'(0)=0.9995$$

The theoretical solution is

$$z(t)=x(t)+iy(t),\quad x,\ y\in\Re$$

$$x(t)=\cos(t)+0.0005t\sin(t),$$

$$y(t)=\sin(t)+0.0005t\cos(t).$$

A two body problem

Consider the two-body system of coupled differential equations; See Vigo-Anguiar and Simos (2001)

$$x''=-\frac{x}{r}\ x(0)=1,\ x'(0)=0$$

$$y''=-\frac{y}{r}\ y(0)=0,\ y'(0)=1$$

where $r=\sqrt[3]{x^2+y^2}$ and whose analytical solution is given by, $x(t)=\cos(t),\ y(t)=\sin(t),$

Results of the computation

In this section, computational results are presented.

Conclusions

In this section, the three problems studied in,[13,15–17] have been re-examined for any step size in the interval [0,1] using the One-step Extended Block Hybrid Formulae (OEBHF) of uniform order six. Analysis of the basic properties of our method shows that it is consistent, convergent, and zero-stable and the stability region shows that it is A-Stable as presented in Figure 1. Tables 1-3 present the exact solution, computed solution and the point global error. The numerical results obtained with the step sizes equal to $h=\frac{1}{n}$ for several values of n in the interval of integration was compared with the analytical solution. The Results shows that the method is efficient, reliable and the exact solution and computed solution are in good conformity.

Citation: Lukman MA, Olaoluwa OE. A one-step extended block hybrid formulae for solving orbital problems. *MOJ App Bio Biomech.* 2019;3(1):37–43. DOI: 10.15406/mojabb.2019.03.00101

Table 1 Computational result of franco and palacios problem $n=10$

x	$y(x_n)$	y_n	$\lvert y(x_n)-y_n\rvert$
0.1	0.99500916111233261815	0.99500916112228124920	9.9486 E-12
	0.09983341831266l664958	0.09983341831267l584052	9.9191 E-15
0.2	0.98008651125674828859	0.98008651126654325209	9.7946 E-12
	0.19866934410175101832	0.19866934410177046764	1.9449 E-14
0.3	0.95538115260284425530	0.95538115261238483140	9.5406 E-12
	0.29552025145925409847	0.29552025145928230703	2.8209 E-14
0.4	0.92113993290280530287	0.92113993291199338516	9.1881 E-12
	0.38941844812514550981	0.38941844812518133795	3.5828 E-14
0.5	0.87770497907026097016	0.87770497907900207362	8.7411 E-12
	0.47942574434856406945	0.47942574434860602881	4.1959 E-14
0.6	0.82551027876125672156	0.82551027876946095331	8.2042 E-12
	0.56464282696966911822	0.56464282696971539790	4.6280 E-14
0.7	0.76507734411305203294	0.76507734412063501589	7.5840 E-12
	0.64421824505945024713	0.64421824505949874569	4.8499 E-14
0.8	0.69701000096723394439	0.69701000097411768415	6.8837 E-12
	0.71735691733594771823	0.71735691733599608107	4.8363 E-14
0.9	0.62198835564150627517	0.62198835564761996156	6.1137 E-12
	0.78332807635359288335	0.78332807635363854491	4.5662 E-14
1	0.54076199953217646308	0.54076199953745719782	5.2807 E-12
	0.84147257008995471110	0.84147257008999494129	4.0230 E-14

Table 2 Computational result of stiefel and bettis Problem $n=360$

x	$y(x_n)$	y_n	$\lvert y(x_n)-y_n\rvert$
0.1	0.9999951220742781944094331 57	0.999995122074278194409433149	8.530 E-27
	0.003123432421368851015389707	0.003143242136885101538970806	6.759 E-29
0.2	0.999980488344701048649077719	0.999980488344701048649077179	1.971 E-26
	0.006246834371010263687228853 3	0.0062468343710102636872288534	8.163 E-29
0.3	0.999956098954032911148979474	0.999956098954032911148979471	3.3554 E-26
	0.009370175377494076871053274 5	0.0093701753774940768710532746	3.378 E-29
0.4	0.99992195414021281667698236 2978	0.99992195414021281667698231 2930	5.0048 E-26
	0.0124934249699846809203866964209	0.0124934249699846809203866963368	8.4100 E-29
0.5	0.99987805423635216163736119 4119	0.99987805423635216163736112 4922	6.9197 E-26
	0.0156165526785382861853006769263	0.0156165526785382861853006766456	2.807 E-28

Citation: Lukman MA, Olaoluwa OE. A one-step extended block hybrid formulae for solving orbital problems. *MOJ App Bio Biomech.* 2019;3(1):37–43.
DOI: 10.15406/mojabb.2019.03.00101

Table Continued

x	$y(x_n)$	y_n	$\lvert y(x_n)-y_n\rvert$
0.6	0.999824399670731457700188569852	0.999824399670731457700188478853	9.0999 E-26
	0.0187395280344001828105329809699	0.0187395280344001828105329804061	5.568 E-28
0.7	0.999760990966796151864594726347	0.999760990966796151864594610892	1.1546 E-25
	0.0218623205703019889331492250672	0.0218623205703019889331492241251	9.421 E-28
0.8	0.999687828743151520153806807434	0.999687828743151520153806664870	1.4256 E-25
	0.0249848998207588843798458101561	0.0249848998207588843798458087324	1.4237 E-27
0.9	0.999604913713556632606079650692	0.999604913713556632606079478368	1.7232 E-25
	0.0281072353223668269641336218113	0.0281072353223668269641336197946	2.0167 E-27
1	0.999512246686917389961209836088	0.999512246686917389961209631351	2.0474 E-25
	0.0312292966140997484838117297791	0.0312292966140997484838117270493	2.7298 E-27

Table 3 Computational result of two-body problem $n=10$

x	$y(x_n)$	y_n	$\lvert y(x_n)-y_n\rvert$
0.1	0.9950041652780257661 0	0.99500416527802721748	1.4513 E-15
	0.09983341664682815230	0.09983341664682821656	6.4256 E-17
0.2	0.98006657784124740767	0.98006657784124163112	5.77655 E-15
	0.19866933079506163312	0.19866933079506121546	4.17660 E-16
0.3	0.95533648912561890854	0.95533648912560601964	1.28889 E-14
	0.29552020666134091870	0.29552020666133957511	1.34359 E-15
0.4	0.92106099400290772840	0.92106099400288508280	2.26456 E-14
	0.38941834230865360559	0.38941834230865049167	3.11392 E-15
0.5	0.87758256189040756578	0.87758256189037271612	3.48496 E-14
	0.47942553860420898381	0.47942553860420300027	5.98354 E-15
0.6	0.82533561490972755019	0.82533561490967829724	4.92529 E-14
	0.56464247339504554223	0.56464247339503535720	1.01850 E-14
0.7	0.76484218728455398613	0.76484218728448842626	6.55598 E-14
	0.64421768723770697745	0.64421768723769105367	1.59237 E-14
0.8	0.69670670934724885284	0.69670670934716542092	8.34319 E-14
	0.71735609089954613508	0.71735609089952276163	2.33734 E-14
0.9	0.62160996827076694921	0.62160996827066445648	1.02492 E-13
	0.78332690962751606037	0.78332690962748338846	3.26719 E-14
1	0.54030230586826205125	0.54030230586813971740	1.22333 E-13
	0.84147098480794042438	0.84147098480789650665	4.39177 E-14

Citation: Lukman MA, Olaoluwa OE. A one-step extended block hybrid formulae for solving orbital problems. *MOJ App Bio Biomech.* 2019;3(1):37–43. DOI: 10.15406/mojabb.2019.03.00101

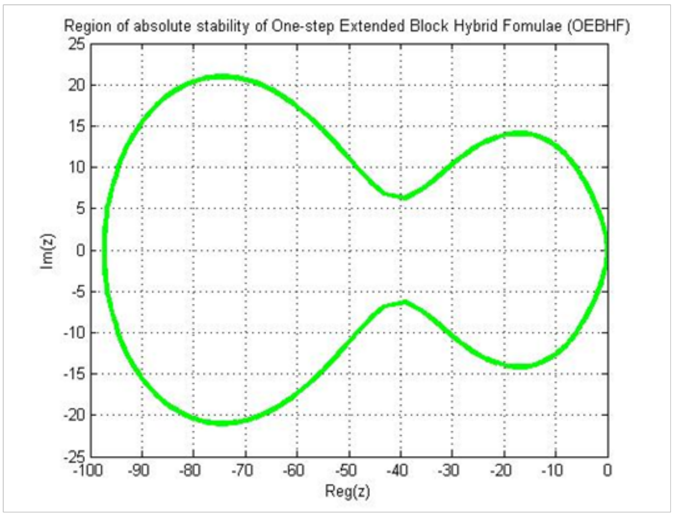

Figure 1 Absolute Stability Region of the (OEBHF).

Acknowledgment

None.

Conflicts of interest

The author declares there is no conflcit of interest.

References

1. Adoghe LO, Omole EO. Comprehensive Analysis of 3-Quarter-Step Collocation Method for Direct Integration of Second Order Ordinary Differential Equations Using Taylor Series Function. *ABACUS (Mathematics Science Series)*. 2018;44(2):311–321.
2. Ehigie JO, Jator SN, Sofoluwe AB, et al. Boundary value technique for initial value problems with continuous second derivative multistep method of Enright. *Journal of Computer Application Mathematics*. 2014;33(1):81–93.
3. Enright WH. Second derivative multistep methods for stiff ordinary differential equations. *SIAM J Numer Anal*. 1974;11(2):321–331.
4. Henrici P. Some Applications of the Quotient-Difference Algorithm. *Proc Symp Appl Math*. 1963. p. 159-183.
5. James A, Adesanya A, Joshua S. Continuous block method for the solution of second order initial value problems of ordinary differential equation. *Int J Pure Appl Math*. 2013;(83):405–416.
6. Lambert JD. Computational Methods in ODEs; John Wiley and Sons: New York, NY, USA.
7. Skwame Y, Sabo J, Kyagya TY. A class of two-step second derivative adam moulton method with two off-step points for solving second order stiff ordinary differential equations. *International Journal of Scientific and Management Research*. 2018. p. 13–20.
8. Skwame Y, Sunday J, Sabo J. On the Development of Two-step Implicit Second Derivative Block Methods for the Solution of Initial Value Problems of General Second Order Ordinary Differential Equations. *Journal of Scientific and Engineering Research*. 2018;5(3):283–290.
9. Tumba P, Sabo J, Hamadina M. Uniformly Order Eight Implicit Second Derivative Method for Solving Second- Order Stiff Ordinary Differential Equations ODEs. *Academic Journal of Applied Mathematical Sciences*. 2018;4:43–48.
10. Aruchunan E, Sulaiman J. Numerical Solution of Second-order Linear Fredholm Integro-differential Equation using Generalized Minimal Residual method. *American Journal of Applied Science*. 2010;7:780–783.
11. Lambert JD, Watson A. Symmetric Multistep method for periodic Initial Value Problems. *Journal of Inst Mathematics & Applied*. 1976;18:189–202.
12. Lambert JD. Numerical Methods for Ordinary Differential Systems of Initial Value Problems. John Willey & Sons, New York, 1991.
13. Owolabi KM. A Robust Implicit Optimal formula for direct Integration of Second order orbital Problems. *Journal of Advances in Physics Theories and Application*. 2015;42:21–26.
14. Simos TE. An Exponentially-fitted Runge-Kutta Method for the Numerical Integration of Initial-value Problems with periodic or oscillating Solutions. *Computational Physics Communications*. 1998;115(1):1–8.
15. Simos TE. Exponentially-fitted and trigonometrically-fitted symmetric linear multistep methods for the numerical integration of orbital Problems. *Physics Letters A*. 2003;315(6):437–446.
16. Stiefel E, Bettis DG. Stabilization of cowell's method. *Numerical Math*. 1969;13(2):154–175.
17. Vigo Anguiar J, Simos TE. Exponentially-fitted and Trigonometrically-fitted symmetric linear multistep methods for the numerical Solution of orbital problems. *Astronomical Journal*. 2003;122:1656–1660.