

Computational analysis of NACA 0010 at moderate to high Reynolds number using 2D panel method

Abstract

Wing structures as found in aircrafts and wind turbine blades are built from airfoils. Computational methods are often used to predict the aerodynamic characteristics of airfoils, typically the force and pressure coefficients along its chord length. In the present work, pressure coefficient distribution of NACA 0010 is evaluated using the 2D panel method for incompressible lifting flows at moderate to high Reynolds number, $Re-3 \times 10^5$, 5×10^5 , 1×10^6 . The analysis was conducted for various AOA (angle of attack), between -40 to 200 for the airfoil. The non-dimensional pressure is illustrated for upper and lower surfaces of airfoil between 00 to 200 angle of attack at specific chord locations of airfoil. The present results from the 2D panel method are validated using the results from Hess & Smith method and inverse airfoil design method implemented for conformal mapped symmetric Jukowski airfoil of 10% thickness at 40 angle of attack.

Keywords: airfoil, panel method, pressure coefficient, angle of attack, chord

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Introduction

NACA airfoils are used in the aircraft industry for producing lift forces on the wing span required during takeoff, maneuvering, cruising and landing conditions necessary for powered flights. The airfoil selection is based on the relevance to a specific application in industry and the service conditions. The pressure distribution not only affects the lift and drag forces which act usually at fixed point on aircraft wings but also change with angle of attack conditions. It also influences aircraft stability which is predominantly related to the pitching, rolling and yawing moment characteristics during operation. The lift and drag forces determine the glide ratio, an important parameter to determine the aircraft wing performance at different flow configuration. Many of the compressible or incompressible flows in aerodynamics can be characterized using the Reynolds number and Mach number.¹ The aerodynamic behavior of airfoil for the incompressible ($M < 0.3$), and compressible ($M > 0.3$) flows is determined using Mach number. Typically the results from experiment study serve as reliable validation method in aircraft industry which can be readily compared with numerically computed or even the actual performance data for wing. Panel methods are modern numerical techniques which are quick to execute and predict fairly accurate results compared to the experimental methods. The experiments are usually cumbersome to implement, in terms of data obtained from measurements, calibration procedures involved in the wind tunnel setup and hence take long duration relative to results obtained from numerical methods. The present analysis deals with the pressure distribution on NACA symmetric profiles using 2D panel method for lifting flows. Hence pressure distribution helps to determine the attitude of geometrically symmetric profiles and the forces acting on them.

Literature review

A comprehensive literature is available on experiment studies conducted on NACA airfoil series (Abbott & Von Doenhoff, 1958) to compare the aerodynamic characteristics for varying Mach and Reynolds number and for viscous or inviscid flows with free and the forced boundary layer transitions.^{2,3} The NACA profiles are also applied in ship industry for construction of rudder that experiences the hydrodynamic forces during service conditions. A program for the

design and analysis of subsonic isolated airfoils was developed by Mark Drela at MIT, known as X-foil software using panel method. Although there are several commercially available online programs namely, www.Aerofoiltools.com, JAVA foil, to calculate the pressure coefficient, most of them utilize the 2D source and vortex panel approximation methods to determine the pressure coefficient, lift and drag forces on airfoil.

Methodology

NACA airfoils were designed (Eastman Jacobs, 1929-47) at NASA Langley field laboratory.² The airfoil geometry for most of the NACA profiles can be divided into x-coordinates known along the chord line and y-coordinates known as the ordinates. The mean line or camber line of the profiles is the average of the distance measured between the upper and lower surfaces of airfoil. The camber for an airfoil however, is designated by the distance between the chord line and its mean line. The chord line is the straight line connecting the leading and trailing edge of airfoil. The shape of mean line is expressed analytically with help of two parabolic arcs drawn tangential to maximum mean line ordinate.¹ The airfoil geometry is selected based upon the parameters shown in Table 1. In the present study NACA 0010 profile is chosen, and with a chord length of 120mm. The numerical investigation is conducted using 2D panel method and described in section B. Figure 1 shows the geometry of NACA 0010 airfoil and its panel approximation. Each panel is made up of pair of end points known as nodes. The total number of panel nodes used is 35 for entire airfoil surface. The coordinates for the airfoil were obtained from the University of Illinois Urbana Champaign website.

Table 1 Geometrical property of 4 digit symmetric & cambered NACA airfoils

Sl. no	NACA Airfoil	t/c, [%]	Maxim camber & position	Design lift coefficient [-]
1	0010	10	-	-
2	0015	15	-	-
3	0024	24	-	-
4	4412	12	4%, 0.4	-

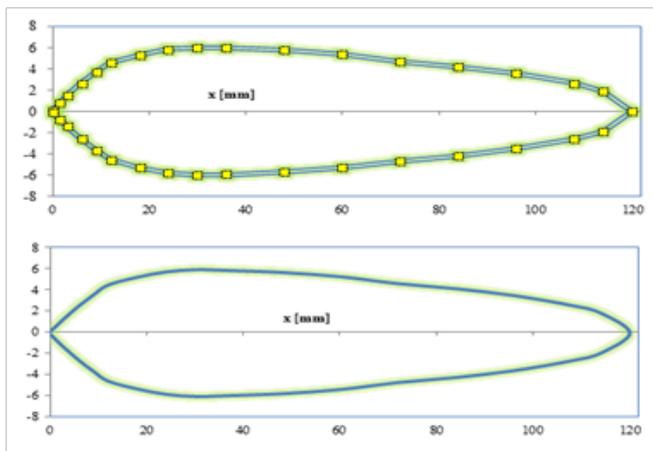


Figure 1 Geometry of NACA 0010 profile of chord length 120mm and its panel approximation.

Airfoil geometry

The important airfoil geometric properties are camber, thickness to chord ratio and chord length of the airfoil for which the different flow configurations are analyzed using the 2D panel method. In general, a particular flow configuration is established for an airfoil using the angle of attack and Reynolds number describing the flow field for a given chord length and thickness. Since NACA 0010 is symmetrical airfoil the camber for such airfoils is zero while thickness to chord ratio is ~10 %. However, in the present study the maximum thickness is scaled to represent 12mm for chosen chord length which is suitable value for the bio inspired profiles. Insects such as the wings of dragonfly have high aspect ratios and typically have chord lengths and thickness of only few mm. NACA 0010 airfoils, General Aviation (GAW) and Eppler series airfoils are most suitable for comparing their aerodynamic performance to that of the corrugated profiles,^{4,5} at various flow configurations. The individual profiles can be distinguished using distinct parameters such as number of digits in series, leading edge radius, and trailing edge angle between sloping surfaces of profiles. The thickness and mean line distributions for the 4 digit symmetric and cambered airfoils are expressed in terms of the t/c ratio. The polynomial equations representing the geometry of NACA 4 digit airfoils are detailed in text.⁶⁻⁸ Figure 2 is the illustration of the airfoil shape parameters with its leading edge radius representing the roundness of airfoil and also the flow stagnation point.

Governing equations for potential flow are most suitable approach for modeling flow around slender bodies of any shape. It involves the superposition of source or a sink or a doublet in uniform distributed flow and does not tend to predict accurate values for compressible flows and for objects with complex shapes. Basic panel methods were developed (Hess & Smith, 1950) at Douglas aircraft for aircraft industry.⁷ These methods were intended for analyzing the steady incompressible 2D lifting flows. Panel methods model the potential flow around the objects surface by distributing source strength as singularity and vortex strength on every panel of the surface in uniform flow stream. A source is point in which the fluid moves radially outward in field at uniform rate while for a sink fluid moves radially inwards at same uniform rate, m²/s. Each source or sink has specific strength, K, and vortex strength denoted by circulation, Γ. Simple 2D uniform lifting flows⁷ can be described using the velocity potential and stream line functions as Eq. (2) & Eq. (1)

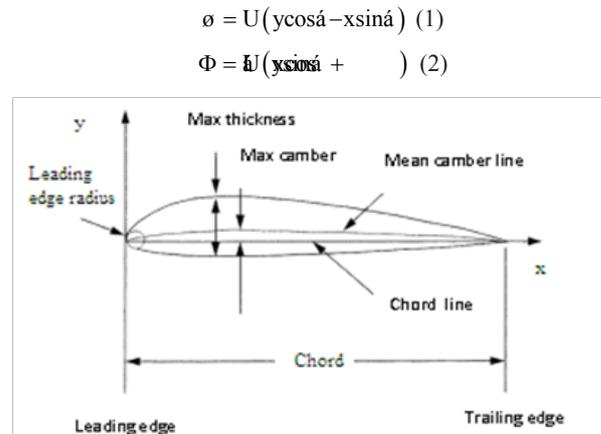


Figure 2 Illustration of airfoil shape parameters.¹⁰

In a uniform flow field, the stream function and velocity potential functions can be described using the Eq.(3)

$$\phi = Uy + D; \psi = Ux + C \quad (3)$$

Where, C & D are constants of integration. In lifting flows the point source or sink is superimposed with vortex strength⁷ which is distributed in all directions of uniform flow field and obeys the continuity equation. The stream function and velocity potential can be expressed in terms of the velocity components as in Eq. (4)

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x} = U \cos \alpha; v = -\frac{\partial \psi}{\partial x} = \frac{\partial \phi}{\partial y} = U \sin \alpha \quad (4)$$

Where, u & v the velocity components of the free stream velocity U. The airfoil geometry is discretized into finite number of panels representing the surface. The panels are shown by series of straight line segments to construct 2D airfoil surface.⁷ Numbering of end points or nodes of the panels is done from 1 to N. The center point of each panel is chosen as collocation points. The periodic boundary condition of zero flow orthogonal to surface also known as impermeability condition is applied to every panel. Panels are defined with unit normal and tangential vectors, \hat{n}_i, \hat{t}_i . Velocity vector, denoted by \mathbf{v}_{ij} are estimated by considering the two panels, i & j representing surface of airfoil, the source on the panel j which induce a velocity on panel i. The perpendicular and tangential velocity components to the surface at the point i, are given by scalar products by Eq.(5) and Eq.(6). These quantities represent the source strength on panel j and expressed mathematically as

$$\hat{\omega}_{ij} \hat{N}_i = \mathbf{v}_{ij} \cdot \hat{n}_i \quad (5)$$

$$\hat{\omega}_{ij} \hat{T}_i = \mathbf{v}_{ij} \cdot \hat{t}_i \quad (6)$$

Where N_{ij} and T_{ij} are known as normal and tangential influence coefficients. It must be noted that the velocity components induced due to source and vortex distribution at a point centered on panel i relative to panel j or point in flow field can be expressed mathematically using Eq (7) – Eq.(9)

$$\mathbf{v}_{ij} = v_{xj} \hat{t}_j + v_{yj} \hat{n}_j \quad (7)$$

$$N_{ij} = v_{yj} \hat{n}_i \quad (8)$$

$$T_{ij} = v_{xj} \hat{t}_i \quad (9)$$

The surfaces represented by the panels are solid rectangular or curvilinear areas upon which the above conditions are applied. The

normal and tangential velocity vectors at each of collocation points consist of source and circulation strengths, which are induced due to free stream velocity, U on panel segment. Together they form the system of linear algebraic equations which are solved for N unknown source strengths, σ_j , using the influence coefficients and written in matrix form as in Eq (10)

$$M.A = B \dots (10)$$

Along with Kutta condition given by Eq. (14) the matrix M contains N+1 x N+1 equations representing the N_{ij} and T_{ij} , known as normal and tangential influence coefficients, A is column matrix of N+1 elements, containing the unknown source and vortex strengths, B is the column matrix of N+1 elements of unit normal velocity vectors. Matrix inversion procedures available in MATLAB are applied to solve for the unknown source strengths and built in function inv() is used with mathematical operator '\'

$$\sum_{j=1}^N \sigma_j T_{i,j} + \tilde{\alpha} T_{i,N+1} + \tilde{U} \cdot \hat{t}_i = v_{t_i} \dots (11)$$

$$C_{p_i} = 1 - \left[\frac{v_{t_i}}{U} \right]^2 \dots (12)$$

The pressure acting at any collocation point i on panel surface can be expressed in non-dimensional form as in Eq. (13)

$$C_{p_i} = 1 - \left[\frac{v_{t_i}}{U} \right]^2 (13)$$

Where v_{t_i} the tangential velocity vector is determined using the influence coefficients, known values of source and circulation strengths as in Eq. (11), U is the free stream velocity in m/s over the airfoil. The impermeability boundary condition is given by Eq. (12) and applied on every panel of airfoil surface. Hence the influence coefficients are important for panel method in order to determine the pressure distribution over the surface of any given airfoil. The trailing edge represents a unique condition for the airfoil. Using panel method, one of the following criteria⁷ is used for airfoils with finite trailing edge thickness

The streamlines leave the trailing edge with a direction along the bisector of the trailing edge angle. The velocity magnitudes on the upper and lower surfaces near the trailing edge of airfoil approach the same limiting values. The trailing edge angle is modeled as the stagnation point for finite value of trailing edge angle hence the source strength must be zero at the trailing edge. The above assumptions are known as the Kutta condition or Trailing edge boundary condition which is essential for solving the matrix system of equations as shown in Eq. (10) and represented in discrete algebraic form as in Eq. (14).

$$C_{p_i} = \left[\frac{P - P_\infty}{\frac{1}{2} \rho U^2} \right] (14)$$

The most commonly used condition is 2nd criterion on airfoils. In the present method 2nd criterion is applied due to its relative simplicity in MATLAB code implementation. Hence, the resulting tangential velocity vector is obtained by adding the known source and circulation strengths with its influence coefficients for each panel in order to calculate the pressure coefficient distribution over the airfoil surface and given by Eq (13). The number of panel nodes used in the simulation is 35 and the maximum panel angle is 76.77° in airfoil

geometry. The MATLAB routine foil.m was developed for NACA 0010 airfoil for which fluid density is assumed as 1.225 kg/m³. It must be noted that the pressure calculated at the control points centered at each panel using the panel method can also be found using Bernoulli's equation as given by Eq (15) from wind tunnel measurements

$$C_{p_i} = \left[\frac{P - P_\infty}{\frac{1}{2} \rho U^2} \right] (15)$$

Where P – stagnation pressure in Pa, P_∞ is the static pressure in Pa for ambient conditions.

Joukowski mapping method

A Joukowski airfoil is obtained by the conformal transformation of cylinder of finite radius using complex mapping functions in complex plane known as $\xi-\eta$ plane, $\zeta = \xi + i\eta$. The conformal parameters for the Joukowski airfoil involve angle of attack, α , thickness and camber (β). Figure 3 shows the Joukowski airfoil obtained using the mapping of cylinder geometry in four steps. Step 1 is solution to the potential flow in the z'' -plane containing the complex potential functions representing the uniform stream, U m/s past a cylinder, a doublet at origin, and also the circulation strength Γ . Step 2 is introducing the angle of incidence, α , by rotating the axes by an angle in the z' -plane. Step 3 determines the thickness and camber of the airfoil shape by shifting the center of circle through a distance by a in z-plane, $x+iy$. Step 4 involves the final Joukowski mapping of the circle in z-plane into the airfoil shape in the $\xi-\eta$ plane. Most of the Joukowski airfoils are characterized using the parameter, β known as camber, ratio R/a known as thickness. The surface in the z'' -plane is circle (cylinder) and expressed mathematical function as Eq. (16)

$$f(z'') = Uz'' + \left[\frac{\tilde{\alpha}UR^2}{z''} \right] - i \left(\frac{\Gamma}{2} \right) \ln(z''/R) (16)$$

Similarly after adding angle of attack, α , the solution to the flow in z-plane is given by Eq (17)

$$f(z') = U \left[z' e^{-i\alpha} + \left[\frac{\tilde{\alpha}R^2 e^{i\alpha}}{z'} \right] \right] - i \left(\frac{\Gamma}{2} \right) \ln \frac{z' e^{-i\alpha}}{R} (17)$$

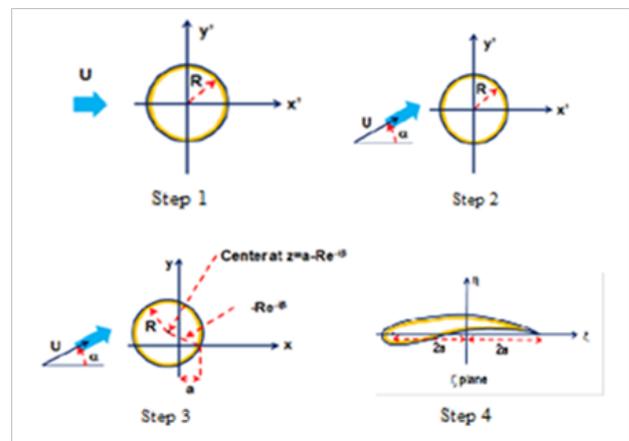


Figure 3 (a) z'' plane (b) z' -plane (c) z-plane (d) $\xi-\eta$ plane³

where $z' = z'' \cdot e^{i\alpha}$. The flow in z-plane after the displacement of cylinder by distance a is given by Eq (18) as

$$f(z) = U e^{-i\alpha} \left[(z-z_0) + \left[\frac{\tilde{a} R^2 e^{2i\alpha}}{(z-z_0)} \right] \right] - i \left(\frac{\tilde{a}}{\tilde{b}} \right) \ln \frac{(z-z_0) e^{-i\alpha}}{R} \quad (18)$$

The final step of jukowski mapping is given by function as in Eq. (19) which is used to represent the Jukowski airfoil in the ξ -plane and given by Eq. (20)

$$\frac{\tilde{x}}{a} = \frac{z}{a} + \frac{a}{z} \quad (19)$$

$$\frac{\tilde{x}}{a} = 1 + \frac{R}{a} \left[e^{i\theta} - e^{-i\beta} \right] + \left[1 + \frac{R}{a} \left[e^{i\theta} - e^{-i\beta} \right] \right]^{-1} \quad (20)$$

Where $z = a$, $z' = z - z_0 = Re^{-i\beta}$, $z'' = Re^{-i(\alpha+\beta)}$.

The parameter R/a , determines the thickness of the airfoil and β is the camber. Further, it must be noted that flow variables, free stream velocity and its components, source and circulation strengths are solved in the complex z -plane using the inverse transformation method.⁹⁻¹¹ It involves solving for the roots of quadratic function as shown in Eq (19). The pressure coefficient, C_p is calculated by taking the first derivative for the complex potential function, W represented by uniform free stream flow, source and sink and circulation strength in ξ -plane ($\xi+i\eta$) as shown in Eq (21) - Eq (23)

$$W = Uze^{-i\alpha} + \frac{UR^2}{z} e^{i\alpha} + k.i.\log z \quad (21)$$

$$|q| = \left| \frac{dW}{dz} \right| = q \quad (22)$$

$$C_p = 1 - \left(\frac{q}{U} \right)^2 \quad (23)$$

Where $k = \Gamma/2\pi = -2U R \cdot \sin(\alpha+\beta)$, Γ is the circulation. k is the vortex strength.

Results and discussion

Flow around the airfoil that has finite chord and span length can be described using the circulation phenomenon. For a given free stream velocity upstream the airfoil as well as the wing tip surface produces bound vortex and also trailing edge vortex downstream of the airfoil.^{8,9} Therefore, due to the nature of the flow between the leading and trailing edge of the airfoil, the strength of the circulation is not uniform. However, the strength of the trailing edge vortex downstream of the airfoil nullifies the bound vortex surrounding airfoil and obeys the Kelvin Helmholtz theorem i.e. net circulation surrounding the airfoil and flow field must be zero. It can be observed from Figure 4. that for constant Reynolds number and at 80% chord the pressure coefficient is increasing on the upper surface and lower value of 1.8, for large angles of attack while it decreases for the rear surface i.e. -0.978. The magnitude of surface velocity on airfoil varies along its chord length and also in span wise direction and influences the C_p values. In the panel method implementation, the angle of attack is defined in matrix form between -4° to 20° . MATLAB software was used for the simulation on 2GB RAM Dell Workstation.

The number of panel nodes representing the NACA 0010 airfoil is 35 in the simulation. In the present study the effect of change in the number of panels on the pressure coefficient, C_p , has not been investigated although it was reported from previous studies⁷ that number of panels would influence the C_p result. The pressure coefficient, C_p , is evaluated for varying Reynolds number at constant chord location, 40 %c as shown in Figure 5. For a given Reynolds

number and for high angle of attack, the pressure gradient between the upper and lower surfaces is found to increase at which flow is expected to become turbulent however, for low angle of attack, the C_p values agree well with each other at different chord locations. Hence at low angle of attack, the pressure coefficient, C_p does not vary significantly at different Reynolds numbers, and particularly for $Re-1 \times 10^6$ (1 million) at 40% chord location. The pressure coefficient, C_p is also computed along chord length of the airfoil and for varying angles of attack, 4° , 12° & 16° as shown in Figure 6 at constant Reynolds number of 3×10^5 . It must be noted that flow separation on airfoils for stall angle of attack, using panel method cannot be modeled accurately since the methodology has draw backs and does not predict the boundary layer characteristics on airfoil. The method as described before evaluates the relative velocity and its components on every panel of the airfoil surface and predicts the pressure coefficient, C_p , by calculating the influence coefficients and tangential velocity vector. The present panel method has been validated using the results obtained from the inverse design method,⁶ Hess and Smith method⁶ and analytical results of (Petrucci, 2007) as shown in Figure 7. The vortex panel method was implemented to verify the inverse design algorithm for the incompressible flows on a conformal mapped symmetric jukowski airfoil having 10 % thickness.⁶ It was revealed that source panel method implementation by Petrucci et al, showed geometric convergence problems for cambered airfoils. In his study, several iterations were required for geometric convergence in case of cambered airfoils in contrast to symmetric airfoils for different flow configurations. But in the present panel method geometric convergence is not relevant. The application of source and vortex panel was done consistently on each panel of airfoil geometry to ensure integrity in every calculation step (Figures 4-7).

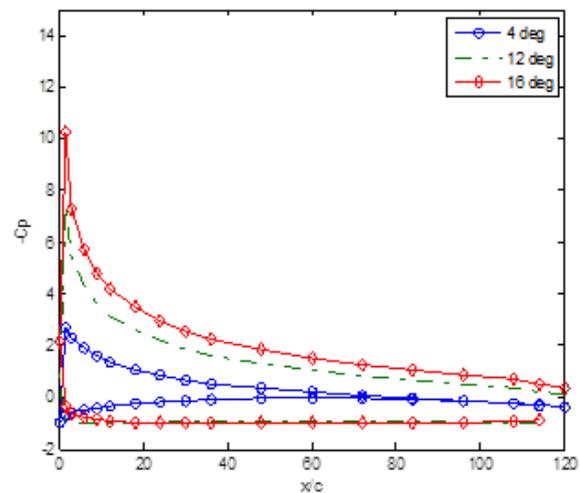


Figure 4 Pressure Coefficient distribution of NACA 0010 profile at $Re = 3 \times 10^5$ at 4, 12 and 16 degree angle of attack.

In the study of Petrucci⁶ the results of the pressure coefficient obtained from inverse design algorithm were also compared with analytical results of the Boundary Integral method that includes boundary layer thickness and viscous effects. The pressure peaks near the leading edge are under predicted in all inverse design methodologies. In Figure 7 the results for C_p , using inverse design method are found to under predict relative to those obtained from present panel method however, they agree well with the results from inverse method (Boundary Integral Formulation). The results from Hess & Smith method⁶ are in better agreement with the present 2D

panel method, in terms of the pressure peaks observed at the leading edge on the upper surface of NACA 0010 airfoil and up to 80% chord length. However, between 80-100%, the pressure coefficient remained gradually reducing and found to vary by ~25 % near the trailing edge section of the airfoil. Thus pressure distribution on airfoil provides essential data for determining the aerodynamic behavior of wings with complex design shapes. The aircraft wings built from airfoils have finite length and thickness with sharp or cusped trailing edge. The thickness of the trailing edge influences the pressure and velocity distribution on the wing at different angles of attack. The velocity contours for the conformal mapped symmetric jukowski airfoil of 10% thickness are evaluated in MATLAB and shown for different angles of attack in Figure 8. MATLAB program Jukowski.m was developed to demonstrate the similarity in results obtained for pressure coefficient and to characterize the flow around airfoil. Similarly the pressure coefficient contours are evaluated for the same flow conditions for the airfoil. Figure 9 shows the pressure coefficient at different angles of attack for the symmetric Joukowski airfoil of equal chord length as that of NACA 0010 airfoil.

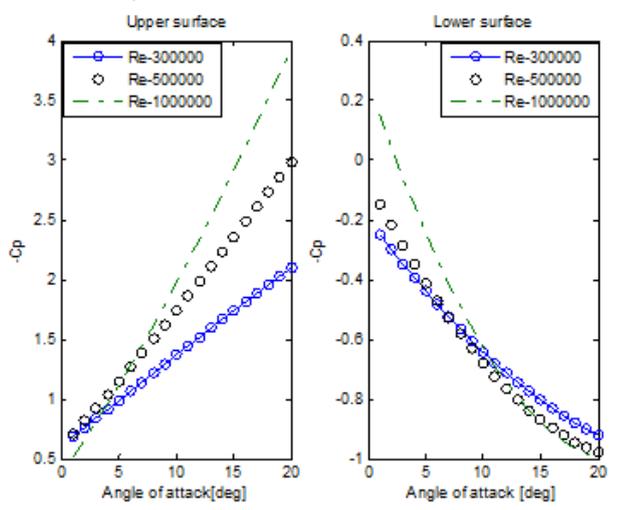


Figure 5 Pressure Coefficient distribution of NACA 0010 profile at $Re = 3 \times 10^5, 5 \times 10^5, 10^6$ at 40% chord location between 0 to 20 degree angle of attack, for upper and lower surfaces.

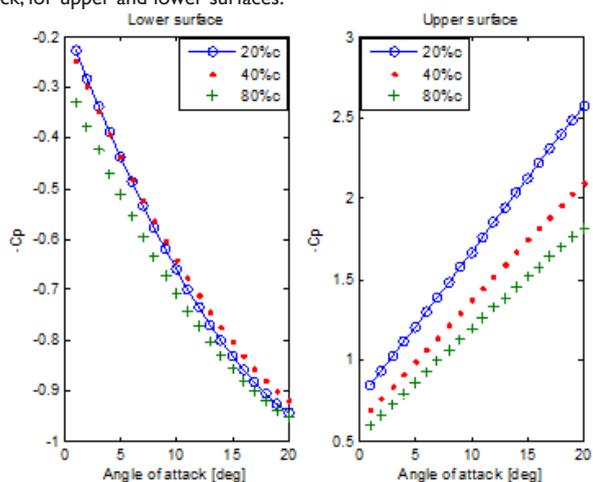


Figure 6 Pressure Coefficient distribution of NACA 0010 profile at $Re = 3 \times 10^5$ at 20%, 40%, 80% chord location between 0 to 20 degree angle of attack, for upper and lower surfaces.

At 0° AOA, the velocity contours and pressure coefficient show symmetry around the airfoil chord length as shown in Figure 8B & Figure 9B. As the angle of attack is varied, at -10° AOA, the velocity and pressure coefficient contours on the upper and lower surface of the airfoil are reversed as shown in Figure 8A and Figure 9A. The maximum deviation in the computed results from present panel method and results obtained from, Hess & Smith, Inverse design methods for conformal mapped symmetric jukowski airfoil are found to be ~20 % near trailing edge of airfoil. It is due to the implementation of kutta condition for the trailing edge panels which accounts for the viscous effects in the steady flow aerodynamics and serves as alternative to the boundary layer viscous phenomenon observed in the unsteady Navier-Stokes solver method. Further it must be noted that towards leading edge of airfoil, i.e. up to 60% chord, the pressure coefficients are found to deviate of the order ~ 0.5 % which indicates the consistency of application in the vortex strength distribution centered on the leading edge panels. Figures 10B–10E shows the streamline contours at different angles of attack for the same free stream velocity, $U, 38\text{m/s}$. The sketch of the mapped airfoil, size of cylinder of radius, $R = 0.3$ units (Figure 10A). The displacement of the cylinder or circle in the real axis (x-axis) is set to 0.03 units to obtain a chord length of 1.2 units equal to ~120mm and thickness 10 % or ~12mm.

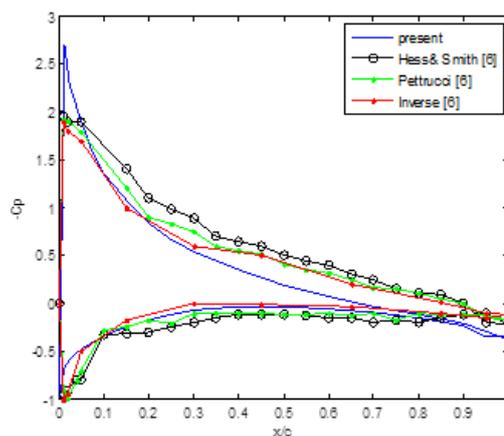


Figure 7 Comparison of pressure coefficient distribution of NACA 0010 profile at $Re = 3 \times 10^5$ for 4° angle of attack with Hess & Smith, Inverse design, Boundary Integral Method results for conformal mapped symmetric Jukowski airfoil of thickness 10%, at 4° angle of attack.

Conclusion

Panel method was implemented on NACA0010 airfoil at moderate to high Reynolds number between -4° to 20° angles of attack. The method involved superimposing the 2D uniform flow stream on combined source and vortex strengths to predict pressure coefficient for symmetric airfoils for incompressible lifting flows. The velocity distribution and pressure coefficient contours are also evaluated using the jukowski mapping method to compare the equivalent results obtained from the panel method implementation at different angles of attack. The present numerical results were found to be close agreement with the Hess and Smith method and inverse design method applied for conformal mapped symmetric Joukowski airfoil of comparable thickness to chord ratio. Sharp pressure peaks at leading edge of airfoil are observed and found to be dependent on the angle of attack as well as Reynolds number. The results for pressure coefficient using present panel method agree well up to 60 % chord length on upper and thoroughly on lower surfaces of airfoil but found

to vary by ~25 % between 60-80 % chord length when compared to those obtained from the inverse design and Hess & Smith results. In comparison to the panel method results, the jukowski mapping

method predicts the pressure coefficient conservatively for the same free stream conditions.

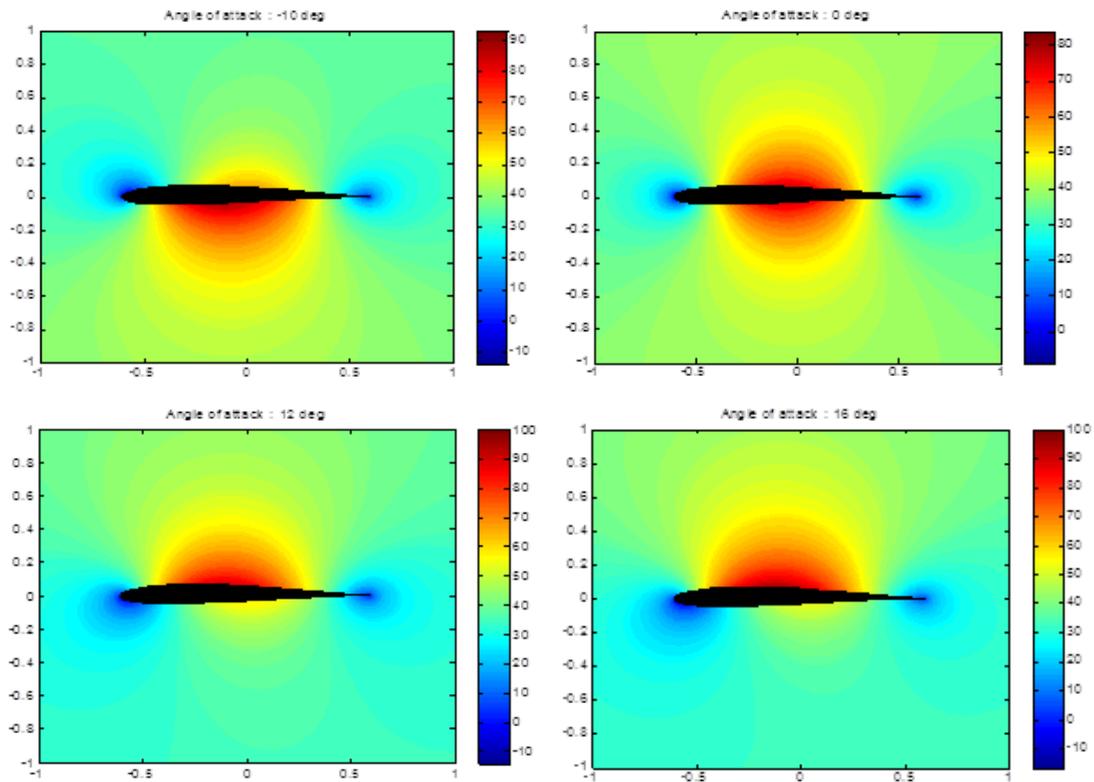


Figure 8 Velocity contours for conformal mapped symmetric Joukowski airfoil at $U = 38\text{m/s}$ (a) at -10° AOA (b) 0° AOA (c) 12° AOA (d) 16° AOA.

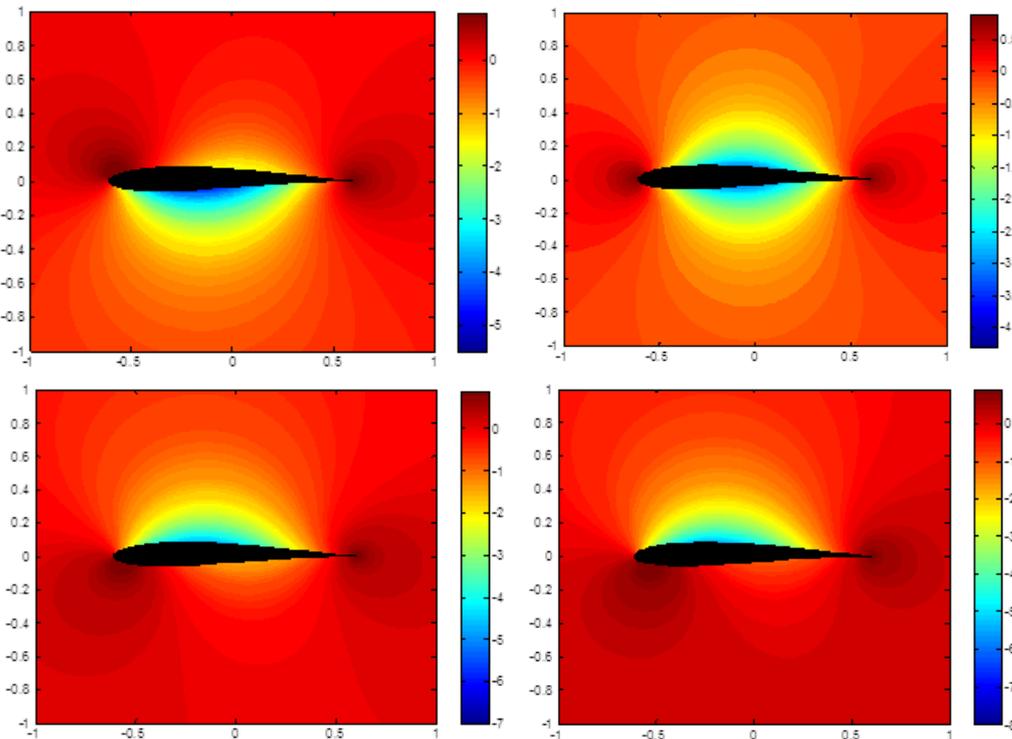


Figure 9 Pressure coefficient contour for conformal mapped symmetric Joukowski airfoil for $U = 38\text{ m/s}$ (a) at -10° AOA (b) 0° AOA (c) 12° AOA (d) 16° AOA.

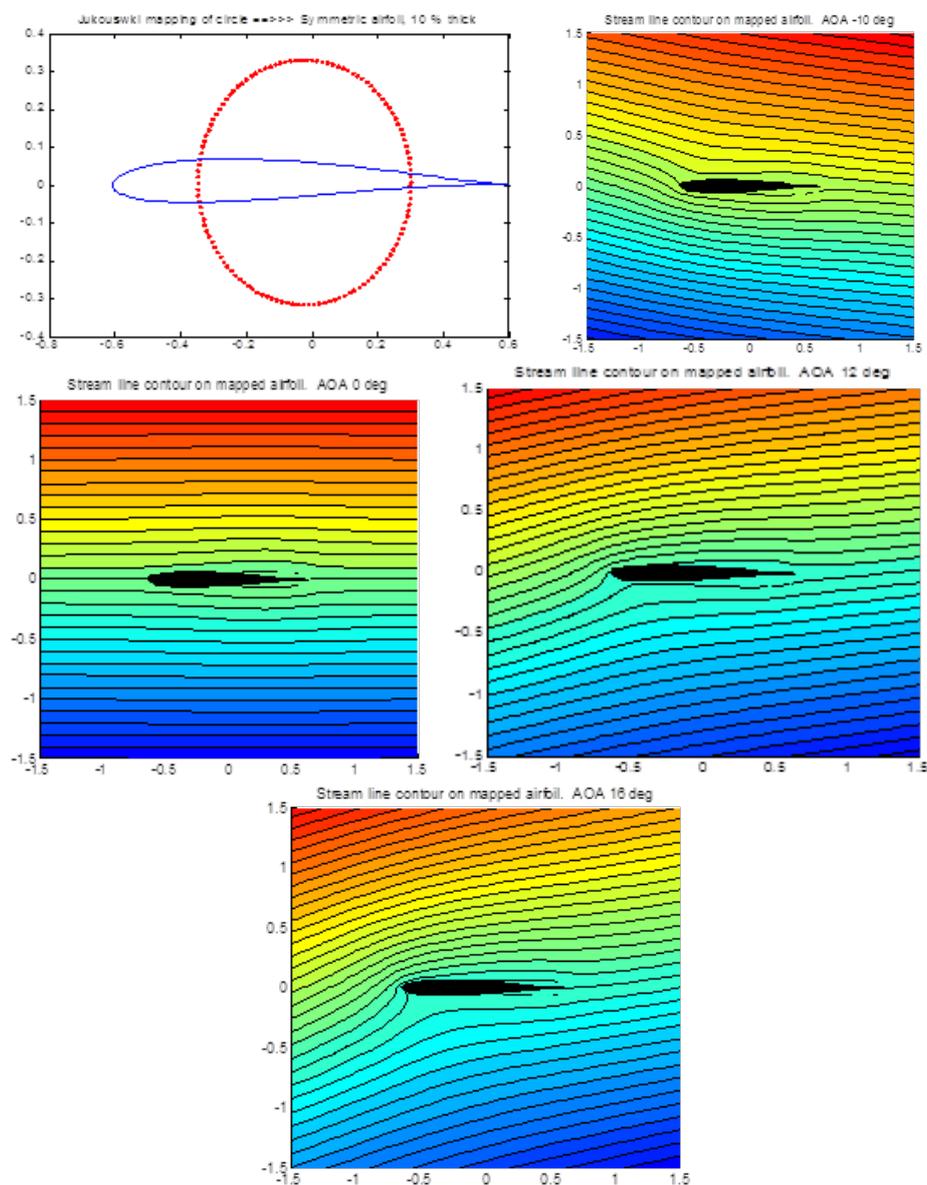


Figure 10 Normalized streamline contours for symmetric jukowski airfoil of 10 % thickness, $U = 38$ m/s (a) Jukowski airfoil and Cylinder of Radius 0.3 units (b) -100 AOA (c) 00 AOA (d) 120 AOA (e) 160 AOA.

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Conflicts of interest

None.

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