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Mathematical modelling of direct drive of card machine drum

Abstract

The paper is concerned with the mathematical modelling of a directly driven card machine drum. The equations of the motion are derived taking account of an interaction of a drum and the electric motor.

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Introduction

The problem of vibration of cylindrical shells is technically important. The equations of the motion can be found in works.[1–6] The natural vibrations have been studied in papers.[7–9] The vibrations of rotating shells have been investigated in papers.[10,11] The phenomena of bifurcation and chaos in externally excited circular cylindrical shells have been reported in work.[12] The stress and transverse deflection of a card machine drum, in a form of a cylindrical shell with wound wire, have been calculated in.[13] The purpose of this work is to formulate a mathematical model of a direct drive of a card machine drum.

Equations of the motion

The rotating shell of mean radius r is shown in Figure 1. Here, m denotes unit mass, g - gravity acceleration and t - time. The rotation angle of shell is denoted as α, the longitudinal and the angular co-ordinates of shell elements are denoted as x and φ. The longitudinal, circumferential and radial components of the shell surface deflection are denoted as u, v and w respectively.

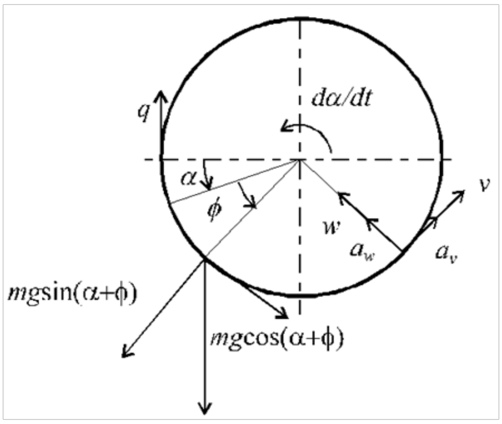

Figure 1 The rotating shell.

The components of the acceleration of the shell element have the form

$$a_v = \frac{\partial^2 v}{\partial t^2} - 2\frac{\mathrm{d}\alpha}{\mathrm{d}t}\frac{\partial w}{\partial t} + \frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2}(r-w) - \left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2 v$$

$$a_w = \frac{\partial^2 w}{\partial t^2} + 2\frac{\mathrm{d}\alpha}{\mathrm{d}t}\frac{\partial v}{\partial t} + \frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2}v + \left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2 (r-w) \qquad (1)$$

Taking into account the inertia, damping and gravity forces, the loading of an element can be written as

$$q_x = -c_1\frac{\partial u}{\partial t} - m\frac{\partial^2 u}{\partial t^2}$$

$$q_\phi = -c_2\frac{\partial v}{\partial t} - m\left(\frac{\partial^2 v}{\partial t^2} - 2\frac{\mathrm{d}\alpha}{\mathrm{d}t}\frac{\partial w}{\partial t} + \frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2}(r-w) - \left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2 v\right) + mg\cos(\alpha+\phi) - q_s\delta(-\alpha)$$

$$q_z = -c_3\frac{\partial w}{\partial t} - m\left(\frac{\partial^2 w}{\partial t^2} + 2\frac{\mathrm{d}\alpha}{\mathrm{d}t}\frac{\partial v}{\partial t} + \frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2}v + \left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2 (r-w)\right) - mg\sin(\alpha+\phi) + q_n\delta(-\alpha)$$

(2)

Here, (c_1, c_1, c_1) are coefficients of damping. The internal forces are assumed to be of the form

$$N'_\phi = m\left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2 r^2$$

$$N'_x = N_{x^1} + N_{x^2}\cos\phi$$

$$N'_{x\phi} = 0 \qquad (3)$$

The internal torque is

$$M_s(x) = m\int_0^x\int_0^{2\pi}\left(\frac{\partial^2 v}{\partial t^2} - 2\frac{\mathrm{d}\alpha}{\mathrm{d}t}\frac{\partial w}{\partial t} + \frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2}(r-w) - \left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2 v\right)(r-w)\,r\,d\phi\,dx \qquad (4)$$

The integration over entire length L of the shell gives the driving torque M_s. The angular acceleration is found as

$$\frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2} = \frac{\dfrac{M_s}{mr} - \displaystyle\int_0^L\int_0^{2\pi}\left(\left(\frac{\partial^2 v}{\partial t^2} - 2\frac{\mathrm{d}\alpha}{\mathrm{d}t}\frac{\partial w}{\partial t} - \left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2 v\right)(r-w)\right)d\phi\,dx}{\displaystyle\int_0^x\int_0^{2\pi}(r-w)^2\,d\phi\,dx} \qquad (5)$$

The components of the shell surface deflection can be expressed in the form

$$u(x.\phi,t) = UT_u, \qquad v(x.\phi,t) = VT_v, \qquad w(x.\phi,t) = WT_w \qquad (6)$$

Where

$$T_u = \begin{bmatrix} T_{un}^{(0)}(t) \\ T_{un}^{(CN)}(t) \\ T_{un}^{(SN)}(t) \end{bmatrix}, \quad T_v = \begin{bmatrix} T_{vn}^{(0)}(t) \\ T_{vn}^{(SN)}(t) \\ T_{vn}^{(CN)}(t) \end{bmatrix}, \quad T_w = \begin{bmatrix} T_{wn}^{(0)}(t) \\ T_{wn}^{(CN)}(t) \\ T_{wn}^{(SN)}(t) \end{bmatrix}, \tag{7}$$

$$\begin{aligned} U &= \left[U_n^{(0)}(x),\ \ U_n^{(CN)}(x)\cos N\phi,\ \ U_n^{(SN)}(x)\sin N\phi\right], \\ V &= \left[V_n^{(0)}(x),\ \ V_n^{(SN)}(x)\sin N\phi,\ \ V_n^{(CN)}(x)\cos N\phi\right], \\ W &= \left[W_n^{(0)}(x),\ \ W_n^{(CN)}(x)\cos N\phi,\ \ W_n^{(SN)}(x)\sin N\phi\right], \end{aligned} \tag{8}$$

$$\begin{aligned} U_n(x) &= \left[U_1(x),\ U_2(x), \dots,\ U_n(x)\right], \\ V_n(x) &= \left[V_1(x),\ V_2(x), \dots,\ V_n(x)\right], \\ W_n(x) &= \left[W_1(x),\ W_2(x), \dots,\ W_n(x)\right], \end{aligned} \tag{9}$$

For $n=1,2,3,\ldots n_{\max}$, $N=1,2,3,\ldots N_{\max}$.

Substituting (6-9) into equation (5) and carrying out the integration (5), one obtains

$$\frac{d^2\alpha}{dt^2} = \frac{\left(\begin{aligned} &\frac{M_s}{mr} - \overline{V}\frac{d^2T_v}{dt^2}r + 2\frac{d\alpha}{dt}\overline{W}\frac{dT_w}{dt}r + \left(\frac{d\alpha}{dt}\right)^2 \overline{V}T_v r \\ &+\left\langle \overline{V^T W}, \frac{d^2T_v}{dt^2}T_w^T \right\rangle - 2\frac{d\alpha}{dt}\left\langle \overline{W^T W}, \frac{dT_w}{dt}T_w^T \right\rangle - \left(\frac{d\alpha}{dt}\right)^2 \left\langle \overline{V^T W}, T_v T_w^T \right\rangle \end{aligned}\right)}{2\pi L r^2 - 2r\overline{W}T_w + \left\langle \overline{W^T W}, T_w T_w^T \right\rangle},$$

$$\int_0^L \int_0^{2\pi} (\ )\, d\phi dx = \overline{(\ )}, \quad \langle A, B\rangle = \sum_r \sum_c A_{rc}B_{rc}\ . \tag{10}$$

The partial differential equations governing the motion of shell element will now be replaced by ordinary differential equations using Ritz-Galerkin. Let's pre-multiply those equations[4,5] by the transposition of functions (8) and then integrate of the resultant expressions by parts with the use of the forces-displacement relationships and boundary conditions

$$\begin{aligned} &\int_0^{2\pi}\int_0^{l} U^T\left(\frac{\partial N_x}{\partial x} + \frac{1}{a}\frac{\partial N_{\phi x}}{\partial \phi} - Q_x\frac{\partial^2 w}{\partial x^2} - N_{x\phi}\frac{\partial^2 v}{\partial x^2}\right) dx d\phi + \\ &\int_0^{2\pi}\int_0^{l} U^T\left(-\frac{Q_\phi}{a}\left(\frac{\partial v}{\partial x} + \frac{\partial^2 w}{\partial x \partial \phi}\right) - \frac{N_\phi}{a}\left(\frac{\partial^2 v}{\partial x \partial \phi} - \frac{\partial w}{\partial x}\right) + q_x\right) dx d\phi = 0\,, \\ &\int_0^{2\pi}\int_0^{l} V^T\left(\frac{1}{a}\frac{\partial N_\phi}{\partial \phi} + \frac{\partial N_{x\phi}}{\partial x} + N_x\frac{\partial^2 v}{\partial x^2} - \frac{Q_x}{a}\left(\frac{\partial v}{\partial x} + \frac{\partial^2 w}{\partial x \partial \phi}\right)\right) dx d\phi + \\ &+\int_0^{2\pi}\int_0^{l} V^T\left(\frac{N_{\phi x}}{a}\left(\frac{\partial^2 v}{\partial x \partial \phi} - \frac{\partial w}{\partial x}\right) - \frac{Q_\phi}{a}\left(1 + \frac{\partial v}{a\partial \phi} + \frac{\partial^2 w}{a\partial \phi^2}\right) + q_\phi\right) dx d\phi = 0\,, \\ &\int_0^{2\pi}\int_0^{l} W^T\left(\frac{\partial Q_x}{\partial x} + \frac{1}{a}\frac{\partial Q_\phi}{\partial \phi} + \frac{\partial}{\partial x}\left(N_{x\phi}\left(\frac{v}{a} + \frac{\partial w}{a\partial \phi}\right)\right) + \frac{\partial}{\partial x}\left(N_x\frac{\partial w}{\partial x}\right)\right) dx d\phi + \\ &+\int_0^{2\pi}\int_0^{l} W^T\left(\frac{N_\phi}{a} + \frac{\partial}{\partial \phi}\left(\frac{N_\phi}{a}\left(\frac{v}{a} + \frac{\partial w}{a\partial \phi}\right)\right) + \frac{\partial}{\partial \phi}\left(\frac{N_{\phi x}}{a}\left(\frac{\partial w}{\partial x}\right)\right) + q_z\right) dx d\phi = 0\ . \end{aligned} \tag{11}$$

As a result of the integration, we obtain a set of three ordinary differential equations of second order

$$K_{11}T_u + K_{12}T_v + K_{13}T_w + K_{12}^{(1)}\left(\frac{d\alpha}{dt}\right)^2 T_v + K_{13}^{(1)}\left(\frac{d\alpha}{dt}\right)^2 T_w + C_{11}\frac{dT_u}{dt} + M_{11}\frac{d^2T_u}{dt^2} = F_1\,,$$

$$\begin{aligned} &K_{21}T_u + K_{22}T_v + K_{23}T_w + K_{22}^{(1)}\left(\frac{d\alpha}{dt}\right)^2 T_v + K_{23}^{(2)}\frac{d^2\alpha}{dt^2}T_w + C_{23}\frac{d\alpha}{dt}\frac{dT_w}{dt} + \\ &+C_{22}\frac{dT_v}{dt} + M_{22}\frac{d^2T_v}{dt^2} + M_{24}\frac{d^2\alpha}{dt^2} + F_{2c}\cos\alpha + F_{2s}\sin\alpha + F_2(\alpha) = 0, \end{aligned}$$

$$\begin{aligned} &K_{31}T_u + K_{32}T_v + K_{33}T_w + K_{32}^{(1)}\left(\frac{d\alpha}{dt}\right)^2 T_v + K_{33}^{(1)}\left(\frac{d\alpha}{dt}\right)^2 T_w + K_{32}^{(2)}\frac{d^2\alpha}{dt^2}T_v + \\ &+C_{32}\frac{d\alpha}{dt}\frac{dT_v}{dt} + C_{33}\frac{dT_w}{dt} + M_{33}\frac{d^2T_w}{dt^2} + F_{3c}\cos\alpha + F_{3s}\sin\alpha + F_3(\alpha) = 0. \end{aligned} \tag{12}$$

Introducing matrices of three times higher order, than the matrices in equations (12), these set can be rewritten as one equation

$$M\frac{d^2T}{dt^2} + C\frac{dT}{dt} + KT + \left(\frac{d\alpha}{dt}\right)^2 K^{(1)}T + \frac{d^2\alpha}{dt^2}K^{(2)}T + C^{(1)}\frac{d\alpha}{dt}\frac{dT}{dt} + F_c\cos\alpha + F_s\sin\alpha + F(\alpha) = 0 \tag{13}$$

Where $T=[T_u, T_v, T_w]$.

Adding now equation governing the driving torque of the motor

$$\frac{dM_s}{dt} = \frac{1}{T_s}\left(C_s\left(\Omega_s - \frac{d\alpha}{dt}\right) - M_s\right) \tag{14}$$

We obtained the set of three equations (10,13,14) from which angle of drum rotation *α,* components of elastic deflection *T* (7-9) and driving torque M_s can be found through numerical integration. Similar calculations were performed by the author for a spindle directly driven by three-phase electric motor.[14]

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Conflict of interest

Authors declare there is no conflict of interest.

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