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A new nonrelativistic investigation for interactions in one–electron atoms with modified inverse–square potential: noncommutative two and three dimensional space phase solutions at planck's and nano–scales

Abstract

In this paper, we present a novel theoretical analytical perform further investigation for the exact solvability of non–relativistic quantum spectrum systems for modified inverse–square potential (m.i.s.) potential is discussed by means Boopp's shift method instead to solving deformed Schrödinger equations with star product, in the framework of both noncommutativity (two –three) dimensional real space and phase (NC: 2D–RSP) and (NC: 3D–RSP). The exact corrections for excited states are found straightforwardly for interactions in one–electron atoms by means of the standard perturbation theory. Furthermore, the obtained corrections of energies are depended on four infinitesimals parameters (,) and (,), which are induced by position–position and momentum–momentum noncommutativity, (NC: 2D–RSP) and (NC: 3D–RSP), respectively, in addition to the discreet atomic quantum numbers: and (the angular momentum quantum number) and we have also shown that, the usual states in ordinary two and three dimensional spaces are cancelled and has been replaced by new degenerated sub–states in the new quantum symmetries of (NC: 2D–RSP) and (NC: 3D–RSP).

Keywords: the inverse–square potential, noncommutative space, phase, star product, boopp's shift method.

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Abbreviations: MIS: Modified Inverse Square potential; NC: 2D–3D–RSP: Noncommutativity (two–three) Dimensional Real Space Phase; CCRs: Canonical Commutations Relations; NNCCRs: New Noncommutative Canonical Commutations Relations; MSE: Modified Schrödinger Equations.

Introduction

It is well–known, that, the modern quantum mechanics, satisfied a big successful in the last few years, for describing atoms, nuclei, and molecules and their spectral behaviors based on three fundamental equations: Schrödinger, Klein–Gordon and Dirac. Schrödinger equation rest the first and the latest in terms of interest, it is playing a crucial role in devising well–behaved physical models in different fields of physics and chemists, many potentials are treated within the framework of nonrelativistic quantum mechanics based on this equation in two, three and D generalized spaces.[1–32] the quantum structure based to the ordinary canonical commutations relations (CCRs) in both Schrödinger and Heisenberg (the operators are depended on time) pictures (CCRs), respectively, as:

$$[x_i, p_j] = i\delta_{ij} \quad \text{and} \quad [x_i, x_j] = [p_i, p_j] = 0 \ldots\ldots\ldots\ldots (1.1)$$

$$[x_i(t), p_j(t)] = i\delta_{ij} \quad \text{and} \quad [x_i(t), x_j(t)] = [p_i(t), p_j(t)] = 0 \quad \ldots\ldots (1.2)$$

Where the two operators $(x_i(t), p_i(t))$ in Heisenberg picture are related to the corresponding two operators (x_i, p_i) in Schrödinger picture from the two projections relations:

$$x_i(t) = \exp(iH(t-t_0))\, x_i \exp(-iH(t-t_0)) \quad \text{and} \quad p_i(t) = \exp(iH(t-t_0))\, p_i \exp(-iH(t-t_0)) \ldots (1.3)$$

Here denote to the ordinary quantum Hamiltonian operator, recently, much considerable effort has been expanded on the solutions of Schrödinger, Dirac and Klein–Gordon equations to noncommutative quantum mechanics, the present paper investigates first the present new quantum structure which based to new noncommutative canonical commutations relations (NNCCRs) in both Schrödinger and Heisenberg pictures, respectively, as follows.[33–60]

$$\left[\hat{x}_i \overset{*}{,} \hat{p}_j\right] = i\delta_{ij}, \left[\hat{x}_i \overset{*}{,} \hat{x}_j\right] = i\theta_{ij} \quad \text{and} \quad \left[\hat{p}_i \overset{*}{,} \hat{p}_j\right] = i\bar{\theta}_{ij}$$
$$\left[\hat{x}_i(t) \overset{*}{,} \hat{p}_j(t)\right] = i\delta_{ij}, \left[\hat{x}_i(t) \overset{*}{,} \hat{x}_j(t)\right] = i\theta_{ij} \quad \text{and} \quad \left[\hat{p}_i(t) \overset{*}{,} \hat{p}_j(t)\right] = i\bar{\theta}_{ij} \quad \ldots\ldots\ldots\ldots\ldots (1.4)$$

Where the two new operators $(\hat{x}_i(t), \hat{p}_i(t))$ in Heisenberg picture are related to the corresponding two new operators $(\hat{x}_i, \hat{p}_i)$ in Schrödinger picture from the two projections relations:

$$\hat{x}_i(t) = \exp(iH_{nc}(t-t_0)) * \hat{x}_i * \exp(-iH_{nc}(t-t_0)) \quad \text{and} \quad \hat{p}_i(t) = \exp(iH_{nc}(t-t_0)) * \hat{p}_i * \exp(-iH_{nc}(t-t_0)) \ .(1.5)$$

Here H_{nc} denote to the new quantum Hamiltonian operator in the symmetries of (NC: 2D–RSP) and (*NC: 3D–RSP*). The very small two parameters $\theta^{\mu\nu}$ and $\bar{\theta}^{\mu\nu}$ (compared to the energy) are elements of two ant symmetric real matrixes and $(*)$ denote to the new star product, which is generalized between two arbitrary functions $f(x,p)$ and $g(x,p)$ to $(f*g)(x,p)$ instead of the usual product $(fg)(x,p)$ in ordinary (two–three) dimensional spaces.[39–63]

$$(f*g)(x,p) \equiv \exp\left(\frac{i}{2}\theta^{\mu\nu}\partial_\mu^x\partial_\nu^x + \frac{i}{2}\bar{\theta}^{\mu\nu}\partial_\mu^p\partial_\nu^p\right)(fg)(x,p) = \left(fg - \frac{i}{2}\theta^{\mu\nu}\partial_\mu^x f\partial_\nu^x g - \frac{i}{2}\bar{\theta}^{\mu\nu}\partial_\mu^p f\,\partial_\nu^p g\right)(x,p)\Big|_{(x^\mu = x^\nu,\, p^\mu = p^\nu)} + O\left(\theta^2, \bar{\theta}^2\right) \ldots.(2)$$

Where the two covariant derivatives $\left(\partial_{\mu}^{x} f(x,p),\partial_{\mu}^{p} f(x,p)\right)$ are denotes to the $\left(\frac{\partial f(x,p)}{\partial x^{\mu}},\frac{\partial f(x,p)}{\partial p^{\mu}}\right)$, respectively, and the two following terms $[-\frac{i}{2}\theta^{\mu\nu}\partial_{\mu}^{x} f(x,p)\partial_{\nu}^{x} g(x,p), -\frac{i}{2}\bar{\theta}^{\mu\nu}\partial_{\mu}^{p} f(x,p)\partial_{\nu}^{p} g(x,p)]$ are induced by (space–space) and (phase–phase) noncommutativity properties, respectively, a Boopp's shift method can be used, instead of solving any quantum systems by using directly star product procedure.[39–66]

$$\left[\hat{x}_i,\hat{x}_j\right]=i\theta_{ij} \text{ and } \left[\hat{p}_i,\hat{p}_j\right]=i\bar{\theta}_{ij} \quad (3.1)$$

The, four generalized positions and momentum coordinates in the noncommutative quantum mechanics $(\hat{x},\hat{y})$ and $(\hat{p}_x,\hat{p}_y)$ are depended with corresponding four usual generalized positions and momentum coordinates in the usual quantum mechanics (x,y) and (p_x,p_y) by the following four relations.[32–55]

$$\begin{cases}\hat{x}=x-\frac{\theta}{2}p_y, \quad \hat{y}=y+\frac{\theta}{2}p_x \\ \hat{p}_x=p_x+\frac{\bar{\theta}}{2}y \text{ and } \hat{p}_y=p_x-\frac{\bar{\theta}}{2}x\end{cases} \quad (3.2)$$

$$\begin{cases}\hat{x}=x-\frac{\theta_{12}}{2}p_y-\frac{\theta_{13}}{2}p_z, \hat{y}=y-\frac{\theta_{21}}{2}p_x-\frac{\theta_{23}}{2}p_z \\ \text{and } \hat{z}=z-\frac{\theta_{31}}{2}p_x-\frac{\theta_{32}}{2}p_y\end{cases} \quad (3.3)$$

and

$$\begin{cases}\hat{p}_x=p_x-\frac{\bar{\theta}_{12}}{2}y-\frac{\bar{\theta}_{13}}{2}z, \hat{p}_y=p_y-\frac{\bar{\theta}_{21}}{2}x-\frac{\bar{\theta}_{23}}{2}z \\ \text{and } \hat{p}_z=p_z-\frac{\bar{\theta}_{31}}{2}x-\frac{\bar{\theta}_{32}}{2}y\end{cases} \quad (3.4)$$

The non–vanish 9–commutators in (*NC–2D: RSP*) and (*NC–3D: RSP*) can be determined as follows:

$$\left[\hat{x},\hat{p}_x\right]=\left[\hat{y},\hat{p}_y\right]=i, \qquad \left[\hat{x},\hat{y}\right]=i\theta_{12} \text{ and } \left[\hat{p}_x,\hat{p}_y\right]=i\bar{\theta}_{12} \quad (3.5)$$

and

$$\left[\hat{x},\hat{p}_x\right]=\left[\hat{y},\hat{p}_y\right]=\left[\hat{z},\hat{p}_z\right]=i, \quad [\hat{x},\hat{y}]=i\theta_{12},[\hat{x},\hat{z}]=i\theta_{13},[\hat{y},\hat{z}]=i\theta_{23}, \quad \left[\hat{p}_x,\hat{p}_y\right]=i\bar{\theta}_{12},\left[\hat{p}_y,\hat{p}_z\right]=i\bar{\theta}_{23},\left[\hat{p}_x,\hat{p}_z\right]=i\bar{\theta}_{13} \quad (3.6)$$

Which allow us to getting the two operators $\hat{r}^2$ and $\hat{p}^2$ on a noncommutative two dimensional space–phase as follows.[32–48]

$$\hat{r}^2=r^2-\theta L_z \text{ and } \hat{p}^2=p^2+\bar{\theta}L_z \quad (4.1)$$

$$\hat{r}^2=r^2-\mathbf{L}\vec{\Theta} \text{ and } \frac{\hat{p}^2}{2\mu}=\frac{p^2}{2\mu}+\frac{\mathbf{L}\vec{\bar{\theta}}}{2\mu} \quad (4.2)$$

Where the two couplings $\mathbf{L}\Theta$ and $\mathbf{L}\vec{\bar{\theta}}$ are given by, respectively:

$$\mathbf{L}\Theta \equiv L_x\Theta_{12}+L_y\Theta_{23}+L_z\Theta_{13} \text{ and } \vec{\mathbf{L}}\vec{\bar{\theta}} \equiv L_x\bar{\theta}_{12}+L_y\bar{\theta}_{23}+L_z\bar{\theta}_{13} \quad (5.1)$$

It is–well known, that, in quantum mechanics, the three components (L_x, L_y and L_z) are determined, in Cartesian coordinates:

$$L_x=yp_z-zp_y, L_y=zp_x-xp_z \text{ and } L_z=xp_y-yp_x \quad (5.2)$$

The study of inverse–square potential has now become a very interest field due to their applications in different fields.[1] this work is aimed at obtaining an analytic expression for the eigenenergies of a inverse–square potential in (NC: 2D–RSP) and (NC: 3D–RSP) using the generalization Boopp's shift method based on mentioned formalisms on above equations to discover the new symmetries and a possibility to obtain another applications to this potential in different fields, it is important to notice that, this potential was studied, in ordinary two dimensional spaces, by authors Shi–Hai Dong and Guo–Hua Sun of the Ref. the Schrödinger equation with a Coulomb plus inverse–square potential in D dimensions.[1] The organization scheme of the study is given as follows: In next section, we briefly review the Schrödinger equation with inverse–square potential on based to Ref.[1] The Section 3, devoted to studying the (two–three) deformed Schrödinger equation by applying both Boopp's shift method to the inverse–square potential. In the fourth section and by applying standard perturbation theory we find the quantum spectrum of the excited states in (NC–2D: RSP) and (NC–3D: RSP) for spin–orbital interaction. In the next section, we derive the magnetic spectrum for studied potential. In the sixth section, we resume the global spectrum and corresponding noncommutative Hamiltonian for inverse–square potential. Finally, the important results and the conclusions are discussed in last section.

Review the eignenfunctions and the energy eigenvalues for inverse–square potential in ordinary two dimensional spaces

Here we will firstly describe the essential steps, which gives the solutions of time independent Schrödinger equation for a fermionic particle like electron of rest mass and its energy moving in inverse–square potential.[1]

$$V(r)=\frac{A}{r^2}-\frac{B}{r} \quad (6)$$

Where A and B are two positive constant coefficients. The above potential is the sum of Colombian $\left(-\frac{B}{r}\right)$ and inverse–square potential $\left(\frac{A}{r^2}\right)$, if we insert this potential into the non–relativistic Schrödinger equation; we obtain the following equation, in two and three dimensional spaces, respectively, as follows:

$$\left\{-\frac{\hbar^2}{2m_0}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2}{\partial\phi^2}\right]-\frac{A}{r^2}+\frac{B}{r}\right\}\Psi(r,\phi)=\mathrm{E}_{2d}\Psi(r,\phi) \quad (7.1)$$

$$\left\{-\frac{\hbar^2}{2m_0}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2(\sin\theta)^2}\frac{\partial^2}{\partial\phi^2}\right]-\frac{A}{r^2}+\frac{B}{r}\right\}\Psi(r,\theta,\phi)=\mathrm{E}_{3d}\Psi(r,\theta,\phi) \quad (7.2)$$

Here $\Psi(r,\phi)$ and $\Psi(r,\theta,\phi)$ is the solution in the (2–3) dimensional in (polar and spherical) coordinates, the complete wave function ($\Psi(r,\phi)$ and $\Psi(r,\theta,\phi)$ separated as follows:

$$\Psi(r,\phi)=R_l(r)e^{\pm i\phi} \quad (8.1)$$

and

$$\Psi(x)=R_l(r)Y_l^l(\theta,\phi) \quad (8.2)$$

Substituting eq. (8.1) and (8.2) into eq. (7.1) and (7.2), we obtain the radial function $R_l(r)$ satisfied the following equation, in (two–three) dimensional spaces.[1]

$$\frac{d^2R_l(\rho)}{d\rho^2}+\frac{2}{\rho}\frac{dR_l(\rho)}{d\rho}+\left(-\frac{1}{4}+\frac{\tau}{\rho}-\frac{2A+l^2}{\rho^2}\right)R_l(r)=0 \quad (9.1)$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)R_l(r)+\left[2(\mathrm{E}-V(r))-\frac{l(l+1)}{r^2}\right]R_l(r)=0 \quad (9.2)$$

Here $\rho=r\sqrt{-8E}$

and $\tau=B\sqrt{-\frac{1}{2E}}$. The proposed solutions of eqs. (9.1) and (9.2) are determined from the unifed relation:

$$R_l(\rho)=\rho^{\lambda}e^{-\frac{\rho}{2}}F(\rho) \quad (10)$$

where $\lambda=\frac{2-D+\sqrt{2A+k^2}}{2}$ and $k=2l+D-2$. We Companie between eqs. (9.1), (9.2) and (10) to obtains.[1]

$$\rho\frac{d^2F(\rho)}{d\rho^2}+(2\lambda+D-1-\rho)+\frac{dF(\rho)}{d\rho}+\left(\tau-\lambda-\frac{D-1}{2}\right)F(\rho)=0 \quad (11)$$

The confluent hypergeometric functions $\varphi(\lambda-\tau+1/2,2\lambda+1;\rho)$ are present the solutions of eq. (11).[1]

$$R(\rho)=\mathrm{N}\rho^{\lambda}e^{-\frac{\rho}{2}}\varphi(\lambda-\tau+(D-1)/2,2\lambda+D-1;\rho) \quad (12)$$

The constraint conditions on the potential parameters are determined from relations.[1]

$$\tau-\lambda-(D-1)/2=n'=0,1,2,\ldots$$

$$n=n'+\frac{\kappa}{2}-D/2+2=n'+l+1 \quad (13)$$

$$\tau=\mathrm{B}\sqrt{\frac{1}{-2\mathrm{E}}}=n-l-1+\lambda+(D-1)/2$$

The normalized wave functions $\Psi(\rho,\phi)$ expressed in terms of the radial functions and spherical harmonic functions read as.[1]

$$\Psi(\rho,\phi)=\left(\frac{4\mathrm{B}}{2n-2m+2s_2-1}\right)\left(\frac{(n-m-1)!}{(2n-2m+2s_2-1)(n-m+2s_2-1)!}\right)^{1/2}\rho^{s_2}e^{\frac{-\rho}{2}}L_{n-m-1}^{2s_2}(\rho)\exp(\pm im\phi) \quad (14.1)$$

$$\Psi(\rho)=\left(\frac{2B}{n}\right)^{\frac{3}{2}}\left[\frac{(n-l-1)!}{2n(n+l)!}\right]^{\frac{1}{2}}\rho^{l}e^{-\frac{\rho}{2}}L_{n-l-1}^{2l+1}(\rho)Y_l^l(\theta,\phi) \quad (14.2)$$

And the corresponding eigenvalues $\mathrm{E}(n,l,D)$ is determined from relation.[1]

$$\mathrm{E}(n,l,D)=-\frac{2\mathrm{B}}{\left(2n-2l-1+\sqrt{8\mathrm{A}+\kappa^2}\right)}=\begin{cases}-\frac{2\mathrm{B}^2}{\left(2n-2m-1+\sqrt{2A+m^2}\right)^2} & \text{for } D=2\\ -2B^2\left\{(2n)^{-2}-\frac{8A}{\kappa}(2n)^{-3}+\frac{16A^2}{\kappa^3}(2n)^{-3}+\frac{48A^2}{\kappa^2}(2n)^{-4}-\ldots\right\} & \text{for } D=3\end{cases} \quad (15)$$

The rest of this section is devoted to the reapply of some essential properties of generalized Laguerre polynomials $L_n^{(\beta)}(\rho)$ which are given by:

$$L_n^{(\beta)}(\rho)\equiv\frac{1}{2\prod i}\oint\frac{\exp\left(-\frac{\rho t}{1-t}\right)}{(1-t)^{\beta+1}t^{n+1}}dt \quad (16)$$

Where β is integer, this can be taking the exciplicitly mathematically forms.[1,65,66,67]

$$L_n^{(\beta)}(\rho)=\frac{\beta(\beta+n+1)}{n!\beta(\beta+1)}{}_1F_1(-n,\beta+1;\rho) \quad (17)$$

The Laguerre polynomials may be defined in terms of hypergeometric functions ${}_1F_1(-n,\beta+1;\rho)$, specifically the confluent hyper geometric functions, as:

$${}_1F_1(-n,\beta+1;\rho)=\sum_{n=0}^{\infty}\frac{a^{(n)}\rho^n}{b^{(n)}{}_{n!}} \quad (18.1)$$

Where $a^{(n)}$ is the Pochhammer symbol, which can be takes the particulars values $a^{(0)}=0$ and $a^{(n)}=a(a+1)\ldots(a+n-1)$, it is important to notice that, the hypergeometric functions have another common notation $\Phi(a,b,\rho)$ which considered as a function of a, $b=0,-1,-2,\ldots$, and the variable ρ. The generalized Laguerre polynomial can also be defined by the following equation:

$$L_n^{(\beta)}(\rho)=\sum_{i=0}^{n}\frac{(-1)^i\binom{n+\beta}{n-i}}{(i)!}\rho^i \quad (18.2)$$

Deformed schrödinger equations and modified inverse–square (m.i.s.) potential in both (nc–2d: rsp) and (nc–3d: rsp):

This section is devoted to constructing of non relativistic modified Schrödinger equations (m.s.e) in both (NC–2D: RSP) and (NC–3D: RSP) for (m.i.s.) potential; to achieve this subject, we apply the essentials following steps.[32–48]

Ordinary two dimensional Hamiltonian operators ($\hat{H}_{is2}(p_i,x_i)$, $\hat{H}_{is3}(p_i,x_i)$) will be replaced by new two dimensional Hamiltonian operators ($\hat{H}_{nc2-is}(\hat{p}_i,\hat{x}_i)$, $\hat{H}_{nc3-is}(\hat{p}_i,\hat{x}_i)$),

Ordinary complex wave function $\Psi(\vec{r})$ will be replacing by new complex wave function $\overline{\Psi}(\vec{\vec{r}})$,

Ordinary energies $E(n,l,2)$ and $E(n,l,3)$ will be replaced by new values $E_{nc2-is}(n,l,2,\ldots)$ and $E_{nc3-is}(n,l,3,\ldots)$, respectively.

And the last step corresponds to replace the ordinary old product by new star product $(*)$, which allow us to constructing the modified two dimensional Schrödinger equation in both (*NC–2D: RSP*) and (*NC–3D: RSP*) as for (*m.i.s.)* potential:

$$\hat{H}_{nc2-is}(\hat{p}_i,\hat{x}_i)*\hat{\Psi}(\vec{\vec{r}})=E_{nc2-is}(n,l,2,..)\hat{\Psi}(\vec{\vec{r}}) \quad (19.1)$$

and

$$\hat{H}_{nc3-is}(\hat{p}_i,\hat{x}_i)*\hat{\Psi}(\vec{\vec{r}})=E_{nc3-is}(n,l,3,..)\hat{\Psi}(\vec{\vec{r}}) \quad (19.2)$$

In order to use the ordinary product without star product, with new vision, as mentioned before, we apply the Boopp's shift method on the above eqs. (19.1) and (19.2) to obtain two reduced Schrödinger in both (*NC–2D: RSP*) and (*NC–3D: RSP*) for (*m.i.s.*) potential:

$$H_{nc2-is}(\hat{p}_i,\hat{x}_i)\psi(\vec{r})=E_{nc2-is}(n,l,2,..)\psi(\vec{r}) \quad (20.1)$$

and

$$H_{nc3-is}(\hat{p}_i,\hat{x}_i)\psi(\vec{r})=E_{nc3-is}(n,l,3,..)\psi(\vec{r}) \quad (20.2)$$

Where the new operators of Hamiltonian $H_{nc2-is}(\hat{p}_i,\hat{x}_i)$ and $H_{nc3-is}(\hat{p}_i,\hat{x}_i)$ can be expressed in three general varieties: both noncommutative space and noncommutative phase (*NC–2D: RSP, NC–3D: RSP*), only noncommutative space (*NC–2D: RS, NC–3D: RS*) and only noncommutative phase (*NC: 2D–RP, NC: 3D–RP*) as, respectively:

$$H_{nc(2-3)-is}(\hat{p}_i,\hat{x}_i)\equiv H\left(p_x+\frac{\bar{\theta}}{2}y,p_y-\frac{\bar{\theta}}{2}x,x-\frac{\theta}{2}p_y,y+\frac{\theta}{2}p_x\right) \quad \text{for NC-2D: RSP and NC-3D: RSP} \quad (21.1)$$

$$H_{nc(2-3)-is}(\hat{p}_i,\hat{x}_i)\equiv H\left(p_x,p_y,x-\frac{\theta}{2}p_y,y+\frac{\theta}{2}p_x\right) \quad \text{for NC-2D: RS and NC-3D: RS} \quad (22.2)$$

$$H_{nc(2-3)-is}(\hat{p}_i,\hat{x}_i)\equiv H\left(p_x+\frac{\bar{\theta}}{2}y,p_x-\frac{\bar{\theta}}{2}x,x,y\right) \quad \text{for NC-2D: RP and NC-3D: RP} \quad (22.3)$$

In recently work, we are interest with the first variety (21.1), after straightforward calculations, we can obtain the five important terms, which will be use to determine the (*m.i.s.*) potential in (*NC: 2D– RSP*) and (*NC: 3D–RSP*), respectively, as:

$$\frac{A}{\hat{r}^2}=\frac{A}{r^4}+\frac{A\theta L_z}{r^4},\quad \frac{B}{\hat{r}}=\frac{B}{r}-\frac{B\theta L_z}{2r^3}\quad \text{and}\quad \frac{\hat{p}^2}{2m_0}=\frac{p^2}{2m_0}+\frac{\mathbf{L}\vec{\vec{\boldsymbol{\theta}}}}{2m_0} \quad (23)$$

and

$$\frac{A}{\hat{r}^2}=\frac{A}{r^4}+\frac{A\vec{L}\vec{\Theta}}{r^4},\quad \frac{B}{\hat{r}}=\frac{B}{r}-\frac{B\vec{L}\vec{\Theta}}{2r^3}\quad \text{and}\quad \frac{\hat{p}^2}{2m_0}=\frac{p^2}{2m_0}+\frac{\bar{\theta}L_z}{2m_0} \quad (24)$$

Which allow us to obtaining the global potential operator $H_{nc2-is}(\hat{p}_i,\hat{x}_i)$ and $H_{nc3-is}(\hat{p}_i,\hat{x}_i)$ for (*m.i.s*) potential in both (*NC: 2D–RSP*) and (*NC: 3D–RSP*), respectively, as:

$$H_{nc2-is}(\hat{p}_i,\hat{x}_i)=\frac{A}{r^2}-\frac{B}{r}+\frac{p^2}{2m_0}+\frac{\bar{\theta}L_z}{2m_0}+\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)\theta L_z \quad (25.1)$$

and

$$H_{nc3-is}(\hat{p}_i,\hat{x}_i)=\frac{A}{r^2}-\frac{B}{r}+\frac{p^2}{2m_0}+\frac{\mathbf{L}\vec{\vec{\boldsymbol{\theta}}}}{2m_0}+\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)\vec{\mathbf{L}}\vec{\Theta} \quad (25.2)$$

It's clearly, that the four first terms are given the ordinary inverse–square potential and kinetic energy in (2D–3D) spaces, while the rest terms are proportional's with infinitesimals parameters (θ , $\bar{\theta}$) and (Θ , $\bar{\theta}$), thus, we can considered as a perturbations terms, we noted by $\hat{H}_{2-pert}(r,A,B,\theta,\bar{\theta})$ and $\hat{H}_{3-pert}(r,A,B,\Theta,\bar{\theta})$ for (*NC: 2D–RSP*) and (*NC: 3D–RSP*) symmetries, respectively, as:

$$\hat{H}_{2-pert}(r,A,B,\theta,\bar{\theta})=\frac{L_z\bar{\theta}}{2m_0}+\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)\theta L_z \quad (26.1)$$

and

$$\hat{H}_{3-pert}(r,A,B,\Theta,\bar{\theta})=\frac{\mathbf{L}\vec{\vec{\boldsymbol{\theta}}}}{2m_0}+\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)\vec{\mathbf{L}}\vec{\Theta} \quad (26.2)$$

The Exact Spin–Orbital Hamiltonian and the Corresponding Spectrum for (m.i.s.) Potential in both (NC: 2D– RSP) and (NC: 3D– RSP) Symmetries for Excited States for One–Electron Atoms

The exact spin–orbital hamiltonian for (m.i.s.) potential in both (NC: 2D– RSP) and (NC: 3D– RSP) symmetries for one–electron atoms

Again, the perturbative two terms $\hat{H}_{2-pert}\left(r,A,B,\theta,\bar{\theta}\right)$ and $\hat{H}_{3-pert}\left(r,A,B,\Theta,\bar{\theta}\right)$ can be rewritten to the equivalent physical form for (*m.i.p.)* potential:

$$\hat{H}_{2-pert}\left(r,A,B,\theta,\bar{\theta}\right)=\left\{\frac{\bar{\theta}}{2m_0}+\theta\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)\right\}\vec{S}\vec{L} \quad (26.3)$$

$$\hat{H}_{3-pert}\left(r,A,B,\Theta,\bar{\theta}\right)=\left\{\frac{\bar{\theta}}{2m_0}+\Theta\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)\right\}\vec{S}\vec{L} \quad (26.4)$$

Furthermore, the above perturbative terms $\hat{H}_{2-pert}\left(r,A,B,\theta,\bar{\theta}\right)$ and $\hat{H}_{3-pert}\left(r,A,B,\Theta,\bar{\theta}\right)$ can be rewritten to the following new equivalent form for (m.i.p.) potential:

$$\hat{H}_{2-pert}\left(r,A,B,\theta,\bar{\theta}\right)=\frac{1}{2}\left\{\frac{\bar{\theta}}{2m_0}+\theta\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)\right\}\left(\overleftrightarrow{J}^2-\overleftrightarrow{L}^2-\overleftrightarrow{S}^2\right) \quad (27.1)$$

$$\hat{H}_{3-pert}\left(r,A,B,\Theta,\bar{\theta}\right)=\frac{1}{2}\left\{\frac{\bar{\theta}}{2m_0}+\Theta\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)\right\}\left(\vec{J}^2-\vec{L}^2-\vec{S}^2\right) \quad (27.2)$$

To the best of our knowledge, we just replace the coupling spin–orbital $\vec{S}\vec{L}$ by the expression $\frac{1}{2}\left(\vec{J}^2-\vec{L}^2-\vec{S}^2\right)$**,** in quantum mechanics. The set ($H_{nc(2-3)-is}\left(\hat{p}_i,\hat{x}_i\right)$, J^2, L^2, S^2 and J_z) forms a complete of conserved physics quantities and the eigenvalues of the spin orbital coupling operator are:

$$p_{\pm}\left(j=l\pm1/2,l,s=1/2\right)\equiv\frac{1}{2}\begin{cases}\left(l+\frac{1}{2}\right)(l+\frac{1}{2}+1)+l(l+1)-\frac{3}{4}\equiv p_+ & \text{for} \quad j=l+\frac{1}{2}\Rightarrow\text{polarization–up}\\ \left(l-\frac{1}{2}\right)(l-\frac{1}{2}+1)+l(l+1)-\frac{3}{4}\equiv p_- & \text{for} \quad j=l+\frac{1}{2}\Rightarrow\text{polarization–}down\end{cases} \quad (27.3)$$

Which allows us to form a diagonal (2×2) and (3×3) two matrixes, with non null elements are [$\left(\hat{H}_{so-is}\right)_{11}$ and $\left(\hat{H}_{so-is}\right)_{22}$] and [$\left(\hat{H}_{so-is}\right)_{11}$, $\left(\hat{H}_{so-is}\right)_{22}$, $\left(\hat{H}_{so-is}\right)_{33}$] for (*m.i.s.*) potential in (*NC: 2D–RSP*) and (*NC: 3D–RSP*), respectively, as:

$$\left(H_{so-ip}\right)_{11}=p_+\left(\frac{\bar{\theta}}{2m_0}+\theta\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)\right)\text{if } j=l+\frac{1}{2} \Rightarrow \text{spin -up}$$

$$\text{and } \left(H_{so-ip}\right)_{22}=p_-\left(\frac{\bar{\theta}}{2m_0}+\theta\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)\right)\text{if } j=l-\frac{1}{2} \Rightarrow \text{spin -down} \quad (28.1)$$

$$\left(\hat{H}_{so-is}\right)_{11}=p_+\left\{\frac{\bar{\theta}}{2m_0}+\Theta\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)\right\}\text{if } j=l+\frac{1}{2} \Rightarrow \text{spin up}$$

$$\left(\hat{H}_{so-is}\right)_{22}=p_-\left\{\frac{\bar{\theta}}{2m_0}+\Theta\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)\right\}\text{if } j=l-\frac{1}{2} \Rightarrow \text{spin down} \quad (28.2)$$

$$\left(\hat{H}_{so-is}\right)_{33}=0$$

Substituting two equations (26.1) and (26.2) into two equations (20.1) and (20.12), respectively and then, the radial parts of the modified Schrödinger equations, satisfying the following important two equations:

$$\frac{d^2R_l(\rho)}{d\rho^2}+\frac{2}{\rho}\frac{dR_l(\rho)}{d\rho}+\left(-\frac{1}{4}+\frac{\tau}{\rho}-\frac{2A+l^2}{\rho^2}-\left\{\frac{\bar{\theta}}{2m_0}+\theta\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)\right\}\vec{S}\vec{L}\right)R_l(r)=0 \quad (29.1)$$

and

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)R_l(r)+\left[2\left(E_{nc3-is}(n,l,3,...)-V(r)\right)-\frac{l(l+1)}{r^2}-\left\{\frac{\bar{\theta}}{2m_0}+\Theta\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)\right\}\vec{S}\vec{L}\right]R_l(r)=0 \quad (29.2)$$

for (*m.i.s.*) potential in (*NC: 2D–RSP*) and (*NC: 3D–RSP*), ii is clearly that the above equations including equations (26.1) and (26.2), the perturbative terms of Hamiltonian operator, which we are subject of discussion in next sub–section.

The exact spin–orbital spectrum for (m.i.s.) potential in both (NC: 2D– RSP) and (NC: 3D– RSP) symmetries for states for one–electron atoms

In this sub section, we are going to study the modifications to the energy levels ($E_{nc-per:u}(\theta,\bar{\theta})$, $E_{nc-per:D}(\theta,\bar{\theta})$) and ($E_{nc-per:u}(\Theta,\bar{\theta})$, $E_{nc-per:D}(\Theta,\bar{\theta})$) for spin up and spin down, respectively, at first order of parameters (θ , $\bar{\theta}$) and (Θ , $\bar{\theta}$), for excited states n^{th} , obtained by applying the standard perturbation theory, using eqs. (14.1) (14.2), (27.1) and (27.2) corresponding (*NC–2D: RSP*) and (*NC–3D: RSP*), respectively, as:

$$E_{nc-per:u}(\theta,\bar{\theta}) \equiv 2\Pi p_{+}\int R^{*}(r)\left[\theta\left(\frac{A}{r^{4}}-\frac{B}{2r^{3}}\right)+\frac{\bar{\theta}}{2m_{0}}\right]R(r)rdr \qquad Si \quad j=l+\frac{1}{2}$$

and

$$E_{nc-per:D}(\theta,\bar{\theta}) \equiv 2\Pi p_{-}\int R^{*}(r)\left[\theta\left(\frac{A}{r^{4}}-\frac{B}{2r^{3}}\right)+\frac{\bar{\theta}}{2m_{0}}\right]R(r)rdr \qquad Si \quad j=l-\frac{1}{2} \qquad (30.1)$$

$$E_{nc-per:u}(\Theta,\bar{\theta}) \equiv -\frac{\alpha p_{+}}{(-8E)^{3/2}}\left(\frac{2B}{n}\right)^{3}\frac{(n-l-1)!}{2n(n+l)!}\int \rho^{2l+2}e^{-\rho}\left[L_{n-l-1}^{2l+1}(\rho)\right]^{2}\left[\Theta\left(\frac{A'}{\rho^{4}}-\frac{B'}{2\rho^{3}}\right)+\frac{\bar{\theta}}{2m_{0}}\right]d\rho$$

$$E_{nc-per:D}(\Theta,\bar{\theta}) \equiv -\frac{\alpha p_{-}}{(-8E)^{3/2}}\left(\frac{2B}{n}\right)^{3}\frac{(n-l-1)!}{2n(n+l)!}\int \rho^{2l+2}e^{-\rho}\left[L_{n-l-1}^{2l+1}(\rho)\right]^{2}\left[\Theta\left(\frac{A'}{\rho^{4}}-\frac{B'}{2\rho^{3}}\right)+\frac{\bar{\theta}}{2m_{0}}\right]d\rho \qquad (30.2)$$

A direct simplification gives:

$$E_{nc-per:u}(\theta,\bar{\theta}) \equiv \frac{2\Pi p_{+}}{(-8E)}\left(\frac{4B}{2n-2m+2s_{2}-1}\right)^{2}\left(\frac{(n-m-1)!}{(2n-2m+2s_{2}-1)(n-m+2s_{2}-1)!}\right)\left(\theta\sum_{i=1}^{2}T_{i-2}+\frac{\bar{\theta}}{2m_{0}}T_{3-2}\right)$$

$$E_{nc-per:D}(\theta,\bar{\theta}) \equiv \frac{2\Pi p_{-}}{(-8E)}\left(\frac{4B}{2n-2m+2s_{2}-1}\right)^{2}\left(\frac{(n-m-1)!}{(2n-2m+2s_{2}-1)(n-m+2s_{2}-1)!}\right)\left(\theta\sum_{i=1}^{2}T_{i-2}+\frac{\bar{\theta}}{2m_{0}}T_{3-2}\right) \qquad (31.1)$$

and

$$E_{nc-per:u}(\Theta,\bar{\theta}) \equiv -\frac{\alpha p_{+}}{(-8E)^{3/2}}\left(\frac{2B}{n}\right)^{3}\frac{(n-l-1)!}{2n(n+l)!}\left(\Theta\sum_{i=1}^{2}T_{i-3}+\frac{\bar{\theta}}{2m_{0}}T_{3-3}\right)$$

$$E_{nc-per:D}(\Theta,\bar{\theta}) \equiv -\frac{\alpha p_{-}}{(-8E)^{3/2}}\left(\frac{2B}{n}\right)^{3}\frac{(n-l-1)!}{2n(n+l)!}\left(\Theta\sum_{i=1}^{2}T_{i-3}+\frac{\bar{\theta}}{2m_{0}}T_{3-3}\right) \qquad (32.2)$$

Where, the 6– terms: (T_{i-2} , T_{i-3} i=1,2), T_{3-2} and T_{3-3} are given by:

$$T_{1-2} = A'\int_{0}^{+\infty} e^{-\rho}\rho^{2s_{2}-3}\left[L_{n-m-1}^{2s_{2}}(\rho)\right]^{2}d\rho$$

$$T_{2-2} = -\frac{B'}{2}\int_{0}^{+\infty} e^{-\rho}\rho^{2s_{2}-2}\left[L_{n-m-1}^{2s_{2}}(\rho)\right]^{2}d\rho \qquad (33.1)$$

$$T_{3-2} = \int_{0}^{+\infty} e^{-\rho}\rho^{2s_{2}+1}\left[L_{n-m-1}^{2s_{2}}(\rho)\right]^{2}d\rho$$

and

$$T_{1-3} = A'\int_{0}^{+\infty} \rho^{2l-2}e^{-\rho}\left[L_{n-l-1}^{2l+1}(\rho)\right]^{2}d\rho$$

$$T_{2-3} = -\frac{B'}{2}\int_{0}^{+\infty} \rho^{2l-1}e^{-\rho}\left[L_{n-l-1}^{2l+1}(\rho)\right]^{2}d\rho \qquad (33.2)$$

$$T_{3-3} = \int_{0}^{+\infty} \rho^{2l+2}e^{-\rho}\left[L_{n-l-1}^{2l+1}(\rho)\right]^{2}d\rho$$

With new notation $A'=(-8E)^2$ and $B'=(-8E)^{3/2}$, know we apply the special integral.[1, 61]

$$J_{n,\alpha}^{(\gamma)}=\int_0^\infty e^{-x}x^{\alpha+\gamma}\left[L_n^\alpha(x)\right]^2dx=\frac{(\alpha+n)!}{n!}\sum_{k=0}^{n}(-1)k\frac{\Gamma(n+\kappa+\gamma)}{\Gamma(-\kappa-\gamma)}\frac{(\alpha+k+\gamma)!}{(\alpha+k)!}\frac{1}{\kappa!(n-\kappa)!}, \quad (34)$$

$\mathrm{Re}(\alpha+\gamma+1)\rangle 0$, γ can be takes: (-3, -2 and $+1$), $\alpha=2s_2$ and $n\to n-m-1$, which allow us to obtaining in (*NC: 2D–RSP*):

$$T_{1-2}=A'J_{n-m-1,2l+1}^{(-3)}=\frac{(2s_2+n-m-1)!}{(n-m-1)!}\sum_{k=0}^{n}(-1)k\frac{\Gamma(n-m+\kappa-4)}{\Gamma(-\kappa+3)}\frac{(2s_2+k-3)!}{(2s_2+k)!}\frac{1}{\kappa!(n-m-1-\kappa)!}, \quad (35.1)$$

$$T_{2-2}=-\frac{B'}{2}J_{n-l-1,2l+1}^{(-2)}=\frac{(2s_2+n-m-1)!}{(n-m-1)!}\sum_{k=0}^{n}(-1)k\frac{\Gamma(n-m+\kappa-3)}{\Gamma(-\kappa+2)}\frac{(2s_2+k-2)!}{(2s_2+k)!}\frac{1}{\kappa!(n-m-1-\kappa)!} \quad (35.2)$$

$$T_{3-2}=J_{n-l-1,2l+1}^{(+1)}=\frac{(2s_2+n-m-1)!}{(n-m-1)!}\sum_{k=0}^{n}(-1)k\frac{\Gamma(n-m+\kappa)}{\Gamma(-\kappa-1)}\frac{(2s_2+k+1)!}{(2s_2+k)!}\frac{1}{\kappa!(n-m-1-\kappa)!} \quad (35.3)$$

For (NC: 3D–RSP) symmetries, we have:

$$T_{1-3}=A'J_{n-l-1,2l+1}^{(-3)}=\frac{(2l+1+n-l-1)!}{(n-l-1)!}\sum_{k=0}^{n}(-1)k\frac{\Gamma(n-l-1+\kappa-3)}{\Gamma(-\kappa+3)}\frac{(2l+1+k-3)!}{(2l+1+k)!}\frac{1}{\kappa!(n-l-1-\kappa)!} \quad (36.1)$$

$$T_{-31}=-\frac{B'}{2}J_{n-l-1,2l+1}^{(-2)}=\frac{(2l+1+n-l-1)!}{(n-l-1)!}\sum_{k=0}^{n}(-1)k\frac{\Gamma(n-l-1+\kappa-2)}{\Gamma(-\kappa+2)}\frac{(2l+1+k-2)!}{(2l+1+k)!}\frac{1}{\kappa!(n-l-1-\kappa)!} \quad (36.2)$$

$$T_{1-3}=J_{n-l-1,2l+1}^{(+1)}=\frac{(2l+1+n-l-1)!}{(n-l-1)!}\sum_{k=0}^{n}(-1)k\frac{\Gamma(n-l-1+\kappa+1)}{\Gamma(-\kappa-1)}\frac{(2l+1+k+1)!}{(2l+1+k)!}\frac{1}{\kappa!(n-l-1-\kappa)!} \quad (36.3)$$

Which allow us to obtaining the exact modifications of fundamental states ($E_{nc-per:u}(\theta,\bar{\theta})$, $E_{nc-per:D}(\theta,\bar{\theta})$) and ($E_{nc-per:u}(\Theta,\bar{\theta})$, $E_{nc-per:D}(\Theta,\bar{\theta})$) produced by spin–orbital effect:

$$E_{nc-per:u}(\theta,\bar{\theta})\equiv\frac{2\Pi p_+}{(-8E)}\left(\frac{4\mathrm{B}}{2n-2m+2s_2-1}\right)^2\left(\frac{(n-m-1)!}{(2n-2m+2s_2-1)(n-m+2s_2-1)!}\right)\left(\theta T_{S2-is}(A,B,n,l)+\frac{\bar{\theta}}{2m_0}T_{3-2}\right)$$
$$E_{nc-per:D}(\theta,\bar{\theta})\equiv\frac{2\Pi p_+}{(-8E)}\left(\frac{4\mathrm{B}}{2n-2m+2s_2-1}\right)^2\left(\frac{(n-m-1)!}{(2n-2m+2s_2-1)(n-m+2s_2-1)!}\right)\left(\theta T_{S2-is}(A,B,n,l)+\frac{\bar{\theta}}{2m_0}T_{3-23}\right) \quad (37.1)$$

and

$$E_{nc-per:u}(\Theta,\bar{\theta})\equiv-\frac{\alpha p_+}{(-8E)^{3/2}}\left(\frac{2B}{n}\right)^3\frac{(n-l-1)!}{2n(n+l)!}\left(\Theta T_{s3-is}(A,B,n,l)+\frac{\bar{\theta}}{2m_0}T_{3-3}\right)$$
$$E_{nc-per:D}(\Theta,\bar{\theta})\equiv-\frac{\alpha p_-}{(-8E)^{3/2}}\left(\frac{2B}{n}\right)^3\frac{(n-l-1)!}{2n(n+l)!}\left(\Theta T_{s3-is}(A,B,n,l)+\frac{\bar{\theta}}{2m_0}T_{3-3}\right) \quad (37.2)$$

Where, the two factors $T_{s2-is}(A,B,n,l)$ and $T_{s3-is}(A,B,n,l)$ are given by, respectively:

$$T_{s2-is}(A,B,n,l)=\sum_{i=1}^{2}T_{i-2}$$
$$T_{s3-is}(A,B,n,l)=\sum_{i=1}^{2}T_{i-3} \quad (38)$$

The exact magnetic spectrum for (m.i.s.) potential in both (NC: 2D– RSP) and (NC: 3D– RSP) symmetries for excited states for one–electron atoms

Having obtained the exact modifications to the energy levels ($E_{nc-per:u}(\theta,\bar{\theta})$, $E_{nc-per:D}(\theta,\bar{\theta})$) and ($E_{nc-per:u}(\Theta,\bar{\theta})$, $E_{nc-per:D}(\Theta,\bar{\theta})$), for exited n^{th} states, produced with spin–orbital induced Hamiltonians operators, we now consider interested physically meaningful phenomena, which produced from the perturbative terms of inverse–square potential related to the influence of an external uniform magnetic field, it's sufficient to apply the following three replacements to describing these phenomena:

$$\frac{L_z\bar{\theta}}{2m_0}+\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)\theta L_z\to\left\{\frac{\bar{\sigma}}{2m_0}+\chi\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)\right\}\vec{H}\vec{L} \quad (39.1)$$

$$\frac{\vec{\mathbf{L}}\vec{\boldsymbol{\theta}}}{2m_0}+\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)\vec{\mathbf{L}}\vec{\Theta}\rightarrow\left\{\frac{\bar{\sigma}}{2m_0}+\chi\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)\right\}\vec{H}\vec{L} \quad (39.2)$$

$$\theta\rightarrow\chi H,\Theta\rightarrow\chi H \quad \text{and } \bar{\theta}\rightarrow\bar{\sigma}H \quad (39.3)$$

Here χ and $\bar{\sigma}$ are infinitesimal real proportional's constants, and we choose the magnetic field $\vec{H}=H\vec{k}$, which allow us to introduce the modified new magnetic Hamiltonians $\hat{H}_{m2-is}\left(r,A,B,\chi,\bar{\sigma}\right)$ and $\hat{H}_{m3-is}\left(r,A,B,\chi,\bar{\sigma}\right)$ in (*NC: 2D–RSP*) and (*NC: 3D–RSP*), respectively, as:

$$\hat{H}_{m2-is}\left(r,A,B,\chi,\bar{\sigma}\right)=\left(\chi\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)+\frac{\bar{\sigma}}{2m_0}\right)\left(\vec{H}\vec{J}-\vec{S}\vec{H}\right) \quad (40.1)$$

and

$$\hat{H}_{m3-is}\left(r,A,B,\chi,\bar{\sigma}\right)=\left(\chi\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)+\frac{\bar{\sigma}}{2m_0}\right)\left(\vec{H}\vec{J}-\vec{S}\vec{H}\right) \quad (40.2)$$

Here $\left(-\vec{S}\vec{H}\right)$ denote to the ordinary Hamiltonian of Zeeman Effect. To obtain the exact noncommutative magnetic modifications of energy ($E_{\text{mag2-is}}\left(\theta,\bar{\theta},n,m,A,B\right)$, $E_{\text{mag-3is}}\left(\Theta,\bar{\theta},n,l,A,B\right)$) for modified inverse–square potential, which produced automatically by the effect of $\hat{H}_{m2-is}\left(r,A,B,\chi,\bar{\sigma}\right)$ and $\hat{H}_{m3-is}\left(r,A,B,\chi,\bar{\sigma}\right)$, we make the following three simultaneously replacements:

$$p_{+}\rightarrow\text{m},(\theta,\Theta)\rightarrow(\chi,\chi) \quad \text{and} \quad \bar{\theta}\rightarrow\bar{\sigma}H \quad (41)$$

In two Eqs. (37.1) and (37.2) to obtain the two values $E_{\text{mag2-is}}\left(\theta,\bar{\theta},n,m,A,B\right)$ and $E_{\text{mag-3is}}\left(\Theta,\bar{\theta},n,l,A,B\right)$ for the exact magnetic modifications of spectrum corresponding n^{th} excited states, in (*NC–2D: RSP*) and (*NC–3D: RSP*), respectively, as:

$$E_{\text{mag2-is}}\left(\theta,\bar{\theta},n,m,A,B\right)\equiv\frac{2\prod mH}{(-8E)}\left(\frac{4\text{B}}{2n-2m+2s_2-1}\right)^2\left(\frac{(n-m-1)!}{(2n-2m+2s_2-1)(n-m+2s_2-1)!}\right)\left(\chi T_{s2-is}\left(A,B,n,l\right)+\frac{\bar{\sigma}}{2m_0}T_{3-2}\left(A,B,n,l\right)\right) \quad (42.1)$$

and

$$E_{\text{mag-3is}}\left(\Theta,\bar{\theta},n,l,A,B\right)=-\frac{mH}{(-8E)^{3/2}}\left(\frac{2B}{n}\right)^3\frac{(n-l-1)!}{2n(n+l)!}\left(\chi T_{s3-is}\left(A,B,n,l\right)+\frac{\bar{\sigma}}{2m_0}T_{3-3}\left(A,B,n,l\right)\right) \quad (42.2)$$

Where m denote to the angular momentum quantum number, $-l\leq m\leq +l$, which allow us to fixing ($2l+1$) values for the orbital angular momentum quantum numbers.

Results of exact modified global spectrum of the lowest excitations states for (m.i.s.) potential in both (nc:2d– rsp) and (nc:3d– rsp) symmetries for one–electron atoms

Let us now resume the eigenenergies of the modified Schrödinger equations obtained in this paper, the total modified energies ($E_{nc-u}\left(\theta,\bar{\theta}\right)$, $E_{nc-D}\left(\theta,\bar{\theta}\right)$) and ($E_{nc-u}\left(\Theta,\bar{\theta}\right)$, $E_{nc-D}\left(\Theta,\bar{\theta}\right)$) of a particle fermionic with spin up and spin down are determined corresponding n^{th} excited states, respectively, for modified inverse–square potential in (*NC: 2D–RSP*) and (*NC: 3D–RSP*), on based to the obtained new results (10.a), (37.1), (37.2), (41.1), (41.2) and (37.b), in addition to the original results (17) of energies we obtain the four new values of global energies:

$$E_{nc-u}\left(\theta,\bar{\theta}\right)\equiv-\frac{2\text{B}^2}{\left(2n-2m-1+\sqrt{2A+m^2}\right)^2}+\frac{2\prod p_{+}}{(-8E)}\left(\frac{4\text{B}}{2n-2m+2s_2-1}\right)^2\left(\frac{(n-m-1)!}{(2n-2m+2s_2-1)(n-m+2s_2-1)!}\right)\left(\theta T_{s-is}\left(A,B,n,l\right)+\frac{\bar{\theta}}{2m_0}T_3\right)$$
$$+\frac{2\prod mH}{(-8E)}\left(\frac{4\text{B}}{2n-2m+2s_2-1}\right)^2\left(\frac{(n-m-1)!}{(2n-2m+2s_2-1)(n-m+2s_2-1)!}\right)\left(\chi T_{s2-is}\left(A,B,n,l\right)+\frac{\bar{\sigma}}{2m_0}T_{3-2}\left(A,B,n,l\right)\right) \quad (43.1)$$

$$E_{nc-D}\left(\Theta,\bar{\theta}\right)\equiv-\frac{2\text{B}^2}{\left(2n-2m-1+\sqrt{2A+m^2}\right)^2}+\frac{2\prod p_{-}}{(-8E)}\left(\frac{4\text{B}}{2n-2m+2s_2-1}\right)^2\left(\frac{(n-m-1)!}{(2n-2m+2s_2-1)(n-m+2s_2-1)!}\right)\left(\theta T_{s-is}\left(A,B,n,l\right)+\frac{\bar{\theta}}{2m_0}T_3\right)$$
$$+\frac{2\prod mH}{(-8E)}\left(\frac{4\text{B}}{2n-2m+2s_2-1}\right)^2\left(\frac{(n-m-1)!}{(2n-2m+2s_2-1)(n-m+2s_2-1)!}\right)\left(\chi T_{s2-is}\left(A,B,n,l\right)+\frac{\bar{\sigma}}{2m_0}T_{3-2}\left(A,B,n,l\right)\right) \quad (43.2)$$

$$E_{nc-u}\left(\Theta,\bar{\theta}\right)\equiv-2B^2\left\{(2n)^{-2}-\frac{8A}{\kappa}(2n)^{-3}+\frac{16A^2}{\kappa^3}(2n)^{-3}+\frac{48A^2}{\kappa^2}(2n)^{-4}-...\right\}-\frac{\alpha p_+}{(-8E)^{3/2}}\left(\frac{2B}{n}\right)^3\frac{(n-l-1)!}{2n(n+l)!}\left(\Theta T_{s-is}(A,B,n,l)+\frac{\bar{\theta}}{2m_0}T_3\right)$$
$$-\frac{mH}{(-8E)^{3/2}}\left(\frac{2B}{n}\right)^3\frac{(n-l-1)!}{2n(n+l)!}\left(\chi T_{S3-is}(A,B,n,l)+\frac{\bar{\sigma}}{2m_0}T_{3-3}(A,B,n,l)\right) \quad (43.3)$$

$$E_{nc-D}\left(\Theta,\bar{\theta}\right)\equiv-2B^2\left\{(2n)^{-2}-\frac{8A}{\kappa}(2n)^{-3}+\frac{16A^2}{\kappa^3}(2n)^{-3}+\frac{48A^2}{\kappa^2}(2n)^{-4}-...\right\}-\frac{p_-}{(-8E)^{3/2}}\left(\frac{2B}{n}\right)^3\frac{(n-l-1)!}{2n(n+l)!}\left(\Theta T_{s-is}(A,B,n,l)+\frac{\bar{\theta}}{2m_0}T_3\right)$$
$$-\frac{mH}{(-8E)^{3/2}}\left(\frac{2B}{n}\right)^3\frac{(n-l-1)!}{2n(n+l)!}\left(\chi T_{s3-is}(A,B,n,l)+\frac{\bar{\sigma}}{2m_0}T_{3-3}(A,B,n,l)\right) \quad (43.3)$$

In this way, one can obtain the complete energy spectra for (m.i.s.) potential in (*NC: 2D–RSP*) and (*NC: 3D–RSP*) symmetries. Know the following accompanying constraint relations:

a. The original spectrum contain two possible values of energies in ordinary two–three dimensional space which presented by equation (15),

The quantum number m satisfied the interval: $-l\leq m\leq+l$, thus we have ($2l+1$) values for this quantum number,

We have also two values for $j=l+\frac{1}{2}$ and $j=l-\frac{1}{2}$.

Allow us to deduce the important original results: every state in usually (two–three) dimensional space will be replace by $2(2l+1)$ sub–states and then the degenerated state can be take $2\sum_{i=0}^{n-1}(2l+1)\equiv2n^2$ values in (*NC: 2D–RSP*) and (*NC: 3D–RSP*) symmetries. It's clearly, that the obtained eigenvalues of energies are real and then the noncommutative diagonal Hamiltonian operators $\hat{H}_{nc2-ip}$ and $\hat{H}_{nc3-ip}$ are Hermitian, furthermore it's possible to writing the two elements [$\left(\hat{H}_{nc2-is}\right)_{11}$, $\left(\hat{H}_{nc2-is}\right)_{22}$] and $\left[\left(\hat{H}_{nc3-is}\right)_{11},\left(\hat{H}_{nc3-is}\right)_{22},\left(\hat{H}_{nc3-is}\right)_{33}\right]$, as follows, respectively:

$$\left(\hat{H}_{nc2-is}\right)_{11}=-\frac{1}{2m_0}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2}{\partial\phi^2}\right)+\frac{A}{r^2}-\frac{B}{r}+p_+\left\{\frac{\bar{\theta}}{2m_0}+\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)\theta\right\}+\left\{\frac{\bar{\sigma}}{2m_0}+\chi\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)\right\}\vec{H}\vec{L}$$
$$\left(\hat{H}_{nc2-is}\right)_{22}=-\frac{1}{2m_0}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2}{\partial\phi^2}\right)+\frac{A}{r^2}-\frac{B}{r}+p_-\left\{\frac{\bar{\theta}}{2m_0}+\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)\theta\right\}+\left\{\frac{\bar{\sigma}}{2m_0}+\chi\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)\right\}\vec{H}\vec{L} \quad (44.1)$$

and

$$\begin{cases}\left(\hat{H}_{nc3-is}\right)_{11}=-\frac{1}{2m_0}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2(\sin\theta)^2}\frac{\partial^2}{\partial\phi^2}\right]+\frac{A}{r^2}-\frac{B}{r}+\\ p_+\left[\Theta\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)+\frac{\bar{\theta}}{2m_0}\right]+\left\{\frac{\bar{\sigma}}{2m_0}+\chi\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)\right\}\vec{H}\vec{L}\quad \text{for } j=\ell+1/2 \Rightarrow\text{spin up}\\ \left(\hat{H}_{nc3-is}\right)_{22}=-\frac{1}{2m_0}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^<\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2(\sin\theta)^2}\frac{\partial^2}{\partial\phi^2}\right]+\frac{A}{r^2}-\frac{B}{r}\\ p_-\left[\Theta\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)+\frac{\bar{\theta}}{2m_0}\right]+\left\{\frac{\bar{\sigma}}{2m_0}+\chi\left(\frac{A}{r^4}-\frac{B}{2r^3}\right)\right\}\vec{H}\vec{L}\quad \text{for } j=\ell-1/2 \Rightarrow\text{spin down}\\ \left(\hat{H}_{nc3-is}\right)_{33}=-\frac{1}{2m_0}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2(\sin\theta)^2}\frac{\partial^2}{\partial\phi^2}\right]+\frac{A}{r^2}-\frac{B}{r}\end{cases} \quad (44.2)$$

On the other hand, the above obtain results (44.1) and (44.2) allow us to constructing the diagonal anisotropic matrixes $\left[\left(\hat{H}_{nc2-is}\right)_{11}\neq\left(\hat{H}_{nc2-is}\right)_{22}\right]$ and $\left[\left(\hat{H}_{nc3-is}\right)_{11}\neq\left(\hat{H}_{nc3-is}\right)_{22}\right]\neq\left(\hat{H}_{nc3-is}\right)_{22}$ of the Hamiltonian operators $\hat{H}_{nc2-ip}$ and $\hat{H}_{nc3-ip}$ for (*m.i.s.*) potential in (*NC: 2D–RSP*) and (*NC: 3D–RSP*) symmetries respectively, as:

$$\hat{H}_{nc2-ip}=\begin{pmatrix}\left(\hat{H}_{nc2-is}\right)_{11} & 0\\ 0 & \left(H_{nc2-is}\right)_{22}\end{pmatrix} \quad (45.1)$$

and

$$\hat{H}_{nc3-is}=\begin{pmatrix} (H_{nc3-is})_{11} & 0 & 0 \\ 0 & (H_{nc3-is})_{22} & 0 \\ 0 & 0 & (H_{nc3-is})_{33} \end{pmatrix} \quad (45.2)$$

Which allows us to obtain the original results for this investigation: the obtained Hamiltonian operators (45.1) and (45.2) can be describing atom which has two permanent dipoles: the first is electric dipole moment and the second is magnetic moment in external stationary electromagnetic field. It is important to notice that, the appearance of the polarization states of a fermionic particle for (m.i.s.) potential indicate to the validity of obtained results at very high energy where the two relativistic equations: Klein–Gordon and Dirac will be applied, which allowing to apply these results of various Nano–particles at Nano scales.

Conclusion

In this study we have performed the exact analytical bound state solutions: the energy spectra and the corresponding noncommutative Hamiltonians for the two and three dimensional Schrödinger equations in polar and spherical coordinates for modified inverse–squire potential by using generalization Boopp's Shift method and standard perturbation theory. It is found that the energy eigenvalues depend on the dimensionality of the problem and new atomic quantum numbers ($j=l\pm 1/1, s=\pm 1/2, l$ and the angular momentum quantum number in addition to two infinitesimals parameters (θ , $\bar{\theta}$) and (Θ , $\bar{\theta}$) in the symmetries of (NC: 2D–RSP) and (NC: 3D–RSP). And we also showed that the obtained energy spectra degenerate and every old state will be replaced by $2(2l+1)$ sub–states. Finally, we expect that the results of our research are valid in the high energies, thus the (m.s.e) can gives the same results of Dirac and Klein–Gordon equations.

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Conflicts of interest

None.

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