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The minimum number of blocks for period-three trefoil knots in the general-angle Rubik's snake tube model

Abstract

We determine the minimum number of blocks for a period-three trefoil in a general-angle circular-tube model of Rubik's snake. Each block has a quarter-circle centerline of radius $1/2$, and the tube radius is $(\sqrt{2}-1)/2$. The total-curvature bound reduces the lower-bound problem to nine blocks. A complete interval cover then shows that every nine-block period-three closure is either unknotted or self-intersecting as a solid tube. An explicit radical construction gives a collision-free twelve-block trefoil. Thus the minimum is exactly twelve. For tubes thin enough, the minimum is proved to be nine even without requiring periodicity.

Keywords: Rubik's snake, general rotation angles, trefoil knot, shortest path

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Introduction

Rubik's snake links a familiar folding toy[1] to mathematical and mechanical questions. Its articulated geometry has motivated protein-folding models,[2] reconfigurable mechanisms,[3,4] and connections with modular robotics and geometric capture.[5,6] Related methods for describing constrained rotations appear in rigid origami. Earlier studies developed design methods,[7] discrete-angle theory,[8] general-angle models,[9] symmetry results,[10] and box constructions;[11] a comic book also introduced the subject to younger readers.[12]

Knot designs led to investigations of trefoils,[13] other prime knots,[14,15] composite knots,[16,17] and torus knots.[18] A recent 2026 paper used local structures to improve several constructions.[19] Here we ask for a global minimum in a specified circular-tube model with unrestricted joint angles and a threefold repeated word for a trefoil knot. The solid and angle constraints differ from the triangular-prism, discrete-angle setting, so the block counts are not directly comparable to 34 blocks in.[13] However, that paper also reported a numerical 24-block trefoil construction in the general angle tube model considered here.

We prove an exact minimum by combining Milnor's total-curvature theorem,[20] a complete exclusion of nine-block closures, and a twelve-block construction. For tubes thin enough, the minimum is proved to be nine even without requiring periodicity. Section 2 defines the model; Sections 3 and 4 establish the lower and upper bounds; Section 5 discusses thin tubes. We conclude in Section 6.

Model and main result

Normalize the shortest side length of a Rubik's snake block to one. A block centerline is a quarter-circle of radius $R = 1/2$; its solid is the sweep of normal disks of radius $r = (\sqrt{2}-1)/2$, the closest distance from the centerline to the rectangular face of the original Rubik's snake. Adjacent blocks share their interface disk and have matching tangents. The interior of different tube pieces must be disjoint. This model uses circular tubes, not the original Rubik's snake with full triangular prisms.

For a joint angle θ as defined in,[9] put $t = \tan(\theta/2)$. The one-step rigid motion is

$$T(t) = \begin{pmatrix} 0 & 1 & 0 & 1 \\ c & 0 & s & 0 \\ s & 0 & -c & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, c = \frac{1-t^2}{1+t^2}, s = \frac{2t}{1+t^2}.$$

This 4-by-4 matrix combines rotation with translation. The c and s are cosine and sine of the joint angle.

The first arc is

$$\gamma_0(u) = (1/2, -1/2, 0) + (-1/2, 0, 0)\cos u + (0, 1/2, 0)\sin u, 0 \le u \le \pi/2.$$

Subsequent arcs are its images under the preceding motions. Strict closure means closure of both position and the complete frame. A period-three configuration has the same subsequence repeated three times.

Theorem 1. At the stated tube radius, a collision-free, strictly closed period-three trefoil has at least twelve blocks, and this bound is attained. More generally, every collision-free period-three closure with fewer than twelve blocks is unknotted.

We prove this theorem by providing a lower bound and constructing an example achieving the bound.

Lower bound

Reduction to nine blocks

Each block contributes $\pi/2$ to total curvature. Tangent continuity therefore gives $TC = N\pi/2$, where TC stands for total curvature. By the total-curvature theorem,[20] a nontrivial embedded knot satisfies $TC > 4\pi$, hence $N > 8$. Since $3 \mid N$, , the only candidate below 12 is a three-angle word repeated three times (9 blocks).

All nine-block closures

Each period corresponds to a rotation and a translation. We separate them and use a 3-by-3 matrix Q and a three dimensional vector v. Strict closure requires $Q^3 = I$ and $(I + Q + Q^2)\,v = 0$. The case

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$Q = I$ is impossible. Thus Q is a rotation through $\pm 2\pi / 3$. With

$$n = (Q_{32} - Q_{23}, Q_{13} - Q_{31}, Q_{21} - Q_{12}),$$

closure is equivalent to $trQ = 0$ (the sum of diagonal equals 1-1/2-1/2=0) and $n \cdot v = 0$.

To include every angle and remove singularity of tangent, use for each joint either $(x_i, y_i) = (z_i, 1)$ or $(1, z_i)$ with $-1 \le z_i \le 1$. Then

$$c_i = \frac{y_i^2 - x_i^2}{x_i^2 + y_i^2}, \qquad s_i = \frac{2x_i y_i}{x_i^2 + y_i^2}.$$

With each having two choices above, the three angles are covered by eight compact cubes $[-1,1]^3$. Define

$$\begin{aligned} P &= x_1x_2y_3 + x_1y_2x_3 + y_1x_2x_3 - 3y_1y_2y_3, \\ S &= x_1x_2x_3 + x_1y_2y_3 + y_1x_2y_3 + y_1y_2x_3, \\ D &= \prod_{i=1}^{3} (x_i^2 + y_i^2), \qquad H = 2S^2 - D. \end{aligned}$$

Direct multiplication gives $trQ = 2S^2 / D - 1$ and $n \cdot v = 2PS / D$. Since $D > 0$, the complete closure set is exactly $P = H = 0$ (cause S cannot be zero). These formulas remain valid on the chart boundaries.

Complete interval exclusion

Bisect the eight cubes and certify each terminal box by one of three conditions, valid throughout the entire box:

No closure. An interval enclosure of P or H excludes zero.

Unknot. Along one period, the polar angle of the axial projection is strictly monotone, and the projection lies in an open half-plane bounded by a line through the axis point. The half-plane condition forces the angular change to be exactly $\pm 2\pi / 3$. Three periods then form a simple polar graph making one turn. Flattening along the axis gives a planar simple closed curve, so the knot is trivial.

Collision. Two nonadjacent tube pieces contain a common rational point strictly in their interiors. For an arc $O + U\cos u + V \sin u$, with $U \perp V$ and $\|U\| = \|V\| = 1/2$, set

$$X = 2(z - O) \cdot U, Y = 2(z - O) \cdot V, Z = 4(z - O) \cdot (U \times V).$$

The sufficient interior test is

$$X > 0, Y > 0, (\sqrt{X^2 + Y^2} - 1/2)^2 + Z^2 < 0.042891 < r^2.$$

The first two inequalities place the normal foot in the interior of the quarter-circle. The last is the squared normal-distance test for the actual tube.

Outward-rounded interval arithmetic verifies all assertions, including the continuous arc conditions. The cover has the following terminal-box counts:

No closure	Unknot	Strict collision	Total
22,188	5,008	2,556	29,752

Completeness is checked recursively from all eight full cubes: every nonterminal box has both children covered, and every terminal assertion is checked again. Consequently every nine-block closure is either unknotted or has a strict tube collision. This is a finite computer-assisted proof over the full parameter domain, not a sample-based exclusion. Together with Section 3.1, it proves $N \ge 12$.

An exact twelve-block trefoil

We only need to construct one example with 12 blocks. For simplicity, consider a palindromic Type II structure [10] with periodic three. With the two conditions for translation and rotation combined with the palindromic condition, the true degree of freedom within a period is 4-3=1. This means we could simply choose one parameter and check if all conditions are indeed satisfied. It turns out that by choosing the half angle tangent for the first angle to be 2, it works. Other half angle tangents can be obtained by solving the closure equations in radicals. Below is the detailed construction.

Set

$$w = \sqrt{\frac{77 - 4\sqrt{91}}{15}}, (t_1, t_2, t_3, t_4) = (2, 2 + w, 2, -2 - w),$$

and repeat the word three times, with $\theta_i = 2 \arctan t_i$. In units of 90°, one period is approximately

$$(1.4096655294, -0.4743557496, 1.4096655294, -1.6558560442).$$

The radical values above, together with the stated inverse-tangent formula, define the construction exactly. The decimal angle sequence is included only for the reader's convenience; its entries are rounded approximations and are not used to establish exact closure or the minimum block count.

For the period motion (Q_4, v_4), direct substitution gives

$$trQ_4 = \frac{75w^4 - 770w^2 + 1491}{25(w^2 - 4w + 5)(w^2 + 4w + 5)} = 0, n_4 \cdot v_4 = 0.$$

The denominator is positive. Hence the period motion has order three with no axial translation, proving exact position and frame closure.

Interval bounds for all 12*(12-1-2)/2=54 nonadjacent arc pairs establish

$$\text{dist}(\gamma_i, \gamma_j) > \sqrt{2} - 1 + 0.01 = 2r + 0.01.$$

We sample each arc at finitely many grid points and rigorously bound the distances between all grid-point pairs on nonadjacent arcs.A derivative bound controls the error between grid points. After subtracting this error, the distances still exceed the tube diameter, ensuring that the full tube pieces has no collision inside.

Figure 1 shows the standard trefoil form (denoted as T(2,3) as a torus knot) in axial view (also verified by Gauss code). Thus the exact radical construction realizes a twelve-block trefoil and attains the lower bound, proving that the minimum is exactly 12. All blocks have the same shape and size.

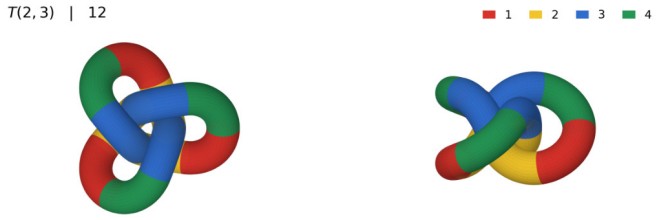

Figure 1 The exact twelve-block trefoil: axial view (left) and oblique view (right). Within each four-block period, the colors are red, yellow, blue and green; colors agree across views.

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A nine-block trefoil for thin enough tubes

If we allow the tube to be thin enough, then the minimum solution is 9 blocks no matter we require periodic three or not. The same argument as 3.1 shows 8 or below is impossible. We now construct 9 blocks.

Let x be the unique root in the interval[21,22] of the quartic polynomial

$$f(x) = x^4 - 24x^3 + 54x^2 + 56x + 9 = 0.$$

The signs at the rational endpoints of the following interval are opposite, and f is strictly increasing on:21,22

$$21.34648954478027 < x < 21.34648954478031.$$

Indeed, $f'(x) = 4x^3 - 72x^2 + 108x + 56 > 0$ on.[21,22] Define the half-angle tangent parameters and actual joint angles by

$$a = \sqrt{x},\ \ b = (3 - x) / (2\sqrt{x}),\ \alpha = 2\arctan a,\ \beta = 2\arctan b.$$

Repeat the three-angle word (α, β, α) three times. Numerically, a ≈ 4.620226135675643 and b ≈ −1.985453634305431. In units of 90°, the joint-angle sequence is

$$(1.72860674, -1.40593966, 1.72860674)^3.$$

The superscript 3 denotes concatenation of three identical words. The algebraic root and inverse-tangent definitions specify the configuration exactly; the decimal angles are for display only. Similar to Section 4, it is straightforward to verify that the exact solution above satisfy the closure condition. For tubes thin enough, for example with radius 0.025 in Figure 2, a similar procedure as in Section 4 verifies there is no collision. The trefoil structure can be clearly seen from Figure 2 with 3 crossings (also verified by Gauss code).

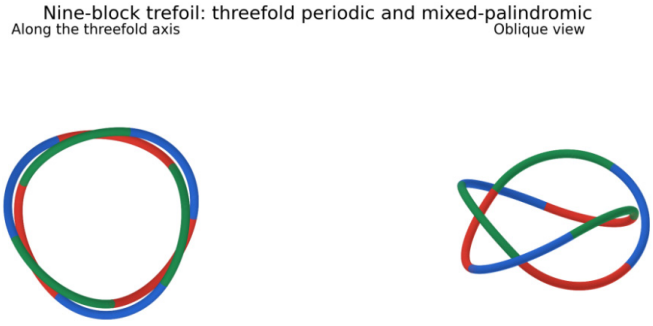

Figure 2 The exact nine-block trefoil at tube radius 1/40, with threefold periodicity and mixed-palindromic symmetry. Left: view along the threefold axis. Right: oblique view. Three colors repeat in the same order within each period, with corresponding blocks colored identically in both views.

The structure of (α, β, α) repeated three times is mixed palindromic since by viewing from the front or the middle the palindromic types are different. The minimum solution for thin enough tubes is 9 no matter we require periodic 3 or not and no matter we require palindromic structure or not.

Conclusions

The minimum is exactly twelve blocks for a period-three trefoil in the stated general-angle tube model with the stated size tubes. The lower bound also applies to every other nontrivial knot under the same periodicity and radius conditions. The 12-block example is palindromic but the minimum proof does not need palindromic condition. For thin enough tubes, the minimum is proved to be exactly nine blocks. Periodicity and palindromic condition are not required in this case.

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Conflicts of interest

There are no conflicts of interest.

References

1. Fenyvesi C. Rubik's snake of "infinite possibilities". The Washington Post. 1981.
2. Iguchi K. A toy model for understanding the conceptual framework of protein folding: Rubik's magic snake model. *Mod Phys Lett B*. 1998;12(13):499–506.
3. Ding X, Lu S, Yang Y. Configuration transformation theory from a chain-type reconfigurable modular mechanism—Rubik's snake. Proceedings of the 13th World Congress in Mechanism and Machine Science. 2011.
4. Liu J, Zhang X, Zhang K, et al. Configuration analysis of a reconfigurable Rubik's snake robot. *Proc Inst Mech Eng C J Mech Eng Sci*. 2019;233(9):3137–3154.
5. Yim M, Roufas K, Duff D, et al. Modular reconfigurable robots in space applications. *Autonomous Robots*. 2003;14(2–3):225–237.
6. Zhang X, Liu J, Feng J, et al. Effective capture of nongraspable objects for space robots using geometric cage pairs. *IEEE/ASME Trans Mechatronics*. 2020;25(1):95–107.
7. Li Z, Hou S, Bishop T. Computational design and analysis of a magic snake. *J Mech Rob*. 2020;12(5):054501.
8. Hou S, Chen Y, Li Z. Some mathematical problems related to the Rubik's snake. *J Mech Rob*. 2021;13(1):014502.
9. Hou S, Atkins S, Chen Y. A Rubik's snake with general rotation angles. *SCIREA Journal of Information Science and Systems Science*. 2021;5(6):123–135.
10. Hou S, Su J, Chen Y. Palindromic, periodic and Möbius Rubik's snakes. *Int Rob Auto J*. 2021;7(3):84–88.
11. Hou S. Designing paths for box shapes using a Rubik's snake. *Int Rob Auto J*. 2022;8(2):66–68.
12. Pan E, Hou S. The legend of the Rubik's snake. Lit House LLC; 2025.
13. Hou S, Su J. Shortest paths of trefoil knot designs using Rubik's snakes. *Int Rob Auto J*. 2022;8(1):18–20.
14. Hou S, Su J. Shortest paths of Rubik's snake prime knots up to 5 crossings. *Int Rob Auto J*. 2022;8(2):47–50.
15. Hou S, Su J, Mufutau R. Shortest paths of Rubik's snake prime knots with up to 6 crossings and application to roller coaster design. *Int Rob Auto J*. 2023;9(1):30–33.
16. Hou S, Su J, Mufutau R. Shortest paths of Rubik's snake composite knots up to 8 crossings. *Int Rob Auto J*. 2023;9(3):110–113.
17. Hou S, Su J, Mufutau R. Shortest paths of Rubik's snake composite knots with 9 crossings. *Int Rob Auto J*. 2024;10(1):25–30.
18. Hou S, Su J. Torus knot designs using a Rubik's snake. *Int Rob Auto J*. 2024;10(2):43–46.
19. Hou S, Su J. Improved results on shortest path of some Rubik's Snake knots. *Int Rob Auto J*. 2026;12(1):18–21.

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20. Milnor JW. On the total curvature of knots. *Ann Math (2)*. 1950;52(2):248–257.

21. Iguchi K. Exactly solvable model of protein folding: Rubik's magic snake model. *Int J Mod Phys B.* 1999;13(4):325–361.

22. Zhang X, Liu J. Prototype design of a Rubik snake robot. In: Ding X, Kong X, Dai JS, editors. Advances in Reconfigurable Mechanisms and Robots II. Mechanisms and Machine Science. Cham: Springer; 2016;36:581–591.

23. Belcastro S-m, Hull T.C. Modelling the folding of paper into three dimensions using affine transformations. *Linear Algebra Appl.* 2002;348(1–3):273–282.

24. Tachi T. Simulation of rigid origami. In: Lang RJ, editor. Origami 4: Fourth International Meeting of Origami Science, Mathematics, and Education. A K Peters; 2009. p. 175–187.

25. Adams CC. The Knot Book: An Elementary Introduction to the mathematical theory of knots. Providence, RI: American Mathematical Society; 2004.

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