Mini Review

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Improved results on shortest path of some Rubik's Snake knots

Abstract

A Rubik's Snake is a toy that was invented over 40 years ago together with the more famous Rubik's Cube. It can be twisted to many interesting shapes including complicated knots. Previously we have studied the shortest paths for Rubik's Snake prime knots with up to 6 crossings and composite knots with up to 9 crossings. Here we provide some improved results for some of the knots.

Keywords: Rubik's Snake, prime knot, composite knot, shortest path

Volume 12 Issue 1 - 2026

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Received: January 9, 2025 | **Published:** January 27, 2026

Introduction

Over 40 years ago, Prof. Rubik invented the Rubik's Snake toy.[1] It consists of right isosceles triangular prisms (called blocks) that, except for the first and last, are connected to two other blocks at the centers of the square faces. The applications of the Rubik's Snake include the study of protein folding[2] and for the construction of reconfigurable modular robots.[3,4] There are more applications of robots presented in.[5,6] In previous papers that the first author published with others, some strategies have been given for the design of a Rubik's Snake,[7] and some mathematical problems concerning a Rubik's Snake have been studied.[8] Rotations that are not integer multiple of 90 degrees were mentioned in[7] but not much theoretical work is presented. On the other hand, there are quite some theoretical work but only concerned with integer multiple of 90 degree rotations in.[8] In,[9] general rotation angles were studied with theoretical work presented. In,[10] more theorems about the Rubik's Snake were presented and proved. In,[11] a counting formula and path designs were discussed for box shapes using a Rubik's Snake. A comic book was also published to make some basic research accessible for kids.[12]

How could a Rubik's Snake form a knot is not reported in the literature until our recent publication in which the shortest Rubik's Snake trefoil knot paths with 34 blocks were presented.[13] We extended the work in,[14–17] to prime knots up to 6 crossings and composite knots up to 9 crossings. We also studied torus knot designs in.[18]

In this paper, we review our previous findings and present improved results for some knots studied in the past. By using the idea of key local structures, a prime knot 5_2 was found to have a shorter path. This was then used to improve the corresponding composite knots. We verified these improved results can no longer be shortened locally.

The rest of the paper is organized as follows: in Section 2, we summarize the previous results. In Section 3, we give the new shortest snake prime knot 5_2 beating the previously published result and explain the construction. In Section 4, we give the new shortest composite knots $3_1\#5_2$ and $4_1\#5_2$. We conclude in Section 5.

Previous results

The simplest non-trivial knot, the trefoil knot, needs at least 34 blocks.[13] The hole structure cannot be further reduced, and the rest of the paths are locally searched exhaustively in each of the two separate parts. The 4_1 knot (also called the figure-8 knot) had a 46-block construction early in[14] but was improved to 44 blocks in[16] using a $[t, -t, t, -t]$ construction and exhaustive search under the constraint. The 5_1 knot had a 52-block construction early in[14] but later improved to 50 blocks in[17] using a non-local change. The 5_2 knot had a 56-block construction early in[14] but later improved to 54 blocks in[16] using a non-local change. The 6_1 knot had a 64-block construction (based on certain 4_1 construction) early in[15] but later improved to 60 blocks in[17] using a $[t, -t, t, -t]$ 8_3 construction and local changes while keeping periodic two (though breaking the $[t, -t, t, -t]$ pattern, as the knot 6_1 does not have such symmetry). The 6_2 knot had a 62-block construction (based on certain 4_1 construction) and 6_3 a 64-block construction (based on certain 3_1 construction) both in.[15]

For composite knots, we refer to[16] and[17] The main idea is to use prime knot structures and exhaustively search locally for connections.

The improved shortest 5_2

The motivation is to carefully study the structure of our 50-block 5_1. It appears that it uses the key hole part of shortest 3_1 and another shortest structure to form a hole. Some local searches can be used to verify the rest. When we first constructed this 5_1, it was based on our previous 52-block construction with a non-local change and we did not study the key components this way.

Now we use the same idea. We include a key pattern [0, 1, 2, 1, 0, 1, 0, 1, 2, 1, 0, 3] in shortest 3_1 path (also in shortest 5_1). We then immediately add [3, 1, 0, 0, 3, 1, 0, 1, 1, 0, 1, 0] to it. This turns one round forming a hole in the middle and cannot be further shortened assuming shared cube for the first and last block. The mathematical reasoning is as follows. Look at the projection of the Rubik's Snake with a hole. The shortest has 8 projection pieces around. Since a Rubik's Snake block cannot go from a face of a unit cube to the opposite face, the 4 sides must have at least two blocks, making it 4*2+4=12 blocks at least. Further, Theorem 1 in [8] stated that a closed loop with integer multiple of 90 degree rotations (integer sequence) must have an even number of blocks and the color on triangular faces for two blocks sharing a unit cube within the same snake must be the same assuming alternating colors for a standard Rubik's Snake toy. Clearly, with alternating colors and the same color for the first and last like GWGWGW...G, we must have odd number of blocks. Therefore

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13 blocks, that is, 12 joints in the sequence is the minimum. Then the rest can be broken into three local exhaustive searches. In the end, we present the following as one of the solutions:

[0,1,2,1,0,1,0,1,2,1,0,3,3,1,0,0,3,1,0,1,1,0,1,0,0,0,3,2,0,0,1,0,1,2,1,0,3,3,0,2,3,0,0,1,0,3,2,3,0,3]

Figure 1 shows the Rubik's Snake 5_2 with 50 blocks and Figure 2 shows the line representation to reveal the knot structure. DT code confirms that it is indeed 5_2.

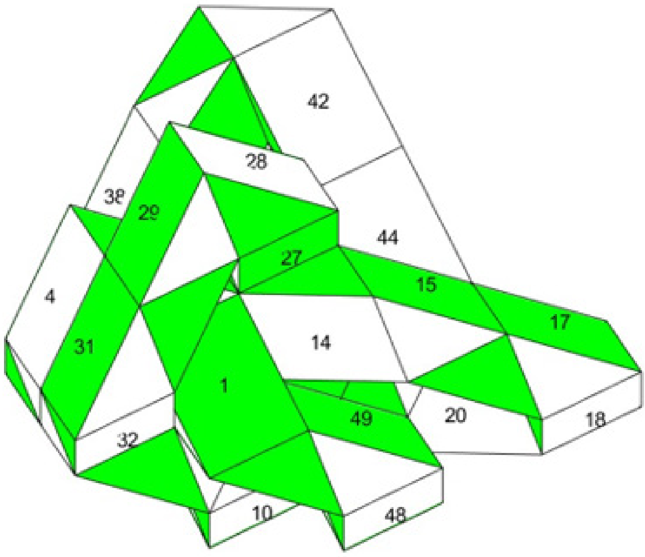

Figure 1 A Rubik's Snake 5_2 knot with 50 blocks with labels.

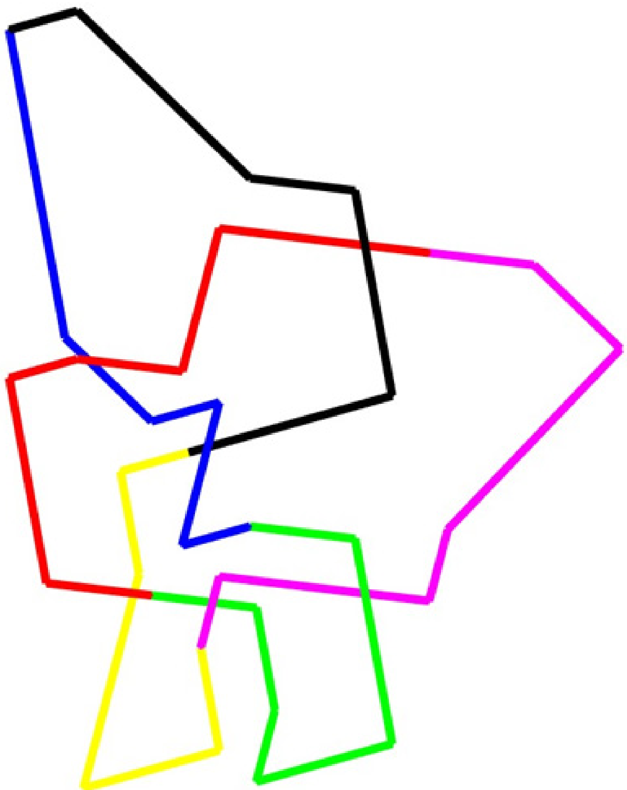

Figure 2 The line representation of a Rubik's Snake 5_2 revealing the prime knot structure

$3_1\#5_2$ and $4_1\#5_2$

Once we have a shorter construction of 5_2, it is natural to expect that the corresponding composite knots $3_1\#5_2$ and $4_1\#5_2$ (and other composite knots with more crossings involving 5_2) can be improved to have shorter paths as well.

Let A and B be partial sequences corresponding to shortest paths we found for two prime knots. Here partial sequence means we omit a part that is not knotted. For a prime knot there are multiple candidates for such partial sequences. We search for closed loops using [A, x, B, y], [A, x, $-B$, y], [A, x, *reverse*(B), y] or [A, x, $-$*reverse*(B), y]. The shortest we found for $3_1\#5_2$ has 72 blocks, improving the previously published result of 76. Below is an example:

[0,1,2,1,0,1,0,1,2,1,0,3,0,0,1,2,0,1,1,0,1,2,1,1,3,0,0,1,0,0,0,3,2,0,0,1,0,1,2,1,0,3,3,0,2,3,0,0,1,0,3,2,3,0,3,0,1,2,1,0,1,0,1,2,1,0,3,3,1,0,0,3]

Figure 3 and Figure 4 shows the snake and path.

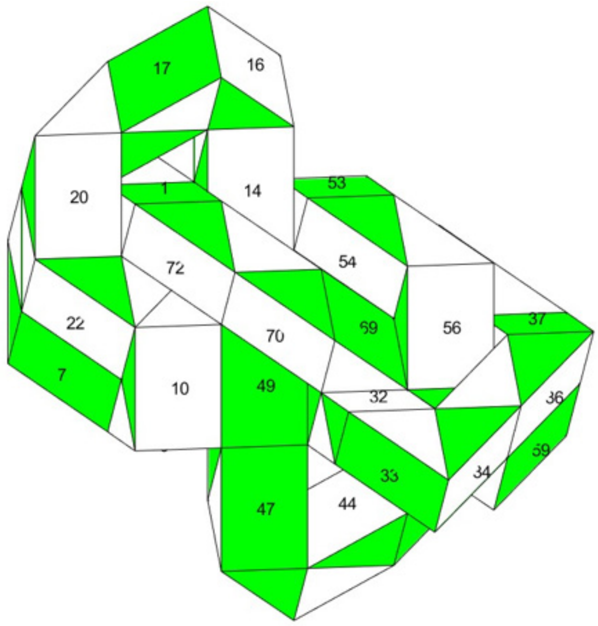

Figure 3 A Rubik's Snake $3_1\#5_2$ knot with 72 blocks with labels

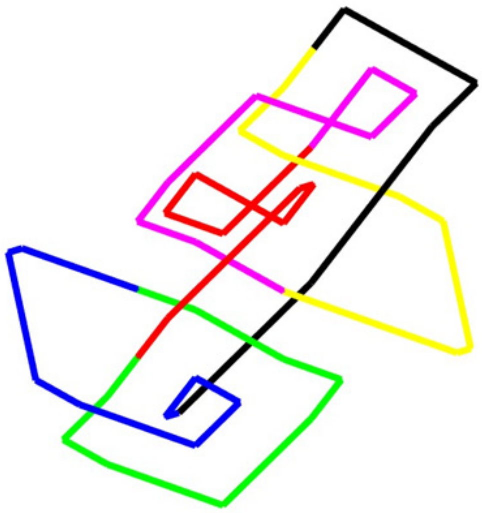

Figure 4 The line representation of a Rubik's Snake $3_1\#5_2$ revealing the knot structure with two components

Citation: Hou S, Su J. Improved results on shortest path of some Rubik's Snake knots. *Int Rob Auto J.* 2026;12(1):18–21. DOI: 10.15406/iratj.2026.12.00312

Similarly, the shortest we found for $4_1\#5_2$ has 82 blocks, improving the previous result of 86. Below is an example:

[0,0,0,1,0,1,1,0,2,1,0,0,0,0,3,0,3,3,0,2,3,0,0,0,0,1,0,1,1,0,2,1,0,0,0,0,3,0,0,3,2,3,0,3,0,0,2,1,0,0,0,3,0,3,3,0,3,1,0,0,3,1,1,0,3,2,3,0,3,0,3,2,3,0,1,0,1,2,1,0,3,0]

Figure 5 and Figure 6 shows the snake and path.

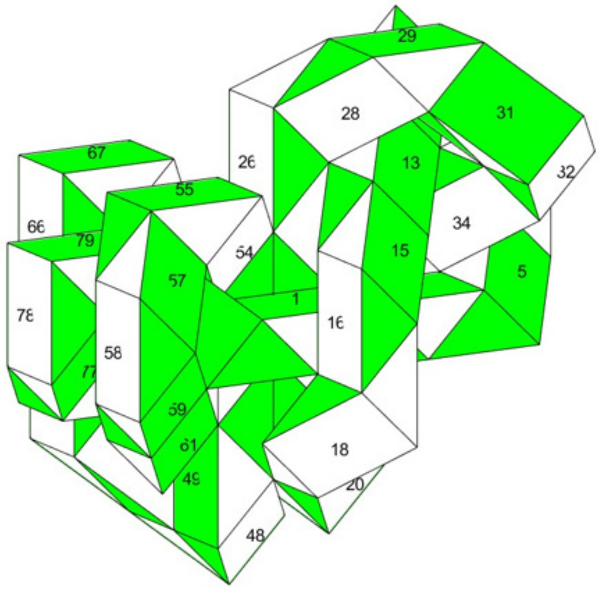

Figure 5 A Rubik's Snake $4_1\#5_2$ knot with 82 blocks with la-bels

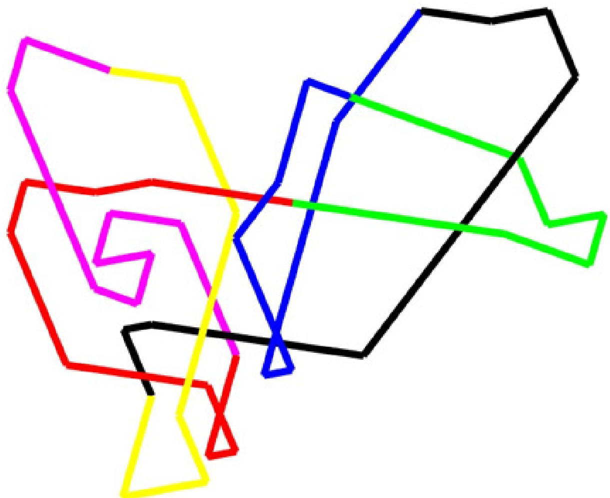

Figure 6 The line representation of a Rubik's Snake $4_1\#5_2$ revealing the knot structure with two components

Conclusions

Finding the shortest path for a Rubik's Snake non-trivial knot is challenging. We used local hole patterns and local searches to improve our previously shortest 5_2 knot with 54 block to only 50 blocks. We then applied it to improve our previously shortest $3_1\#5_2$ from 76 blocks to 72 blocks and improve $4_1\#5_2$ from 86 blocks to 82 blocks. The complete improved list of length of shortest paths found is in Table 1. The prime knot results up to 6 crossings can be found in the second column and the composite knot results up to 9 crossings can be found in the third column. We verified that no local improvement is possible for any of these results. We will study more complicated knots in the future.

Table 1 The complete improved list of length of shortest paths found for prime knots up to 6 crossings and composite knots up to 9 crossings

Shortest A	Shortest B	Shortest composite
3_1:34	3_1:34	$3_1\#3_1$: 34+34-12=56
4_1:44	4_1:44	$4_1\#4_1$: 44+44-12=76
3_1:34	4_1:44	$3_1\#4_1$: 34+44-10=68
3_1:34	5_1:50	$3_1\#5_1$: 34+50-12=72
3_1:34	52: 50 (new)	3_1#52: 34+50-12=72 (new)
3_1:34	6_1: 60	$3_1\#6_1$: 34+60-10=84
3_1:34	62: 62	3_1#62: 44+62-12=84
3_1:34	63: 64	3_1#63: 34+64-10=88
4_1:44	5_1:50	$4_1\#5_1$: 34+50-12=82
4_1:44	52: 50 (new)	4_1#52: 34+50-12=82 (new)
3_1:34	3_1:34	$3_1\#3_1\#3_1$: 34+34+34-20=82

Acknowledgment

We thank Kyle Xu for testing some Rubik's Snake knot designs. S. Hou's research is partially supported by the Walter Koss Endowed Professorship. The title of the professorship is made available through the State of Louisiana Board of Regents Support Funds.

Conflicts of interest

There are no conflicts of interest.

References

1. Fenyvesi C. Rubik's snake of 'infinite possibilities'. The Washington Post. 1981.
2. Iguchi K. A toy model for understanding the conceptual framework of protein folding: Rubik's magic snake model. *Mod Phys Lett B*. 1998;12(13):499–506.
3. Ding X, Lu S, Yang Y. Configuration transformation theory from a chain--type reconfigurable modular mechanism-rubik's snake. *The 13th World Congress in Mechanism and Machine Science.* 2011.
4. Liu J, Zhang X, Zhang K, et al. Configuration analysis of a reconfigurable rubik's snake robot. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*. 2019;233(9):3137–3154.
5. Yim M, Roufas K, Duff D, et al. Modular reconfigurable robots in space applications. *Autonomous Robots*. 2003;14(2-3):225–237.
6. Zhang X, Liu J, Feng J, et al. Effective capture of nongraspable objects for space robots using geometric cage pairs. *IEEE/ASME Trans-actions on Mechatronics.* 2020;25(1):95–107.
7. Li Z, Hou S, Bishop T. Computational design and analysis of a magic snake. *J Mech Rob*. 2020;12(5):054501.
8. Hou S, Chen Y, Li Z. Some mathematical problems related to the rubik's snake. *J Mech Rob*. 2021;13(1):014502.
9. Hou S, Atkins S, Chen Y. A rubik's snake with general rotation angles. *SCIREA Journal of Information Science and Systems Science*. 2021;5(6):123–135.
10. Hou S, Su J, Chen Y. Palindromic, periodic and mobius¨ rubik's snakes. *International Robotics and Automation Journal*. 2021;7(3): 84–88.
11. Hou S. Designing paths for box shapes using a rubik's snake. *International Robotics and Automation Journal*. 2022;8(2):66–68.
12. Pan E, Hou S. The legend of the rubik's snake. Lit House LLC. 2025.

13. Hou S, Su J. Shortest paths of trefoil knot designs using rubik's snakes. *International Robotics and Automation Journal.* 2022;8(1):18–20.
14. Hou S, Su J. Shortest paths of rubik's snake prime knots up to 5 crossings. *International Robotics and Automation Journal.* 2022;8(2):47–50.
15. Hou S, Su J, Mufutau R. Shortest paths of rubik's snake prime knots with up to 6 crossings and application to roller coaster design. *International Robotics and Automation Journal*. 2023;9(1):30–33.
16. Hou S, Su, J, Mufutau R. Shortest paths of rubik's snake composite knots up to 8 crossings. *International Robotics and Automation Journal*. 2023;9(3):110–113.
17. Hou S, Su J, Mufutau R. Shortest paths of rubik's snake composite knots with 9 crossings. *International Robotics and Automation Journal.* 2024;10(1):25–30.
18. Hou S, Su J. Torus knot designs using a rubik's snake. *International Robotics and Automation Journal*. 2024;10(2):43–46.