Research Article

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On the analytical mechanics and nonlinear stability theory methods in mathematical modeling of the controlled dynamics parallel manipulators by incomplete state information

Abstract

Despite the fact that parallel kinematic manipulators are becoming more and more widespread due to their advantageous operational characteristics, modern methods of mathematical control theory have not yet found application in their control systems. A necessary condition for the these methods effective use is the availability of an adequate mathematical model in analytical form, obtained by rigorous methods and including the all components automatic system. Despite intensive research, there are currently no general methods for obtaining such models. Almost all known results are based on the inverse kinematics problem consideration, the which solution in general analytical form, as is known, is impossible. In the studies published to date on modeling the specific technical devices dynamics with parallel kinematics, modern information technologies for processing symbolic information are used to one degree or another. In this article, we develop the such rigorous methods application of theoretical non-free systems mechanics and nonlinear stability theory for nonlinear mathematical dynamics modeling of systems with geometric constraints, which in the general case eliminate the need to consider the inverse kinematics problem. The application of vector-matrix equations obtained in analytical form in redundant coordinates in the general stabilization problem provides the possibility of reducing the volume of measurement information by reducing the dimensionality of the controlled subsystem from which the stabilizing control is determined. The controlled subsystem does not contain dependent coordinates, therefore, to form a stabilizing control, only information on independent coordinates and their velocities is sufficient. The analytical form model of made it possible to specify in the work the observability conditions that create the possibility for further reducing the measurement information volume by forming a stabilizing control using an phase state estimate of the controlled subsystem obtained using the evaluation system constructed in the article. The coefficients of the stabilizing control and the parameters for the state estimate system are determined from the corresponding linear-quadratic problems solution by Krasovskii's method.

Keywords: mathematical dynamics model, geometric constraints, measurement information, state estimate linear-quadratic problem

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Introduction

On the peculiarities of applying control theory with incomplete state information in manipulators with parallel kinematics

Automatically controlled devices make up the vast majority of modern technological equipment. Skilled application of the modern mathematical control theory results with incomplete information about the controlled object state for the provides ample opportunities to improve the its operation quality. A necessary condition for the effective implementation of these potential opportunities is an adequate mathematical model that includes all components of the automatic system. Based on the nonlinear mathematical model obtained by strict methods, general methods of control theory with incomplete information[1–3] can reduce the actuators number in mechanisms and reduce the measuring sensors number. The manipulators with parallel kinematics considered in this article constitute a special class of technical devices whose dynamics are constrained by geometric constraints. The presence of parallel kinematic chains, on the one hand, complicates obtaining the adequate mathematical their dynamics models,[4–12] and on the other hand, provides certain advantages in developing control systems. It is sufficient to ensure a given behavior with respect to independent coordinates, and the change in dependent coordinates is determined by relationships expressing geometric constraints.

However, in the many technical practice problems of controlling manipulators with parallel kinematics (e.g. Delta robot,[4,5,8–14]), the working elements dynamics (manipulator grippers) is described by dependent coordinates. As a result, the traditional use[8–12] of classical Lagrange equations with constraint multipliers[15–17] not only increases the mathematical model dimensionality, but also leads to the need for an analytical solution to the inverse kinematics problem, which greatly complicates the study. With this approach to modeling the controlled dynamics of modern complex technical devices with parallel kinematics, it is impossible to obtain a nonlinear mathematical model in analytical form. The study is limited only to computer simulation based on a model obtained using software for processing symbolic information.[5,8]

Methods for rigorous modeling of the dynamics of non-free systems

In the middle of the 20th century, much more effective (but still not used by specialists in technical practice) general methods were developed in analytical mechanics,[18–20] significantly simplifying the dynamics descript them, in our opinion, is the M.F. Shulgin's proposed[20] transition to constraint multipliers free equations in redundant coordinates. The use of the dynamics equations in the M.F. Shulgin's form not only reduces the mathematical model dimensionality by a double constraints number due to the constraint multipliers exclusion and velocities of dependent coordinates from consideration.[21–27] It should be especially noted that with this approach, in general case, there is no need to consider the inverse kinematics problem.

Reducing the nonlinear mathematical dynamics model dimensionality in the analytical form for the mechanical component of an automatically controlled object with geometric connections allows us to include in the consideration the mathematical model of actuators and, thus, move on to strict mathematical modeling of controlled dynamics, in particular, manipulators with parallel kinematics.

Let us consider the most general case of a mechanical system with n the freedom degrees, the which state is described by coordinates $q_1,\ldots,q_{n+m}$ the which number is greater than the degrees freedom number, since m geometric constraints are imposed on the system.

$$F(q)=0;\quad F'(q)=\left(F_1(q),...,F_m(q)\right);\qquad det\left\|\frac{\partial\left(F_1(q),...F_m(q)\right)}{\partial\left(q_{n+1},...q_{n+m}\right)}\right\|\neq 0; \tag{1}$$

Let us assume that the kinetic energy has the most general form

$$T(q,\dot q)=1/2a_{ij}(q)\dot q_i\dot q_j+a_i(q)\dot q_i+T_0(q);\qquad i,j=\overline{1,n+m}; \tag{2}$$

The system is under the action of potential forces with energy $\Pi(q))$ and non-potential forces Q_i related to the coordinates. Kinetic and potential energies and non-potential forces satisfy the differential equations theory conditions, guaranteeing the solutions existence and uniqueness to the motion equations.

For the further consideration convenience, in accordance with the different nature of the dependence of kinetic (2) and potential energies, constraints (1) and non-potential forces on coordinates, we introduce matrices and vectors

$$q=\begin{pmatrix}q_1\\ ...\\ q_{n+m}\end{pmatrix}=\begin{pmatrix}r\\ s\end{pmatrix};\quad r=\begin{pmatrix}q_1\\ \ldots\end{pmatrix}\quad s=\begin{pmatrix}q_{n+1}\\ ...\\ q_{n+m}\end{pmatrix}\quad F(q)=\begin{pmatrix}(F_1q)\\ ...\\ F_m(q)\end{pmatrix};$$

$$Q(q,\dot q)=\begin{pmatrix}Q_1\\ ...\\ Q_{n+m}\end{pmatrix};\quad \mathrm{Q_r}(q,\dot q)=\begin{pmatrix}Q_1\\ ...\\ Q_n\end{pmatrix};\ \mathrm{Q}_s(q,\dot q)=\begin{pmatrix}Q_{n+1}\\ ...\\ Q_{n+m}\end{pmatrix};$$

Due to constraints (1), the conditions are imposed on the variations of coordinates

$$\frac{\partial F(q)}{\partial r}\delta r+\frac{\partial F(q)}{\partial s}\delta s=0;\quad \delta\mathrm{s}=-\left(\frac{\partial F}{\partial s}\right)^{-1}\left(\frac{\partial F}{\partial s}\right)\delta r=B(q)\delta r;$$

$$B(q)=-\left(\frac{\partial F}{\partial s}\right)^{-1}\left(\frac{\partial F}{\partial s}\right);\quad \delta s-B(q)\delta r=0; \tag{3}$$

As equations of motion, we will use Shulgin's equations in redundant coordinates

$$\frac{d}{dt}\frac{\partial T^*}{\partial\dot r}-\frac{\partial T^*}{\partial r}=Q_r^*+B'(q)\left(\frac{\partial T^*}{\partial s}+Q_r^*\right);\dot s=B(q)\dot r; \tag{4}$$

These equations are derived from the traditionally used equations obtained from the d'Alembert-Lagrange principle [15-19] with constraint multipliers λ

$$\frac{d}{dt}\frac{\partial T}{\partial\dot q}-\frac{\partial T}{\partial r}=Q+b'\lambda;\quad b=\left(\frac{\partial F(q)}{\partial q}\right);\ (5)$$

Using (3) we can separate equations (5) for dependent and independent coordinates

$$\frac{d}{dt}\frac{\partial T}{\partial\dot r}-\frac{\partial T}{\partial r}=Q-B'(q)\lambda;\quad \frac{d}{dt}\frac{\partial T}{\partial\dot s}-\frac{\partial T}{\partial r}=Qs+\lambda;;\ (6)$$

Expressing the multipliers from the second equation (6)

$$\frac{d}{dt}\frac{\partial T}{\partial\dot s}-\frac{\partial T}{\partial r}-Qs=\lambda;\ (7)$$

After substituting (7) for the first equation of system (6), we obtain

$$\frac{d}{dt}\frac{\partial T}{\partial\dot r}-\frac{\partial T}{\partial r}=Qr-B'(q)\left(\frac{d}{dt}\frac{\partial T}{\partial\dot s}-\frac{\partial T}{\partial r}-Qs\right);\ (8)$$

Similar equations were obtained by Suslov GK[19] and Lurye AI.[18] Moreover, both of these researchers considered more complex systems, on which in addition to geometric constraints, non-integrable (non-holonomic) constraints were also imposed. Therefore, the modeling of the dynamics of such systems ended with equations of the form (8). Shulgin MF,[20] developing the analytical mechanics of non-holonomic systems, was able to simplify the mathematical model for the case of only geometric constraints.

To do this, dependent velocities $\dot s$ from equations (1) differentiated with respect to time

$$\dot s=B'(q)\dot r;\ (9)$$

should be excluded from the kinetic energy and non-potential forces. Denoting the result of eliminating dependent velocities using (12) from the kinetic energy (3) through $T^*(q_1,...,q_{n+m},\dot q_1,...,\dot q_n)$, , and from the acting forces through Q_i^*. .

From a comparison of the corresponding derivatives

$T^*(q_1,...,q_{n+m},\dot q_1,...,\dot q_n)$, and $T(q_1,...,q_{n+m},\dot q_1,...,\dot q_{n+m})$ taking into account the integrability of the constraints (9), in the general case, a nonlinear mathematical model of the system dynamics is obtained in the M.F. Shulgin equations form,[20] free of multipliers in redundant coordinates. These equations can be considered as the P.V. Voronets equations[28] in the integrability case of the nonholonomic constraints.[29,30]

The obtained general analytical form of the nonlinear dynamics mathematical model allows, in the general case, to present the mathematical dynamics model for any systems with geometric constraints (under the action of arbitrary potential and non-potential forces that do not violate the conditions for the existence and uniqueness of the differential equations solutions) in the Shulgin equations form (4) in explicit vector form[13,17,21–27] without resolving the inverse problem of kinematics (cf.[4,6,8]

In normal form, the model dimension (4) is equal to *2n+m*, i.e., by the analytical mechanics methods of systems with redundant coordinates, compared to the traditional approach, it is reduced by the geometric constraints (1) number *m*.

Application of the developed mathematical model and nonlinear stability theory to the general stabilization problem for non-free systems

Let the system admit an equilibrium position

$$\mathrm{r} = \mathrm{r}_0 = const; \quad s = s_0 = const \quad (10)$$

Let us introduce disturbances,

$$r = r_o + x; \quad \dot{r} = x_1; \quad s = s_o + y$$

compose the equations of the disturbed motion and single out the first approximation in them

$$\begin{aligned} &\dot{x} = x_1; \\ &\dot{x}_1 = C_x + C_{x1} + H_y + X^{(2)}(x, x_1, \mathrm{y}); \qquad (11) \\ &\dot{y} = B_{(x,y)} x_1 = B(0) x_1 + B^{(1)}(x, y) x_1; \quad B(0) = B(r_0, s_0); \end{aligned}$$

Here the superscript in brackets denotes the order of the lowest terms in the expansion of the corresponding expression, and the matrices *C,G,H* are expressed in a known manner[14,21–30] through the kinetic energy coefficients (2), the constraints equations (1), and the coefficients in the potential and non-potential forces expansion.

After the replacement,[31]

$$y = z + B(0)x; \; (12)$$

the characteristic equation of the first approximation system

$$\begin{aligned} &\dot{x} = x_1; \\ &\dot{x}_1 = (C + HB(0))x + G_{x1} + H_Z + X^{(2)}(x, x_1, z+)B(0)x; \quad (13) \\ &\dot{z} = B^{(1)}(x, z + B(0)x)x; \end{aligned}$$

will take the form

$$\begin{vmatrix} E_n\lambda & -E_n & 0 \\ -C - HB(0) & E_n\lambda - G & -H \\ 0 & 0 & E_m\lambda \end{vmatrix} = 0 \quad (14)$$

Equation (14) for any equilibrium of a system with geometric constraints in the general case has *m* zero roots corresponding to the variable *z*. Its remaining roots are determined by the equation

$$\begin{vmatrix} E_n\lambda & -E_n \\ -C - HB(0) & E_n\lambda - G \end{vmatrix} = 0; \; (15)$$

If the real parts of all roots of equation (15) are negative, then in the complete nonlinear system (13) there is a special case of *m* zero roots.[32–34] According to theorem 1[21] proved on the basis of Kamenkov's theorem,[33] in the complete nonlinear system (13) we obtain asymptotic stability, despite the presence of m zero roots.

To ensure such an arrangement of roots, it is sufficient to solve the stabilization problem for a linear controlled subsystem

$$\dot{\xi} = M + Nu; \xi' = (x', x_1')$$

$$M = \begin{pmatrix} 0 & -E_n \\ C + HB(0) & G \end{pmatrix} = 0; N = \begin{pmatrix} 0 \\ N_1 \end{pmatrix}; \quad (16)$$

Provided that

$$rank\ [N\, M\, N \ldots M^{2n-1} N] = 2n; \; (17)$$

we obtain stabilizing control as a linear function of only x, x_1:

$$u = K_1 x + K_2 x_1; \qquad (18)$$

Matrices K_1, K_2 can be determined from the solution by Krasovsky's method[34] of the linear-quadratic problem for subsystem (16), in particular with the simplest criterion

$$\int_0^\infty (\xi'\xi + u'u)dt \to min; \qquad (19)$$

Thus, using the nonlinear stability theory methods,[32–34] the system dimension for determining the stabilizing control (18) that ensures asymptotic stability in a complete closed nonlinear system

$$\begin{aligned} &\dot{x} = x_1; \\ &\dot{x}_1 = (C + HB)x + G_{x1} + H_Z + N_1(K_1 x + K_2 x) + X^{(2)}(x, x_1, z + Bx); \\ &\dot{z} = B^{(1)}(x, z + B(0)x)x_1; \end{aligned} \qquad (20)$$

is only 2n, i.e. in the general case it is reduced by the geometric constraints number (the dependent coordinates number). In this case, information about only variables x, x_1 is sufficient to form the control (18).

The explicit analytical form (20) presence of the controlled dynamics nonlinear mathematical model provides grounds for applying the mathematical control theory results with incomplete information about the state for systems with geometric constraints.

General stabilizing problem a given configuration with incomplete state information

Let us assume that in the system (20) only the following measurement information about phase variables is available

$$\sigma = \sum \xi = \sum{}_1 x + \sum{}_2 x_1; dim\sigma = s < 2n; \qquad (21)$$

In order to obtain information about the full phase vector of the subsystem (16) using measurement (21), it is necessary to construct a state estimation system

$$\begin{aligned} &\dot{\hat{\xi}} = M\hat{\xi} + D(\sum \xi = \sigma) + Nu; \\ &\dot{\mu} = M'u + \sum{}' w; w = D'\mu; \end{aligned} \qquad (22)$$

The matrix *D* can be determined from the solution by Krasovskii's method of the dual linear-quadratic problem for the system

if the observability condition (controllability for (22)) is met

$$rank[\sum{}' M' \sum{}' \ldots M'^{2n-1} \sum{}'] = 2n; \qquad (23)$$

The dynamics of a complete nonlinear system, closed by a stabilizing control, constructed based on the state estimate from the measurement (21) is described by the model

$$\begin{aligned} &\dot{x} = x_1; \\ &\dot{x}_1 = (C + HB)x + Gx_1 + Hz + N_1(K_1\hat{x} + K_2 x_1) + X^{(2)}(x, x_1, z + Bx); \\ &\dot{\mathrm{z}} = \mathrm{B}^{(1)}(x, z + B(0)x)x_1 \;\; \dot{\mathrm{z}} = \mathrm{B}^{(1)}(x, z + B(0)x)x_1 \;\; (24) \end{aligned}$$

$$\dot{\hat{x}} = \hat{x}_1;$$

$$\begin{aligned} &\dot{\hat{x}}_1 = (C + HB)\hat{x} + G\hat{x}_1 + Hz + N_1(K_1\hat{x} + K_2\hat{x}_1) + \\ &D(\Sigma'_1 \hat{x} + + \Sigma'_2 \hat{x}_1 - \sigma) \end{aligned}$$

$$\dot{z} = B^{(1)}(x, z + B(0)x)x_1; \sigma = \Sigma_1 x + \Sigma_1 x_1;$$

When conditions (17), (23) are met, the problem of stabilizing equilibrium (10) to asymptotic stability for all variables is solved by linear control

$$\hat{u} = \hat{K}_1 + K_2\hat{x}_1; \quad (25)$$

formed based on the assessment of the phase state by measuring (21). In the presence of cyclic coordinates, it is possible to further simplify the study by reducing the dimensionality of the control problem by switching to Routh variables.

Discussion

In accordance with the different dependence nature of the system parameters and the acting forces on different coordinates, we divide the independent coordinates vector into the following vectors

$$\mathrm{r} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}; \alpha = \begin{pmatrix} q_1 \\ \cdots \\ q_k \end{pmatrix}; \beta = \begin{pmatrix} q_{k+1} \\ \cdots \\ q_l \end{pmatrix}; \gamma = \begin{pmatrix} q_{l+1} \\ \cdots \\ q_n \end{pmatrix}$$

$$\mathrm{F}(q) = \begin{pmatrix} F_1(q) \\ \cdots \\ F_m(q) \end{pmatrix} Q(q,\dot{q}) \begin{pmatrix} Q\alpha \\ Q\beta \\ Q\gamma \end{pmatrix};$$

Let us assume that the coordinates β,γ are cyclic by definition of Shulgin, that is, the following conditions are met:

1. the coordinates β,γ are not explicitly included in the expression of the function *L*, (but it clearly depends on β,γ ˙),

2. the coordinates β,γ are not explicitly contained in the expressions of the finite constraints (1), i.e.

$$\frac{\partial L^*}{\partial \beta} = \frac{\partial L^*}{\partial \gamma} = 0; \frac{\partial F}{\partial \beta} = \frac{\partial F}{\partial \gamma} = 0; Q_\beta(\alpha,s,\dot{q}) = Q_\gamma(\alpha,s,\dot{q}) = 0; \quad (26)$$

Under these conditions, the constraints (1) are reduced to the form

$$\dot{\mathrm{s}} = \mathrm{B}_\alpha(\alpha,s)\dot{\alpha}; \mathrm{B}_\alpha(\alpha,s) = -\left(\frac{\partial F}{\partial s}\right)^{-1}\left(\frac{\partial F}{\partial \alpha}\right);$$

As is known, in the cyclic coordinates presence it is more convenient[16,36,37] to model the dynamics in Routh variables. For this purpose, we introduce impulses and the Routh function

$$p_\beta = \frac{\partial T^*}{\partial \dot{\beta}}; p\gamma = \frac{\partial T^*}{\partial \dot{\gamma}} \quad (27)$$

The dynamics mathematical model for systems with geometrical connections in Routh variables is

$$\frac{d}{dt}\frac{\partial R}{\partial \dot{\alpha}} - \frac{\partial R}{\partial \alpha} = \tilde{Q}_\alpha + B'(\alpha,s)\left(\frac{\partial R}{\partial s} + \tilde{Q}_s\right) \quad (28)$$

Under conditions (26) the system allows stationary motion

$$\dot{p}_\beta = p_{\beta 0} = const; p_\gamma = p_{\gamma 0} = const; \alpha = \alpha_0 = const; s = s_0 = const; \quad (29)$$

Further, implementing a similar algorithm, disturbances are introduced, composing equations of the disturbed motion with a distinguished first approximation and applying the results of the theory of critical cases and the theory of stability under constantly acting disturbances. Variants of application of stabilizing controls are considered both for positional coordinates and for the smallest possible part of cyclic coordinates.[38] It is with the latter variant that the division of the vector of cyclic coordinates into some two subvectors is associated, one of which remains uncontrolled.

Conclusion

In contrast to the works published to date on modeling the devices dynamics with parallel kinematics, the article develops such an application of non-free systems analytical mechanics that in the general case eliminates the need to consider the inverse kinematics problem to obtain an analytical form of a nonlinear model for the systems dynamics with geometric constraints. The use vector-matrix equations obtained in analytical form in redundant coordinates creates the possibility for strictly justified application of the nonlinear stability theory methods and mathematical control theory with incomplete information about the state. Using the example of a general stabilization problem, the reducing possibilities for the measurement information volume by reducing the dimension of the controlled subsystem, by which the stabilizing control is determined, as well as further reducing the measurement information volume by forming a stabilizing control using an estimate of the controlled subsystem phase state obtained using the estimates system constructed in the article are discussed. The problems of applying the proposed approach to considering the dynamics of systems with cyclic coordinates are discussed.

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Conflicts of interest

Authors declare that there is no conflict of interest.

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