

Designing paths for box shapes using a Rubik's Snake

Abstract

A Rubik's Snake is a toy that was invented over 40 years ago together with the more famous Rubik's Cube. It starts with a straight ruler and could be twisted into many interesting shapes. One important type of shapes is a box shape. Many complicated shapes can be constructed as a combination of box shapes. Therefore it is a fundamental problem to study the box shape. We constructed a path for a general box shape and derived a counting formula. Some other properties and application were also discussed.

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Introduction

The Rubik's Snake is a toy that was invented by Prof. Rubik in 1981.¹ It consists of right isosceles triangular prisms called blocks that, except for the first and last block, are connected to two other blocks at the centers of the square faces.

The Rubik's Snake has been used as a tool for the study of protein folding² and for the construction of reconfigurable modular robots.³⁻⁵ There are more applications of robots presented in.^{6,7} Some ideas in the study of Rubik's Snake such as the use of rotation matrix is also used in rigid Origami folding.^{8,9} In previous papers that the author collaborated with others, strategies have been given for the design of a Rubik's Snake,¹⁰ and some mathematical problems concerning a Rubik's Snake have been studied.¹¹ Rotations that are not integer multiple of 90 degrees are mentioned in¹⁰ but not much theoretical work is presented. On the other hand,¹¹ has quite some theoretical work but is only concerned with integer multiple of 90 degree rotations. In,¹² Rubik's Snakes with general rotation angles were studied with theoretical work presented. In,¹³ theorems about palindromic, periodic and Möbius Rubik's Snakes were proved.

Although some Rubik's Snake shape design problems^{14,15} were discussed, a fundamental problem of constructing the box shape is not seen in the literature. We will derive a counting formula for box shape in Section 2. Then we will discuss a period 2 general solution for a box with parameter $(1, 1, n)$ in Section 3. We will present a method to construct a path for the general (x, y, z) case in Section 4. The non-Möbius property is studied in Section 5. We will present an application in Section 6. We conclude in Section 7.

A counting formula for a Rubik's Snake with box shape

A box shape of Rubik's Snake is a closed loop¹¹ with positive integer parameters (x, y, z) (related to sizes in three orthogonal directions) that forms a box. This can be demonstrated by Figure 1 in which we have $x = 2, y = 3, z = 5$. It can be seen that there are 164 blocks. The goal of this section is to present a general counting formula for parameters (x, y, z) .

The idea is as follows. We observe that there are 6 "faces" having shared triangles for each square. Overall there are $2(2xy + 2yz + 2zx)$ blocks associated with these 6 "faces". This counts for $4xy + 4yz + 4zx$. Then we observe that we have 12 "edges" with $x + y + x + y + x + y + x + y + z + z + z + z$ blocks. Adding all these up, we come up with the counting formula: $N = 4(x + y + z + xy + yz + zx)$, where N is the number of blocks.

To verify the formula, plug in $x = 2, y = 3, z = 5$. Then $N = 4(2 + 3 + 5 + 6 + 15 + 10) = 164$, as expected from Figure 1.

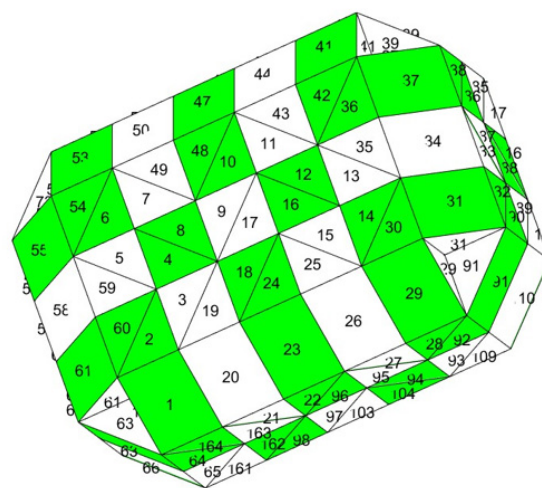


Figure 1 A box with parameters 2,3,5.

A period 2 general solution for a box with parameters $(1, 1, n)$

In,¹¹ the degree sequence is defined for rotations of a Rubik's Snake. 0 means going straight. 1 means turning left by 90 degrees, 3 (or -1) means turning right by 90 degrees and 2 means turning 180 degrees. A collection of such numbers is a degree sequence. For a closed loop the rotation from the last to the first block is counted. Periodicity was also discussed. This is a powerful property to help simplifying problems.

For box with parameters $(1, 1, n)$, we came up with the following general formula for the degree sequence of the snake path:

For odd n , the sequence is $[1, 2, 3, 3, 2, 1]$ repeated $(n-1)/2$ times followed by $[1, 3, 1, 3, 3, 1]$, then $[1, 2, 3, 3, 2, 1]$ repeated $(n-1)/2$ times, followed by $[1, 3, 3, 1, 3, 1]$. Finally, repeat the entire sequence to make it period 2.

For even n , the sequence is $[1, 2, 3, 3, 2, 1]$ repeated $n/2 - 1$ times (for $n = 2$ this means it does not appear) followed by $[1, 2, 3, 3, 1, 3, 1, 3]$, then $[3, 2, 1, 1, 2, 3]$ repeated $n/2 - 1$ times (for $n = 2$ this means it does not appear), followed by $[3, 2, 1, 1, 3, 1, 3, 3, 1]$. Finally, repeat the entire sequence to make it period 2.

Figure 2 shows an example of $(1, 1, 5)$ solution.

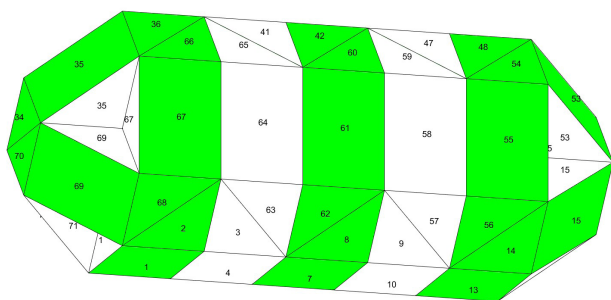


Figure 2 A box with parameters 1,1,5.

Constructing a path for a general (x, y, z) box

The idea to construct a path for a general (x, y, z) box is as follows. We could use a path to fill in one face (for example the top) including edges then use a zig zag pattern to go along the sides back and forth. Finally we could fill in the bottom face including edges and go back to the first block. By using this idea, we came up with a solution for the (2, 3, 5) box in Figure 1. Note that the solution is not unique but one is enough to make a construction. For other parameters the construction is similar. This is a systematic way for general (x, y, z). Our sequence for the (2, 3, 5) box is

[1,0,2,0,2,2,0,2,0,2,0,2,0,2,0,2,0,0,1,3,2,1,1,2,3,3,2,
1,1,3,3,2,1,1,2,3,3,1,3,1,1,2,3,3,2,1,1,2,3,3,2,1,1,3,3,
2,1,1,2,3,3,1,1,2,3,1,0,2,0,2,0,1,3,0,2,0,2,0,2,0,2,0,3,
1,0,2,0,2,0,1,3,0,2,0,2,0,0,1,3,2,1,1,2,3,3,2,1,1,3,1,3,
3,2,1,1,2,3,3,1,1,2,3,3,2,1,1,2,3,3,2,1,1,3,1,3,3,2,1,1,
2,3,1,0,0,2,0,2,0,2,2,0,0,2,0,2,0,0,3,1,0,2,0,3]

We also show an intermediate step after the zig zag pattern of sides are filled and edges of bottom face filled. In Figure 3, we can clearly see that one face is not yet filled except the edges and a path needed to go back to the first block is reserved. It is the first 142 of the 164 blocks in Figure 1. The sequence is also the first 142 numbers of the above sequence, though the 142nd number does not really matter in the figure, as the 143rd block is not shown.

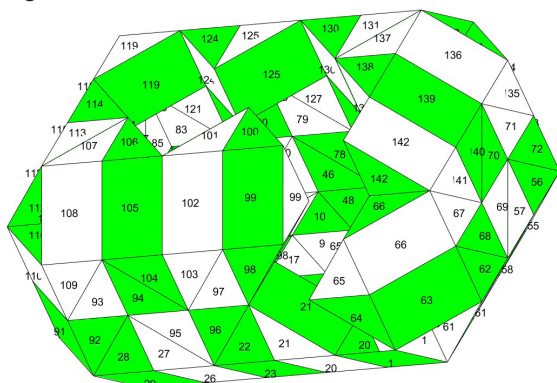


Figure 3 An intermediate step for a box with parameters 2,3,5.

The non-Möbius property of box shapes

In,¹¹ the Möbius strip of a Rubik's Snake was introduced. It was further studied in.¹³ For any box shape, we argue that it is impossible to be a Möbius strip. The reason is as follows. We start a trail from one face on the outside of the box. Based on the definition of the Möbius

strip trail, the trail on the next block stays on the outside of the box. Therefore, it stays on the outside forever and never reaches inside. That implies after going one round in this closed loop, the trail is back to the original face, not the opposite face. Therefore a box is never a Möbius strip.

Based on this result and the conclusion in,¹¹ we conclude that the sum of degrees in a degree sequence of a box is always a multiple of 4.

As an example, we compute the sum of all numbers in the sequence in the previous section. The answer is 252, which is indeed a multiple of 4.

Another example is the general solution for (1, 1, n) we discussed earlier. We listed several parts of that sequence such as [1, 2, 3, 3, 2, 1] etc. Those all add up to even numbers. Note that we used period 2. Therefore the total sum is 2 times an even number. The result is clearly a multiple of 4.

Application

By using the counting formula, we find that for $x=1, y=2, z=2$, $N = 4(1 + 2 + 2 + 2 + 4 + 2) = 52$. If we use a 72-block Rubik's Snake (one of the typical number of blocks sold in stores), then we are left with $72 - 52 = 20$ blocks. Therefore we could make a tortoise with 4 blocks for a head, 4 times 4 blocks for legs and the rest for the body. The solution is shown in Figure 4.

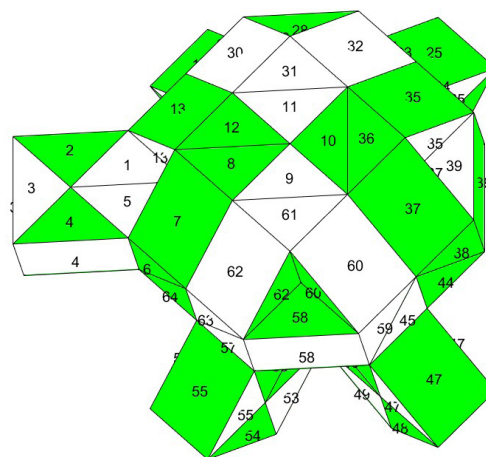


Figure 4 A tortoise shape design.

Conclusion

Designing a box shape using a Rubik's Snake is a fundamental problem. Many interesting shapes are built using such basic shapes. We presented a counting formula for a general box shape. We had a period 2 solution for the special case of parameters (1, 1, n). We then explained a method to build a general box shape. We showed that the box shape is never a Möbius strip, and therefore the sum of the degree sequence is a multiple of 4. We applied our results to build a tortoise shape. Clearly many other shapes could be built and the counting formula is quite useful to distribute the right number of blocks for each part.

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Conflicts of interest

Author declares that there is no conflict of interest.

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