Opinion

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# Paper new definition of the definite integral of fuzzy valued function linearly generated by structural elements

## Opinion

Fuzzy function limit has different forms because of different fuzzy distance. The result of fuzzy distance can be real number or fuzzy number. The fuzzy distance in this paper is a fuzzy number. It is illustrated concretely that $\tilde{a}$ and $\tilde{b}$ are arbitrary two fuzzy numbers, the distance

$$\tilde{d}\left(\tilde{a},\tilde{b}\right)=\bigcup_{\lambda\in[0,1]}\lambda\left[\sup_{\lambda\le\mu\le1}\left|\tilde{a}_{\mu}^{-}-\tilde{b}_{\mu}^{-}\right|,\sup_{0\le\lambda\le\mu}\left(\left|\tilde{a}_{\mu}^{-}-\tilde{b}_{\mu}^{-}\right|\vee\left|\tilde{a}_{\mu}^{+}-\tilde{b}_{\mu}^{+}\right|\right)\right]$$

the fuzzy distance is required to satisfy the level convergence in defining the fuzzy limit. In other words, for fuzzy sequence $\{\tilde{A}_n\}, n=1,2,...$, if there is $\lim_{n\to\infty}\tilde{A}_n$, then be related to $\lambda\in(0,1]$, the cut set of $\tilde{A}_n$ is $\tilde{A}_{n_\lambda}=\left[\tilde{A}_{n_\lambda}^{-},\tilde{A}_{n_\lambda}^{+}\right]$, further function sets $\left\{A_{n_\lambda}^{-}\right\}$ and $\left\{A_{n_\lambda}^{+}\right\}$ for any really positive $\varepsilon$, there is a positive integer N, when p, q > N, such that $\left|A_{p\lambda}^{-}-A_{q\lambda}^{-}\right|<\varepsilon$, and $\left|A_{p\lambda}^{+}-A_{q\lambda}^{+}\right|<\varepsilon$. The limit existence if and only if $\lim_{n\to+\infty}\tilde{A}_n=\tilde{A}_0$, that is for any really positive number $\varepsilon$, there exist positive integer N, when n > N, such that $\tilde{d}\left(\tilde{A}_n,\tilde{A}_0\right)<\varepsilon$. As a form of fuzzy number, fuzzy set E is the fuzzy structural element over the field R of real numbers, if its membership function E(x) has following: (1) E(0)=1, and E(1+0)=E(-1-0)=0; (2) If $x\in[-1,0)$, then E(x) is increasingly monotonic function being right continuous, and if $x\in(0,1)$ then E(x)is decreasing monotonic function being left continuous; (3) If $x\in(-\infty,-1)\cup(1,+\infty)$, then E(x)=0. We can easily understand that structural element E itself is also fuzzy number. If $\tilde{A}=a+rE\left(a\in R,r\in R^{+}\right)$ then e A is a fuzzy number linearly generated by E. Based on extension principle, $\tilde{A}=\bigcup_{\lambda\in[0,1]}\lambda\tilde{A}_{\lambda}=\bigcup_{\lambda\in[0,1]}\lambda\left[a+rE_{\lambda}^{-},a+rE_{\lambda}^{+}\right]$, all fuzzy numbers linearly generated by E is denoted as the symbol $\varepsilon(E)$, and write $\varepsilon(E)=\left\{\tilde{A}\middle|\tilde{A}=a+rE,\forall a\in R,r\in R^{+}\right\}$. Similarly, the fuzzy valued function linearly generated by E in this paper defined in the real field can be expressed as $\tilde{f}(x)=h(x)+\omega(x)E,\forall x\in R$ and $\omega(x)$ are bounded function, even $\omega(x)>0\}$ the symbol $\tilde{N}\left(E_f\right)$ to denote all of fuzzy valued function linearly generated by E, and write $\tilde{N}\left(E_f\right)=\left\{\tilde{f}(x)\middle|\tilde{f}(x)=h(x)+\omega(x)E,\forall x\in X,\omega(x)>0\right\}$. For writing convenience, there must be $\tilde{A}\in\varepsilon(E)$, and $\tilde{f}(x)\in\tilde{N}\left(E_f\right)$ all in this paper.

Let the definition domain of $\tilde{f}(x)U^{0}\left(x_0,\delta'\right)$. If define the limit $\lim_{x\to x_0}\tilde{f}(x)=\tilde{A}$ special attention should be paid to (1) and (2), for the property of fuzzy distance in this paper, it is necessary to strengthen the condition of E(x) to uniform convergence, that is to say with regard to any $\lambda\in(0,1]$, the cut set of E(x) is $E(x)_{\lambda}=\left[E(x)_{\lambda}^{-},E(x)_{\lambda}^{+}\right]$, for any positive, $\varepsilon$ there is a positive integer N, when m, n > N, such that $\left|E_m(x)_{\lambda}^{-},E_n(x)_{\lambda}^{-}\right|<\varepsilon$, $\left|E_m(x)_{\lambda}^{+},E_n(x)_{\lambda}^{+}\right|<\varepsilon$. So the existence of $\lim_{x\to x_0}\tilde{f}(x)$ can be expressed as: for any really positive $\varepsilon$ there exist positive $\delta(<\delta')$, whenever $x',x''\in U^{0}\left(x_0,\delta'\right)$, such that $d\left(\tilde{f}(x'),\tilde{f}(x'')\right)<\varepsilon$, even $\lim_{x\to x_0}\tilde{f}(x)=\tilde{A}$ for any really positive number $\varepsilon>0$, there exists positive $\delta(<\delta')$ such that $0<\left|x-x_0\right|<\delta$, then $d\left(\tilde{f}(x),\tilde{A}\right)<\varepsilon$. In this paper, the limit definition of the fuzzy valued function linearly generated by structure elements is widely used, and the different representation between the fuzzy number and the real number is clear. For the definite integral of fuzzy valued function linearly generated by structural elements, the method of definition is to divide the first step of the fuzzy valued function

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linearly generated by the structural element on the interval of the defined domain, the second step is that for each approximate rectangle obtained by cutting, the approximate rectangular area is calculated, and sum all fuzzy rectangle areas, the third step is to and the fuzzy limit of the summation. Then the new definition is used to study the basic properties of the definite integral of fuzzy valued function linearly generated by structural elements defined on the interval [a, b]. They are Newton Leibniz formula, addition together with multiplication, interval additively, boundedness, local protection and the first mean value theorem for integrals. In order to discuss some the integral condition of the fuzzy valued function linearly generated by structural elements. It is defined that Darboux sum of fuzzy valued function linearly generated by structural elements. Meanwhile, some theorems of fuzzy Darboux sum are discussed. Then the first integrable condition and the second integrable condition of fuzzy valued functions for linearly generated by structural elements on [a, b] are given. Immediately following, the integrable condition of $\tilde{f}(x)$ is continuous, or bounded function with finite discontinuous points, or monotonic function on [a, b] is kicked something around. Whether it's for the properties of definite integral of $\tilde{f}(x)$ defined on [a, b] or the integral condition of e $\tilde{f}(x)$ defined on the interval [a, b], because $\varepsilon$ is a real number, the discussion of the fuzzy limit, monotonicity, continuity and discontinuity can be guaranteed and has practical significance. The study of this paper has a rich role in the theory of fuzzy calculus, and can be applied to fuzzy comprehensive evaluation model, hierarchical principle, language quantifiers and so on. Thank you for your attention to this article. If there is something to be discussed, please put forward.

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## Conflict of interest

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