Research Article

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Improved estimation of reservoir shape factor incorporating skin factor using the direct synthesis technique

Abstract

The Direct Synthesis Technique (*TDS*), introduced by Tiab,[1] is a well test interpretation methodology noted for its practicality and accuracy, utilizing distinctive features observed on log-log plots of pressure and its derivative to enhance reservoir characterization. The shape factor, C_A, originally proposed by Dietz,[2] is crucial for estimating average reservoir pressure. While several authors have explored its estimation using the *TDS* Technique, previous equations often yielded results in the range of 10^{-13}, diverging from the expected 1-100 due to the omission of the skin factor.

This study addresses this gap by incorporating the skin factor into new simplified equations. The new equations provide more accurate and reliable estimates of the shape factor compared to conventional analyses. The proposed expressions are validated through their application to vertical and horizontal wells in homogeneous reservoirs, naturally fractured reservoirs, and hydraulically fractured wells in homogeneous reservoirs. Results show a significant improvement, aligning closely with those obtained from conventional straight-line analysis methodologies. Two examples are given to show the results of the new equations. This advancement enhances the accuracy and reliability of reservoir characterization using the *TDS* Technique.

Keywords: shape factor, pressure derivative, pseudosteady-state, conventional analysis

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Nomenclature

- A Well-drainage area, Ac
- a_r Slope of the semilog plot ΔP vs time
- B Oil volume factor, rb/STB
- b_r Intersection of the semilog plot ΔP vs time
- C_A Dietz reservoir shape factor
- c_t Total system compressibility, psi^{-1}
- c_p Slope of the cartesian plot of ΔP vs time
- C Wellbore storage coefficient, bbl/psi
- d Characteristic distance, ft^2
- d_p Intersection of cartesian plot of ΔP vs time
- h Reservoir thickness, ft
- k Formation permeability, md
- L_w Effective length, ft
- $m(P)$ Pseudopressure, psi^2/cp
- P Pressure, psi
- P_i Initial reservoir pressure, psi
- $\overline{P}$ Average reservoir pressure, psi
- q Oil flow rate, BPD
- q_g Gas flow rate, MSCF/d
- r_w Wellbore radius, ft
- s Skin factor
- s_b Reservoir boundary skin factor
- s_m Mechanical or infinite skin factor
- s_z Vertical skin factor
- s_x x-direction skin factor due to partial penetration effects in the x-direction parallel to the wellbore
- t Drawdown time, hr
- T Temperature, °R
- Δt Shut-in time, hr
- ΔP Pressure drop, psi
- t_D Dimensionless time
- t_D*P_D Dimensionless pressure derivative
- $t*\Delta P'$ Pressure derivative, psi
- $t*\Delta m(P')$ Pseudopressure derivative, psi^2/cp
- $t_D*m(P)_D'$ Dimensionless pseudopressure derivative
- x_f Half-fracture length, ft

Greeks

- Δ Change, drop
- ϕ *Porosity, fraction*
- μ Viscosity, cp
- ω Storativity ratio, for naturally fractured reservoir
- λ Dimensionless interporosity coefficient

Suffices

D Dimensionless

DA Dimensionless based on area

Dxf Dimensionless based on half-fracture length

e External

i Initial, intersection of early unit-slope and radial lines

int Intersection

rpssi Intersection between radial flow and pseudosteady flow

RiR' Intersection between radial flow of derivate and radial flow of semilog graphic

pss Pseudosteady state

w Well

wf Well flowing

ws Well static

1hr 1 hour

Introduction

Tiab (1995)[1] introduced the Direct Synthesis Technique (*TDS*), a well test interpretation methodology noted for its practicality and accuracy. This technique leverages distinctive features observed on log-log plots of pressure and its derivative to enhance reservoir characterization. Comprehensive details of this technique are elaborated in Escobar's works (2015, 2019), and a state-of-the-art review by Escobar et al,.[3]

The shape factor, C_A, originally proposed by Dietz,[2] plays a crucial role in estimating average reservoir pressure. Several authors have explored its estimation using the *TDS* Technique. Initial efforts were presented by Chacon et al,.[4] followed by Escobar et al,.[5] who extended the concept to naturally fractured reservoirs, providing expressions for average reservoir pressure. Subsequent studies by Escobar et al,.[6] addressed multirate tests in both homogeneous and naturally fractured formations. Escobar et al,.[7] further extended these insights from vertical to horizontal wells to approximate average reservoir pressure. More recently, Escobar et al,.[8–10] developed methodologies specific to buildup, drawdown, and multirate tests in homogeneous and naturally fractured reservoirs.

Despite the advancements, previous equations have shown limitations, often yielding results on the order of 10-13, whereas the shape factor C_A is ideally expected to range between 1 and 100. This discrepancy is often attributed to the neglect of the skin factor in these models. In this study, we address this gap by incorporating the skin factor into simplified equations, contrasting with the complexities of prior research. An illustrative example is provided to validate the improved accuracy and reliability of our proposed approach.

Formulation

The dimensionless pressure and pressure derivative for a vertical oil well are given by:

$$P_D(t_D, r_D, C_D, \text{Geometry}, ...) + s = \frac{kh\Delta P}{141.2\,q\,B\mu} \tag{1}$$

and the pressure derivative is given by:

$$t_D * P_D' = \frac{kh(t*\Delta P')}{141.2\,q\,B\mu} \tag{2}$$

The dimensionless pressure and pseudopressure derivative for vertical gas wells are given by:

$$m(P)_D + s = \frac{hk\left[m(P_i) - m(P)\right]}{1422.52 q_g T} \tag{3}$$

$$t_D * \Delta m(P)_D' = \frac{hk\left[t*\Delta m(P)'\right]}{1422.52 q_g T} \tag{4}$$

For a horizontal well reservoir thickness, *h*, is replaced by the effective well length, L_w and the permeability is given by the average permeability $\bar{k} = \sqrt{k_x k_y}$ .The dimensionless time based upon area, wellbore radius and half-fracture length are, respectively, given by:

$$t_{DA} = \frac{0.0002637kt}{\phi\mu c_t A} \tag{5}$$

$$t_D = \frac{0.0002637kt}{\phi\mu c_t r_w^2} \tag{6}$$

$$t_{Dxf} = \frac{0.0002637kt}{\phi\mu c_t x_f^2} \tag{7}$$

According to Escobar et al,.[8] for natural fractured reservoirs the dimensionless time is expressed by:

$$t_{Dd} = \frac{0.0002637\bar{k}t\omega}{(\phi c_t)_f \mu d^2} \tag{8}$$

Being *d* a characteristic length which is replaced by either *A*, r_w, x_f or L_w. A practical way of obtaining the $(\phi c_t)_f$, product was given by Tiab,.[11]

$$(\phi c_t)_f = (\phi c_t)_m \left(\frac{\omega}{1-\omega}\right) \tag{9}$$

and,

$$(\phi c_t)_{f+m} = (\phi c_t)_m \left(1 + \frac{\omega}{1-\omega}\right) \tag{10}$$

The pseudosteady-state pressure behavior for various well-reservoir scenarios has been characterized by several key studies: Ramey et al.,[12] for wells in homogeneous reservoirs, DaPrat[13] for wells in naturally fractured reservoirs, and Russell and Truit[14] for hydraulically fractured wells in homogeneous reservoirs. These studies provide the foundational equations for understanding pressure behavior in these contexts, as follows:

$$P_D(t_{DA}) = 2\pi t_{DA} + 0.5\left[\ln\left(\frac{2.2459\,A}{C_A r_w^2}\right)\right] \tag{11}$$

$$P_D(t_{DA}) = 2\pi t_{DA} + 0.5\left[\ln\left(\frac{2.2459\,A}{C_A r_w^2}\right)\right] + \frac{2\pi\,(1-\omega)^2}{\lambda A} \tag{12}$$

$$P_D(t_{DA}) = 2\pi t_{DA} + 0.5\left[\ln\left(\left[\frac{x_e}{x_f}\right]^2 \frac{2.2459}{C_A}\right)\right] \tag{13}$$

The pseudosteady-state pressure behavior for horizontal wells has been characterized for different reservoir geometries. Ozkan[15] provided the equations for horizontal wells in both cylindrical and rectangular reservoirs as follows:

$$P_D = 2\pi t_{DA} + \frac{1}{2}\ln\left\{\frac{8.9834Ae^{2[1+s_z+s_x+s_m+s_b]}}{C_A L_w^2}\right\} \tag{14}$$

$$P_D = 2\pi t_{DA} + \frac{1}{2}\ln\left\{\frac{2.2458A}{C_A L_w^2}\right\} \tag{15}$$

By substituting the dimensionless quantities from Equations (1) and (5) into Equation (11), the result is:

$$P_{wf} = -\left[\frac{0.23395qB}{\phi c_t Ah}\right]t + P_i - \frac{70.6q\mu B}{kh}\left[\ln\frac{A}{r_w^2} + \ln\left(\frac{2.2458}{C_A}\right) + 2s\right] \tag{16}$$

From the slope, m^*, and intercept, P_{INT}, of Equation (16) the well-drainage area and Dietz shape factor can be obtained from either:

$$A = -\frac{0.23395qB}{Ah\phi c_t m^*} \tag{17}$$

$$C_A = 5.456\frac{m}{m^*}e^{2.303\frac{P_{1hr}-P_{INT}}{m}} \tag{18}$$

Equations (16) to (18) were already reported by Earlougher.[16] Tiab[1] found that the permeability and skin factor are found from:

$$k = \frac{70.6q\mu B}{h(t^*\Delta P')_r} \tag{19}$$

$$s = 0.5\left(\frac{\Delta P_r}{(t^*\Delta P')_r} - \ln\left[\frac{k\,t_r}{\phi\mu c_t r_w^2}\right] + 7.43\right) \tag{20}$$

Additionally, Tiab (1994)[1] provided an equation for determining the well-drainage area:

$$A = \frac{kt_{rpssi}}{301.77\phi\mu c_t} \tag{21}$$

Since the semilog slope, m, is related to the pressure derivative during radial flow, $(t^*\Delta P')r$, as $m= \ln(10)(t^*\Delta P')r$ and m^* is the Cartesian pressure derivative at a late point during pseudosteady-state, $(t^*\Delta P')_{pss}$. Also, $P_{1hr} = P_i - \Delta P_r + \ln(10)(t^*\Delta P')_r \log(t_r)$ and $P_{int} = P_i - \Delta P_{pss} + (t^*\Delta P')_{pss}$ for drawdown, and $P_{1hr} = P_{wf} + \Delta P_r - \ln(10)(t^*\Delta P')_r \log(t_r)$ and $P_{int} = P_{wf} + \Delta P_{pss} - (t^*\Delta P')_{pss}$ for buildup. Then, Equation (18) transforms into:

$$C_A = 12.563\frac{t_{pss}(t*\Delta P')_r}{(t*\Delta P')_{pss}}e^{\frac{-\Delta P_r + \ln(10)(t*\Delta P')_r\cdot\log(t_r) + \Delta P_{pss} - (t*\Delta P')_{pss}}{(t*\Delta P')_r}} \tag{22.a}$$

$$C_A = 12.563\frac{t_{pss}(t*\Delta P')_r}{(t*\Delta P')_{pss}}e^{\frac{\Delta P_r - \ln(10)(t*\Delta P')_r\cdot\log(t_r) - \Delta P_{pss} + (t*\Delta P')_{pss}}{-(t*\Delta P')_r}} \tag{22.b}$$

Equation (22a) or (22b) avoids the need to construct the Cartesian plot.

Let's address the inclusion of the mechanical skin factor, s, in the modified equation. To account for this, we adjusted Equation (1) by subtracting the skin factor. Neglecting it could lead to inaccurate calculations. By substituting the adjusted Equation (1) and Equation (5) into Equation (11) and solving for the shape factor, C_A, we obtain:

$$\frac{kh\Delta P_{pss}}{70.6qB\mu} - 2s = \frac{kt_{pss}}{301.77\phi\mu c_t A} + \left[\ln\left(\frac{2.2459A}{C_A r_w^2}\right)\right] \tag{23}$$

Using Equations (19) and (21) the above equation becomes:

$$\frac{\Delta P_{pss}}{(t^*\Delta P')_r} - 2s = \frac{t_{pss}}{t_{rppsi}} + \left[\ln\left(\frac{2.2459A}{C_A r_w^2}\right)\right] \tag{24}$$

Solving for the shape factor, C_A,

$$C_A = \frac{2.24592A}{r_w^2\exp\left[\frac{\Delta P_{pss}}{(t^*\Delta P')_r} - \frac{t_{pss}}{t_{rpssi}} - 2s\right]} \tag{25}$$

By the same token, Equation (12) and (13) provide:

$$C_A = \frac{2.2458\,A}{r_w^2\exp\left(\frac{\omega t_{pss}(\phi c_t)_t}{t_{rpssi}(\phi c_t)_f}\left(\frac{\Delta P_{pss}}{(t^*\Delta P')_r} - 2s - 1 - \frac{3792.188\phi\mu c_t(1-\omega)^2}{\lambda kt_{Pwf}}\right)\right)} \tag{26}$$

$$C_A = \frac{2.24592A}{x_f^2\exp\left[\frac{\Delta P_{pss}}{(t^*\Delta P')_r} - \frac{t_{pss}}{t_{rpssi}} - 2s\right]} \tag{27}$$

For gas wells, Equations (25), (26) and (27) become:

$$C_A = \frac{2.24592A}{r_w^2\exp\left[\frac{\Delta m(P)_{pss}}{[t^*\Delta m(P)']_r} - \frac{t_{pss}}{t_{rpssi}} - 2s\right]} \tag{28}$$

$$C_A = \frac{2.2458\,A}{r_w^2\exp\left(\frac{\Delta m(P)_{pss}}{[t^*\Delta m(P)']_r} - \frac{\omega t_{pss}(\phi c_t)_t}{t_{rpssi}(\phi c_t)_f} - 2s - 1 - \frac{3792.188\phi\mu c_t(1-\omega)^2}{\lambda kt_{pss}}\right)} \tag{29}$$

$$C_A = \frac{2.24592A}{x_f^2\exp\left[\frac{\Delta m(P)_{pss}}{[t^*\Delta m(P)']_r} - \frac{t_{pss}}{t_{rpssi}} - 2s\right]} \tag{30}$$

The logarithmic pressure derivative of Equations (11) through (15) is given as:

$$t_D * P_D' = 2\pi t_{DA} \tag{31}$$

Dividing Equation (11) by Equation (31), replacing the dimensionless quantities of Equations (1) an (5). Then, replacing in such result Equations (19) and (21) and solving for the shape factor,

$$C_A = \frac{2.24592A}{r_w^2\exp\left[\frac{t_{pss}}{t_{rpssi}}\left(\frac{\Delta P_{pss}}{(t^*\Delta P')_{pss}} - \frac{2s(t^*\Delta P')_r}{(t^*\Delta P')_{pss}} - 1\right)\right]} \tag{32}$$

By the same token, from Equations (12) and (13), we obtain:

$$C_A = \frac{2.2458\,A}{r_w^2\exp\left(\frac{\omega t_{pss}(\phi c_t)_t}{t_{rpssi}(\phi c_t)_f}\left(\frac{\Delta P_{pss}}{(t^*\Delta P')_{pss}} - \frac{2s(t^*\Delta P')_r}{(t^*\Delta P')_{pss}} - 1 - \frac{3792.188\phi\mu c_t(1-\omega)^2}{\lambda kt_{Pwf}}\right)\right)} \tag{33}$$

$$C_A = \frac{2.24592A}{x_f^2\exp\left[\frac{t_{pss}}{t_{rpssi}}\left(\frac{\Delta P_{pss}}{(t^*\Delta P')_{pss}} - \frac{2s(t^*\Delta P')_r}{(t^*\Delta P')_{pss}} - 1\right)\right]} \tag{34}$$

For gas wells,

$$C_A = \frac{2.24592A}{r_w^2\exp\left[\frac{t_{pss}}{t_{rpssi}}\left(\frac{\Delta m(P)_{pss}}{[t^*\Delta m(P)']_r} - \frac{2s[t^*\Delta m(P)']_r}{[t^*\Delta m(P)']_{pss}} - 1\right)\right]} \tag{35}$$

By the same token, from Equations (12) and (13), we obtain:

$$C_A = \frac{2.2458\,A}{r_w^2\exp\left(\frac{\omega t_{pss}(\phi c_t)_t}{t_{rpssi}(\phi c_t)_f}\left(\frac{\Delta m(P)_{pss}}{[t^*\Delta m(P)']_r} - \frac{2s[t^*\Delta m(P)']_r}{[t^*\Delta m(P)']_{pss}} - 1 - \frac{3792.188\phi\mu c_t(1-\omega)^2}{\lambda kt_{Pwf}}\right)\right)} \tag{36}$$

$$C_A = \frac{2.24592A}{x_f^2\exp\left[\frac{\omega t_{pss}(\phi c_t)_t}{t_{rpssi}(\phi c_t)_f}\left(\frac{\Delta m(P)_{pss}}{[t^*\Delta m(P)']_r} - \frac{2s[t^*\Delta m(P)']_r}{[t^*\Delta m(P)']_{pss}} - 1\right)\right]} \tag{37}$$

For horizontal wells, using Equations (14) and (15), we obtain:

$$C_A = \frac{8.9834Ae^{2[1+s_z+s_x+s_m+s_b]}}{L_w^2\exp\left[\frac{\Delta P_{pss}}{(t^*\Delta P')_r} - \frac{t_{pss}}{t_{rpssi}} - 2s\right]} \tag{38}$$

$$C_A = \frac{2.2458A}{L_w^2 \exp\left[\frac{\Delta P_{pss}}{(t*\Delta P')_r} - \frac{t_{pss}}{t_{rpssi}} - 2s\right]} \tag{39}$$

For naturally fractured reserv oirs, Escobar et al,.[9] made use of Equation with Equations (14) and (15) so,

$$C_A = \frac{8.9834Ae^{2[1+s_z+s_x+s_m+s_b]}}{L_w^2 \exp\left[\frac{\Delta P_{pss}}{(t*\Delta P')_r} - \frac{\omega t_{pss}(\phi c_t)_t}{t_{rpssi}(\phi c_t)_f} - 2s\right]} \tag{40}$$

$$C_A = \frac{2.2458A}{L_w^2 \exp\left[\frac{\Delta P_{pss}}{(t*\Delta P')_r} - \frac{\omega t_{pss}(\phi c_t)_t}{t_{rpssi}(\phi c_t)_f} - 2s\right]} \tag{41}$$

For horizontal gas wells, Equation (38) through (41) will become:

$$C_A = \frac{8.9834Ae^{2[1+s_z+s_x+s_m+s_b]}}{L_w^2 \exp\left[\frac{\Delta m(P)_{pss}}{[t*\Delta m(P)']_r} - \frac{t_{pss}}{t_{rpssi}} - 2s\right]} \tag{42}$$

$$C_A = \frac{2.2458A}{L_w^2 \exp\left[\frac{\Delta m(P)_{pss}}{[t*\Delta m(P)']_r} - \frac{t_{pss}}{t_{rpssi}} - 2s\right]} \tag{43}$$

$$C_A = \frac{8.9834Ae^{2[1+s_z+s_x+s_m+s_b]}}{L_w^2 \exp\left[\frac{\Delta m(P)_{pss}}{[t*\Delta m(P)']_r} - \frac{\omega t_{pss}(\phi c_t)_t}{t_{rpssi}(\phi c_t)_f} - 2s\right]} \tag{44}$$

$$C_A = \frac{2.2458A}{L_w^2 \exp\left[\frac{\Delta m(P)_{pss}}{[t*\Delta m(P)']_r} - \frac{\omega t_{pss}(\phi c_t)_t}{t_{rpssi}(\phi c_t)_f} - 2s\right]} \tag{45}$$

As performed before, dividing Equations (14) and (15) by the pressure derivative given by Equation (31), it will result:

$$C_A = \frac{8.9834Ae^{2[1+s_z+s_x+s_m+s_b]}}{L_w^2 \exp\left[\frac{t_{pss}}{t_{rpssi}}\left(\frac{\Delta P_{pss}}{(t*\Delta P')_{pss}} - \frac{2s(t*\Delta P')_r}{(t*\Delta P')_{pss}} - 1\right)\right]} \tag{46}$$

$$C_A = \frac{2.2458A}{L_w^2 \exp\left[\frac{t_{pss}}{t_{rpssi}}\left(\frac{\Delta P_{pss}}{(t*\Delta P')_{pss}} - \frac{2s(t*\Delta P')_r}{(t*\Delta P')_{pss}} - 1\right)\right]} \tag{47}$$

For naturally fractured reservoirs,

$$C_A = \frac{8.9834Ae^{2[1+s_z+s_x+s_m+s_b]}}{L_w^2 \exp\left[\frac{\omega t_{pss}(\phi c_t)_t}{t_{rpssi}(\phi c_t)_f}\left(\frac{\Delta P_{pss}}{(t*\Delta P')_{pss}} - \frac{2s(t*\Delta P')_r}{(t*\Delta P')_{pss}} - 1\right)\right]} \tag{48}$$

$$C_A = \frac{2.2458A}{L_w^2 \exp\left[\frac{\omega t_{pss}(\phi c_t)_t}{t_{rpssi}(\phi c_t)_f}\left(\frac{\Delta P_{pss}}{(t*\Delta P')_{pss}} - \frac{2s(t*\Delta P')_r}{(t*\Delta P')_{pss}} - 1\right)\right]} \tag{49}$$

For gas horizontal wells, Equations (46) through (49) become:

$$C_A = \frac{8.9834Ae^{2[1+s_z+s_x+s_m+s_b]}}{L_w^2 \exp\left[\frac{t_{pss}}{t_{rppsi}}\left(\frac{\Delta m(P)_{pss}}{[t*\Delta m(P)']_r} - \frac{2s[t*\Delta m(P)']_r}{[t*\Delta m(P)']_{pss}} - 1\right)\right]} \tag{50}$$

$$C_A = \frac{2.2458A}{L_w^2 \exp\left[\frac{t_{pss}}{t_{rpssi}}\left(\frac{\Delta m(P)_{pss}}{[t*\Delta m(P)']_r} - \frac{2s[t*\Delta m(P)']_r}{[t*\Delta m(P)']_{pss}} - 1\right)\right]} \tag{51}$$

For naturally-fractured reservoirs,

$$C_A = \frac{8.9834Ae^{2[1+s_z+s_x+s_m+s_b]}}{L_w^2 \exp\left[\frac{\omega t_{pss}(\phi c_t)_t}{t_{rpssi}(\phi c_t)_f}\left(\frac{\Delta m(P)_{pss}}{[t*\Delta m(P)']_r} - \frac{2s[t*\Delta m(P)']_r}{[t*\Delta m(P)']_{pss}} - 1\right)\right]} \tag{52}$$

$$C_A = \frac{2.2458A}{L_w^2 \exp\left[\frac{\omega t_{pss}(\phi c_t)_t}{t_{rpssi}(\phi c_t)_f}\left(\frac{\Delta m(P)_{pss}}{[t*\Delta m(P)']_r} - \frac{2s[t*\Delta m(P)']_r}{[t*\Delta m(P)']_{pss}} - 1\right)\right]} \tag{53}$$

Recently, Tiab (2024)[17], published new equations to estimate the reservoir shape factor and skin factor which are:

$$C_A = 12.65\left(\frac{a_R}{c_P}\right)\exp\left[\frac{b_R - d_P}{a_R}\right] \tag{54}$$

$$s = 4.2166 - 0.5\ln\left(\frac{k\Delta t_{RiR'}}{\phi \text{ì}\, c_t r_w^2}\right) \tag{55}$$

All the developed equations for the estimation of the reservoir shape factor are applied to both draw dawn and buildup pressure tests. They can also be easily extended to multirate testing as described by Escobar et al.,[8] As mentioned above, $m*$, which Tiab[17] called c_p, can be replaced the cartesian pressure derivative at a late point during the pseudosteady-state, $(t*\Delta P')_{pss}$. Refer to Figure 1 to observe that, $a_r = (t*\Delta P')_r$, d_P is the $\Delta P_{int} = \Delta P_{pss} + (t*\Delta P')_{pss}$. Then, Equation (54) becomes:

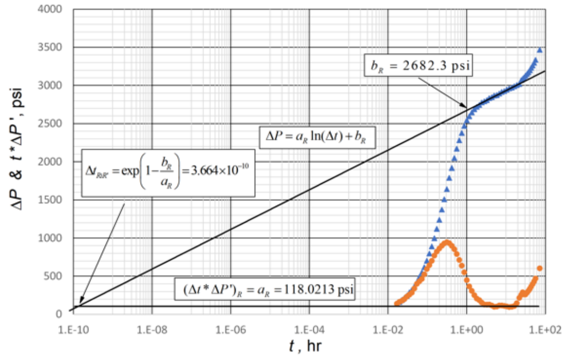

Figure 1 Semilog plot of pressure drop (blue curve) and pressure derivative (orange curve) versus time of example 1.

$$C_A = 12.65\left(\frac{t_{pss}(t*\Delta P')_r}{(t*\Delta P')_{pss}}\right)\exp\left(\frac{\Delta P_{1hr} - \left[\Delta P_{pss} + (t*\Delta P')_{pss}\right]}{(t*\Delta P')_r}\right) \tag{56}$$

Equation (56) avoids building the cartesian plot of pressure versus time.

Finally, for comparison purposes we bring Equation (1.12) from Djebrouni et al,.[4]:

$$C_A = \frac{2.2458A}{r_w^2 \exp\left(\frac{kt_{pss}}{301.77\phi\mu c_t A}\left(\frac{(\Delta P)_{pss}}{(t*\Delta P')_{pss}} - 1\right)\right)} \tag{57}$$

Examples

The *TDS* Technique, as established by Tiab,[17] represents a highly versatile and practical approach for interpreting pressure and rate transient analysis. This method utilizes characteristic points, intercepts, and features discernible on log-log plots of pressure and pressure derivative versus time. The use of mnemotechnical subscripts in the *TDS* Technique enhances its practicality; for example, '*r*' denotes radial dimensions, while '*pss*' signifies pseudosteady-state conditions. These subscripts are not just placeholders but are crucial for understanding the dynamics of fluid flow in reservoirs, as demonstrated in the example below. By applying the *TDS* Technique, engineers can derive meaningful insights into the reservoir's behavior, aiding in more accurate predictions and efficient management of hydrocarbon extraction processes.

Example 1

Figure 1, 2 present the pressure and pressure derivative versus time data for a drawdown test which initial pressure is 6009 psi. Other relevant information is provided in Table 1. Estimate reservoir parameters and the shape factor by *TDS* and conventional techniques.

Solution by *TDS* technique: The following information was read from Figure 2:

t_r = 7.7608 hr ΔP_r = 2916.22 psi $(t*\Delta P')_r$ = 118.0213 ps

t_{pss} = 72 hr ΔP_{pss}=3468.218 psi $(t*\Delta P')_{pss}$ = 605.1 psi

t_{rpssi}=14.2685 hr

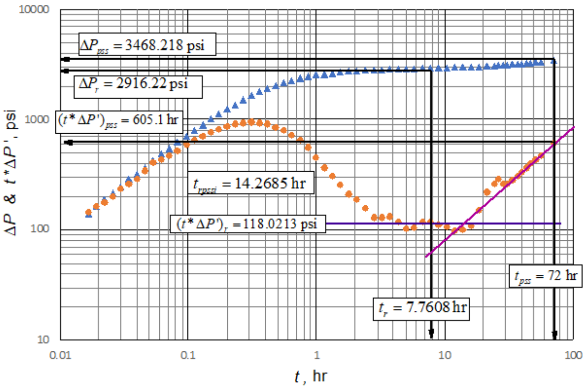

Figure 2 Pressure drop (blue curve) and pressure derivative (orange curve) versus time log-log plot of example 1.

Find reservoir permeability using Equations (19), which will be used to find the skin factor:

$$k=\frac{70.6q\mu B}{h(t*\Delta P')_r}=\frac{(70.6)(200)(3.2)(1.23)}{(60)(117.577599)}=7.878md$$

Find skin factor using Equations (20), which will be used to find the well-drainage area:

$$s=0.5\left(\frac{\Delta P_r}{(t*\Delta P')_r}-\ln\left[\frac{k\,t_r}{\phi\mu c_t r_w^2}\right]+7.43\right)$$

$$s=0.5\left(\frac{2916.22}{117.578}-\ln\left[\frac{(7.878)(7.76)}{(0.1)(3.2)(1\times10^{-6})(0.3)^2}\right]+7.43\right)=5.474$$

Use Equation (21) to determine the well-drainage area which will be used to find the shape factor:

$$A=\frac{kt_{rpssi}}{301.77\phi\mu c_t}=\frac{(7.878)(14.2685)}{301.77(0.1)(3.2)(1\times10^{-6})}=1164035.505\text{ ft}^2$$

Determine the reservoir shape factor with Equations (25) and (32):

$$C_A=\frac{2.2459A}{r_w^2\exp\left[\frac{\Delta P_{pss}}{(t*\Delta P')_r}-\frac{t_{pss}}{t_{rpssi}}-2s\right]}=\frac{2.2459(1164035.505)}{0.3^2\exp\left[\frac{3020.068}{117.578}-\frac{72}{14.2685}-2(5.43)\right]}=35.7$$

$$C_A=\frac{2.2459A}{r_w^2\exp\left[\frac{t_{pss}}{t_{rpssi}}\left(\frac{\Delta P_{pss}}{(t*\Delta P')_{pss}}-\frac{2s(t*\Delta P')_r}{(t*\Delta P')_{pss}}-1\right)\right]}$$

$$C_A=\frac{2.24592(1164035.505)}{0.3^2\exp\left[\frac{72}{14.2685}\left(\frac{3468.218}{601.5}-\frac{2(5.43)(118.022)}{601.5}-1\right)\right]}=47.18$$

Now used the latest Equation published by Tiab[17] to estimate the shape factor, Equation (54). The following information was read from Figure 1:

a_R = 116.0145 psi b_R = 2682.3 psi

This equation requires finding the slope, m^* -called c_P by Tiab (2024)[17], of a cartesian plot of ΔP versus time which intercept, ΔP_{int}, is called by Tiab (2024) as d_P. Although such plot is not presented here, the values are:

$c_P = m^*$ = 8.3463 psi/hr $\Delta P_{int} = d_P$ = 3145.8 psi ΔP_{1hr}2680 psi

$$C_A=12.65\left(\frac{a_R}{c_P}\right)\exp\left[\frac{b_R-d_P}{a_R}\right]=12.65\left(\frac{118.0213}{8.3463}\right)\exp\left[\frac{2682.3-3145.8806}{118.0213}\right]=3.622$$

From Figure 1, ΔP_{1hr} =2680 psi. Use the Equation (56) to re-estimate the shape factor,

$$C_A=12.65\left(\frac{t_{pss}(t*\Delta P')_r}{(t*\Delta P')_{pss}}\right)\exp\left(\frac{\Delta P_{1hr}-\left[\Delta P_{pss}+(t*\Delta P')_{pss}\right]}{(t*\Delta P')_r}\right)$$

$$C_A=12.65\left(\frac{(72)(118.0213)}{605.1}\right)\exp\left(\frac{2680-[3375.577+605.1]}{(116.0145)}\right)=3.43$$

Estimate the reservoir shape factor using Equation (57). Needless to remind that it was already published by Djebrouni et al,.[4]:

$$C_A=\frac{2.2458A}{r_w^2\exp\left(\frac{kt_{pss}}{301.77\phi\mu c_t A}\left(\frac{(\Delta P)_{pss}}{(t*\Delta P')_{pss}}-1\right)\right)}$$

$$C_A=\frac{2.2458(1164035.505)}{0.3^2\exp\left(\frac{(7.878)(72)}{301.77(0.1)(3.2)(1\times10^{-6})(1164035.505)}\left(\frac{(3468.2)}{(218.1219)}-1\right)\right)}=4360361.252$$

The intersection point between the radial flow regime and the semilog trend of the pressure drop during infinite-acting behavior can be identified in Figure 1, as follows:

$$\Delta P_{RiR'}=\exp\left[1-\frac{b_R}{(t^*\Delta P')_r}\right]=\exp\left[1-\frac{2682.3}{(118.0213)}\right]=3.66415\times10^{-10}$$

This intersection allows for the re-estimation of the skin factor using Equation (55), which closely aligns with the value estimated from Equation (20),

$$s=4.2166-0.5\ln\left[\frac{k\Delta t_{RiR'}}{\phi\mu c_t r_w^2}\right]=4.2166-0.5\ln\left[\frac{(7.878)(3.66415\times10^{-10})}{(0.1)(3.2)(1\times10^{-6})(0.3)^2}\right]=5.366$$

Solution by straight-line conventional analysis: The following information was read from Figure 3, 4:

m^* = 8.3463 psi/hr m = -264.316 psi/cycle

P_{1hr}= 3333.9882 psi P_{int} = 3141.7364 psi

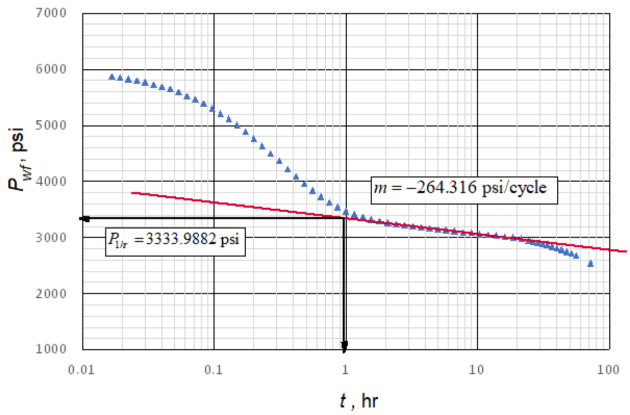

Figure 3 Semilog plot of pressure versus of example 1.

The well-drainage area was obtained from equation (17) and Dietz shape factor could be obtained from equation (18) either:

$$A=-\frac{0.23395qB}{Ah\phi c_t m^*}=-\frac{(0.23395)(200)(1.23)}{(60)(0.1)(1\times10^{-6})(8.404)}=1141355.306\text{ ft}^2$$

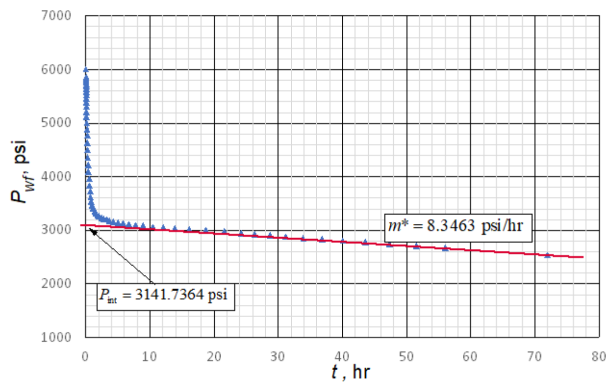

Figure 4 Cartesian plot of pressure versus of example 1.

$$C_A = 5.456\frac{m}{m^*}e^{2.303\frac{P_{1hr}-P_{INT}}{m}} = 5.456\frac{(264.316)}{8.3463}e^{2.303\frac{(3333.9882)-3142.7364)}{(-264.316)}} = 32.36$$

Use Equation (22a), proposed here, to estimate the reservoir shape factor. The idea is not to build neither semilog nor cartesian plot, then: $P_{1hr} = P_i - b_r = 6009 - 2682.9 = 3326.7$, $P_{pss} = P_i - \Delta P_{pss} = 6009 - 3468.218 = 2540.782$ psi, then:

$$C_A = 12.563\frac{t_{pss}(t*\Delta P')_r}{(t*\Delta P')_{pss}}e^{\frac{-\Delta P+\ln(10)(t*\Delta P')_r\cdot\log(t_r)+\Delta P_{pss}-(t*\Delta P')_{pss}}{(t*\Delta P')_r}}$$

$$C_A = 12.563\frac{72(118.0213)}{605.1}e^{\frac{-2916.22+\ln(10)*(118.0213)+3468.218-605.1}{118.0213}} = 873$$

Example 2

Figure 5 presents a log-log plot of the pressure and pressure derivative versus time data for a buildup test which well-flowing pressure is 2980 psi. Reservoir, fluid and well data are provided in Table 1. Estimate the shape factor by *TDS* and conventional techniques. For space-saving purposes, it is also given $m^* = 0.1$ psi/hr, $m = 46.98$ psi/cycle, $P_{1hr} = 3297.68$ psi and $P_{int} = 3370$ psi.

$t_r = 7.7608$ hr $\Delta P_r = 2916.22$ psi $(t^*\Delta P')_r = 118.0213$ psi

$t_{pss} = 72$ hr $\Delta P_{pss} = 3468.218$ psi $(t^*\Delta P')_{pss} = 605.1$ psi

$t_{rpssi} = 14.2685$ hr

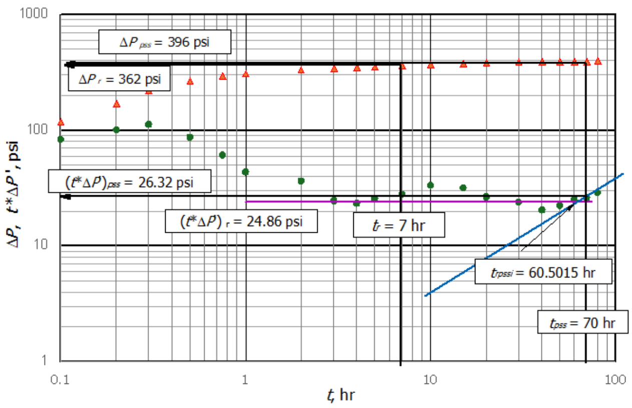

Figure 5 Pressure drop and pressure derivative versus time log-log plot of example 2.

Solution by *TDS*: The following data were read from Figure 5,

$t_r = 7$ hr $\Delta P_r = 362$ psi $(t^*\Delta P')_{pss} = 26.32$ psi

$\Delta P_{pss} = 396$ psi $t_{pss} = 70$ hr $(t^*\Delta P')_r = 24.86$ psi

$t_{rpi} = 60.5015$ hr

Table 1 Input data for given examples

Parameter	Example1	Example2
h, ft	60	44
r_w, ft	0.33	
q, bbl/D	200	340
B, rb/STB	1.23	1.24
μ, $\chi\pi$	3.2	0.76
φ, %	10	12
c_t, 1/psi	1×10^{-6}	36×10^{-6}

Permeability, skin factor and well-drainage area of 23.4 md, 1.034 and 1428942 ft^2 were estimated, respectively, with Equations (19), (29) and (21). The reservoir shape factor was determined with Equations (25) and (32), respectively, to be 86.14 and 20.35.

Solution by straight-line conventional analysis: A reservoir shape factor of 34.65 was found with Equation (18).

Comments on the results

The new equations presented in this study for estimating the reservoir shape factor provided reasonable values compared to conventional analyses. A summary of results is given in Table 2 for both *TDS* and straight-line conventional analysis. Needless to say that any minor change inside the exponential will alter radically the answer. It was observed that previous equations, represented here by Equation (57), deviated significantly from conventional methodologies due to their omission of the skin factor.

Table 2 Summary of C_A Results

Equation	Example1	Example2
TDS		
25	35.7	86.14
32	47.18	20.35
54	3.622	
56	3.43	
57	4360361.25	
Conventional		
18	32.36	34.65
22a	873	

Conclusion

New expressions are introduced in this paper to estimate the reservoir shape factor from buildup or drawdown tests pressure tests using the *TDS* Technique. These expressions are applied to vertical hydrocarbon wells under three scenarios: homogeneous reservoirs, naturally fractured reservoirs, and hydraulic fractured wells in homogeneous reservoirs. Also, expressions for hydrocarbon horizontal wells in anisotropic homogeneous and heterogeneous formations are presented. The results are successfully compared with those obtained from the straight-line conventional analyses.

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Conflicts of interest

The authors declare that there is no conflicts of interest.

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