Research Article

Mathematical modelling for characterising and forecasting the water supplying system condition in Lajeado (RS)

Abstract

Characterising water consumption in relation to total population growth is essential for managing water resources in a particular region. This study aims at characterising the water supply system called Companhia Riograndense de Saneamento (CORSAN) at Lajeado (RS) from 2000-2007. And by this characterisation we wish to write to forecast future water consumption for the population in CORSAN SAA at Lajeado, to check whether it is probable or not a water scarcity collapse. In the characterising stage we used collected data tabulation and pairing by Microsoft® Excel 2003. In the 2008-2032 data forecasting we employed the LAB Fit Curve Fitting by the chi-square method through time series whose data refer to the 2000-2007 period. We used this software to indicate the most useful function taking into consideration, the reduced chi-square value and correlation and determination coefficients. This study showed the brink of collapse if the current rate of consumption and discharge are kept, providing for authorities the chance to take preventive actions. In the 2008-2032 data forecasting we employed the LAB Fit Curve Fitting which provides the mathematical model functions ordered by the least chi-square method, providing the increase profile, the chi-square value fit into the non-rejection zone, and correlation and determination coefficients. Results for characterising stage were fulfilled, and in the future forecasting stage we have found that the CORSAN physical structure at Lajeado set in 2007 will supply the population up to mid-2026 provided that the same levels of production and water consumption are maintained.

Keywords: water supply, mathematical modelling, consumption projection

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Introduction

Water is essential for human life, economic development, health, and welfare in all societies.[1,2] Thus, since ancient times societies are built closer to rivers, lakes and shallow streams.[1] Today the total quantity of water on Earth is about 1.386billion km^3; out of these, 97.5 percent are saltwater and 2.5 percent freshwater. Out of 2.5 percent of freshwater, 99.6 percent are not proper drinking water. Hence, freshwater available on the Earth's surface for human intake is only 0.4 percent in lakes, humid areas and rivers (LLamas, 1991). While Brazil concentrates 12-16 percent of all the potable water in the world, there is badly distributed and water shortage is due to continuing pressures caused by the population's multiple uses.[2] (Clarke et al., 2005). The population density in Brazil in 2007 was 22.3hab/km^2; in Rio Grande do Sul was 37.56hab/km^2 and in Lajeado came up to 746.39hab/km^2, which is 19.89 times greater than the very state's population density.[3] So it is worth noting that Lajeado is one of the densest towns in the state, while its territory is essentially urban, with a rate of 97 percent, its urban demand increasing particularly when its last districts became towns in 2000.[4] In tune with this, this study wants to forecast water consumption, characterise CORSAN SAA, and write a mathematical model to simulate the water consumption tendency line in relation to variables for Lajeado (RS). The specific objectives were:

i. Characterising the treated and consumed water volume, and population increase in Lajeado;

ii. Drawing a tendency line for production increase and water consumption in relation to the population growth;

iii. Identifying the mean value for the water consumption for each inhabitant/day.

The significance of this study for CORSAN at Lajeado consists of how the water supply system may manage the water demand and draw a tendency line check whether a water shortage will come out or not according to its population growth. This research issue emerged because of lack of projects and research in this field in the region of Vale do Taquari (region where this CORSAN is situated), a region that is strongly dependant of the Taquari River for water supply in Lajeado. Considering the population growth, the short water supply, and the increasing water demand for inhabitants at Lajeado, particularly after some districts having emancipated, we wonder whether a water scarcity collapse is probable or not, when the same outflow and infrastructure are the same.

Theoretical reference

For Nucci[5] & Narchi[6] urban water demand match the total water rate for the various uses on a marked urban area. The demand for urban water is an important factor for planning and management.[6] To check water demand it is necessary to collect time series data. For Trautwein[7] time series data consist of a set of observations of variables in a sequence along the time which is dependent on each other. When time series are statistically combined to determine a future estimation, this process is considered a forecasting method.[7] In Brazil the forecasting method was not largely used. Usually System of Water Supply (SWS) are designed from forecasting and projection on per capita water consumption, and this is a major factor for projection, but cannot be considered as the only one as it suffers variations from external factors.[8] Long-term forecasting is less susceptible to consumption variation and over the course of time may be set, by introducing the observed data to adapt the projected data to the observed ones.[8] One forecasting method occurs by mathematical modelling.[9] Mathematical modelling as an art of changing a position in current reality into a mathematical problem takes answers in an easy language.[9] Today there are about tens of software's working with mathematical modelling. One that comes to the foreground in Brazil is LAB Fit Curve Fitting software for its wide use in teaching and research laboratories and for providing statistical programme certifications by the project of the Statistical Reference Datasets Project (SRDP) in the National Institute of Standards and Technology (NIST).[10] LAB Fit Curve Fitting Software 7.2.31.[10] was developed to analyse and work with data in a large way as it has both aspects of fitting a function with its graphic representation, a menu with basic statistical calculation, one with error programming calculation and for having a library with more than 200 functions in on independent variable and 280 in two ones. These authors also add that functions are classified according to the lower reduced x^2 value (x^2 red) provide the correlation coefficient (r), determination coefficient (r^2), freedom degrees (FD) and x^2 value. The x^2 test is also used to assess discrepancy between observed and expected data frequencies.[11]

Another datum provided by LAB Fit Curve Fitting Software is that the correlation coefficient is the one which measures the dependence degree, that is, it depicts the linear relation between data pairs in quantitative time series[11,12] This coefficient may assume any value between -1 and 1, and when the value is r=1 there is a perfect positive correlation. Otherwise, the coefficient correlation value is r=-1, then there is a perfect negative correlation.[13,14] The Pearson's determination coefficient (r^2), also provided by LAB Fit Curve Fitting Software, is defined as the variation rate in which one variable is explained according to the other one.[12] The determination coefficient variation goes from 0 to 1, and the higher the r^2 value is, the better the function matches to the dispersive graphic.[13,14]

Methodology

The studied area lies on the lower Northeast in the mid-East of the Brazilian state of Rio Grande do Sul, in the region called Vale do Taquari in the municipal area of Lajeado. Lajeado lies between 29º 24' 06" and 29º 29' 52" South latitude coordinates and between 51° 55' 06" and 52° 06' 42" West coordinates (PML, 2009). Raw water catchment for treating and supplying the town is carried in Taquari River and then it is conducted to the Water Treatment Plant (WTP), where it goes through flocculation, decantation, filtration, chlorination, fluoridation and procedure (where physical, chemical and bacteriological analyses are conducted). After water treatment, it is pumped into sinks to be distributed by sewage for every home.[15] The total area of Lajeado is 90.40 km^2.[4] However, districts of Planalto, Igrejinha, Centenário, Imigrante, Conventos, São Bento and Floresta[16] are not supplied by the Riograndense Sanitary Company (CORSAN). This study is confined to districts supplied by CORSAN Water Supplying System (SAA) in Lajeado between 2000 and 2007. In this study neither water quality nor the population's sociological behaviour relating to water consumption will be taken into consideration. Data collection and data concerning SAA, such as yearly Captured raw water (CRW) yearly captured treated water (CTW) and extension of the water distribution network, were obtained at the National Information System about Sanitation (SNIS) between 2000 and 2007; rate of economic joints between 2001 and 2007; and average m^3 fare between 2002 and 2007 shows in Figure 1 & Figure 2.

Data collected at SAA at CORSAN in Lajeado between 2000 and 2007 was the consumed water rate (C), and the same data in 2007 was the consumed water rate classified in areas. Collected data concerning the population (habitants) obtained at the Brazilian Institute of Geography and Statistics (IBGE) between 2000 and 2007 was the total population, and data obtained at SNIS between 2000 and 2007 was the population supplied by SAA at CORSAN. Data from intervening variables, such as humidity (between 2003 and 2006), rainfall rate (between 2003 and 2007) and yearly average temperature (between 2003 and 2006), all referring Lajeado, were collected at the Hydrometeorological Information Centre (CIH) at UNIVATES. In every period, collecting date above was the only one we obtained at the Database at SNIS, CORSAN, IBGE and CIH. As it will be necessary to work with paired data variables, the sampling period had to be reduced. After collection data were tabularised by using Microsoft® Office Excel 2003 software.[17] During characterisation of SAA at CORSAN in Lajeado we used BioEstat 5.0[18] software to determine the correlation and determination coefficients. During forecasting future data of SAA at CORSAN in Lajeado we used the LAB Fit Curve Fitting,[19] and it suggests among a set of available functions which is the best tendency line for provided time series. After conducting regressions, we will construct a future projection to check whether a water shortage will come out or not if situation remains the same.

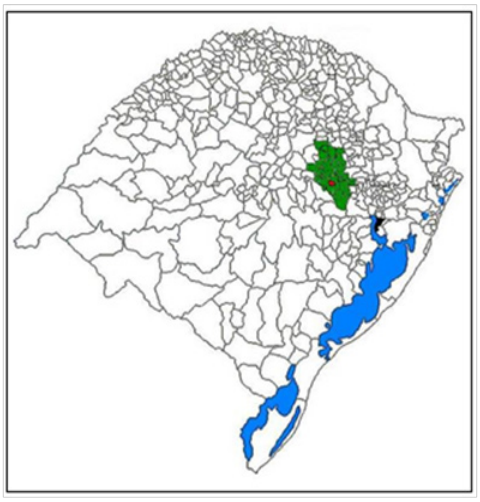

Figure 1 Geographical positions of Lajeado in the state of Rio Grande do Sul. Source: PML[4] the author's adaption.

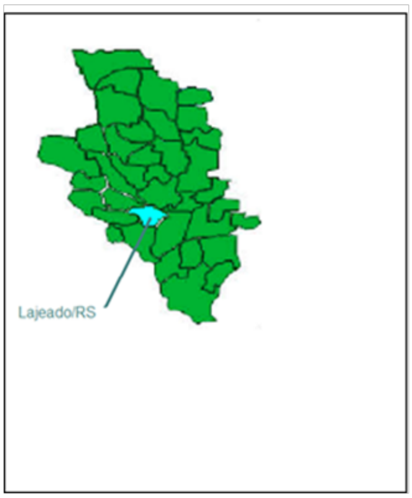

Figure 2 Geographical positions of Lajeado in the region of Vale do Taquari (RS).

Source: http://www.bdr.univates.br/, the author's adaption.

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Results

During characterisation of SAA at CORSAN in Lajeado, we have found the following results showed in Table 1.

Discussion and conclusions

Table 1-based discussion and conclusion

Concerning variables it is worth noting that growth in the CTW volume period is 22.52 percent and C is 18.98 percent, and with data concerning losses (%) that increased in 4.18 percent we have found that CTW volume growth is larger than C growth, which is explained by the increase in losses. The pressure inside water pipes also affects water distribution, based on the higher pressure and the higher water loss, to maintain the same C volume demand. The increase in supplied population rate is larger than the total population growth in 2.53 times. Hence, a decrease in non-supplied population rate, an increase in the extension of the distribution network and an increase in the per capita consumption occur due to an enlargement of the supplying area. The correlation among variables is highlighted for its high degree of correlation between CWR and CTW, CWR and C, and CTW and C in an SAA: this is usually in a normal situation. The correlation of C volume with yearly average temperature and yearly humidity average has a median intensity. Therefore, the C yearly volume behaviour is also explained in relation to other factors, which is beyond the scope of this study. In relation to intervening variables, per capita consumption is explained in relation to the humidity. Concerning the C volume by areas we have found that the area with the largest consumption of water is the domestic ones, corresponding to 84 percent of the sites in Lajeado. This result derives from the fact that the town is basically an urban-supplied area, where there is naturally a higher rate of domestic economies. The lower consumption rate is the industrial area with one percent as most industries do not use water in their productive processes, while those using it take water from deep private wells. During the forecasting of time series at SAA at CORSAN in Lajeado Table 2 results were found.

Table 1 During characterisation of SAA at CORSAN in Lajeado

Period	Variable	Growth period	Yearly growth average	Analysis
Between 2000 and 2007	CRW	16.46%	2,35%	
Between 2000 and 2007	CTW	22.52%	3,22%	
Between 2000 and 2007	C	18.98%	2,71%	
Between 2000 and 2007	Losses	4.18%	0,52%	
Between 2001 and 2007	Extension	8.44%	1,41%	
Between 2000 and 2007	Total population	5.30%	0,76%	
Between 2000 and 2007	Supplied population	13.46%	1,92%	
Between 2000 and 2007	Not supplied population	-14.73%	-2,10%	
Between 2000 and 2007	per capita consumption	6.15%	1,02%	
Period	**Relation between variables**	**R**	**r^2**	**Correlation**
Between 2000 and 2007	CRW x CTW	0.98	0,96	High
Between 2000 and 2007	CRW x C	0.83	0,68	High
Between 2000 and 2007	CTW x C	0.87	0,76	High
Between 2003 and 2006	C x Temperature	0.58	0,34	Median
Between 2003 and 2007	C x Rainfall	0.04	0,002	No
Between 2003 and 2007	C x Relative humidity in the air	0.68	0,46	Median
Between 2000 and 2007	per capita C x Total Population	0.64	0,41	Median
Between 2000 and 2007	C per capita x Supplied population	0.08	0,006	No
Between 2003 and 2006	C per capita x Temperature	0.46	0,21	Low
Between 2003 and 2007	C per capita x Rainfall	0.12	0,01	No
Between 2003 and 2006	C per capita x Relative humidity	0.83	0,70	High
Between 2002 and 2007	C per capita x Average fare	0.56	0,42	Median

Table Continued

Period	C by area	Participation	C per capita/Area $\left(\frac{L}{hab.day}\right)$
2007	Residential	84%	127
2007	Commercial	13%	20
2007	Public	2%	3
2007	Industrial	1%	2

Figure 3 & Table 2 based discussion and conclusion

Functions (9, 33, 382, 181, 343) suggested by LAB Fit Curve Fitting Software, with the criterion of the lowest red. x^2 and approved by the criterion of providing a profile of growth and correlation and determination coefficients, gave a solid result, so that they were used to make the time series future forecasting, while functions 140 and 304 provided no solid result and were rejected. The collapse due to water shortage at SAA at CORSAN in Lajeado (RS) in relation to the population growth will occur in the mid-2026 (Table 1). We recommend enlarge the SAA discharge before 2026, as in this year the SAA will be on its highest working. Projects to enlarge the SAA will have to be done before 2026, because having any change in the setting (change in the supplied population's culture, climate, taxes, etc.) may cause an anticipation of the lack of discharge. So for any change in the SAA or the behaviour of the population, it is necessary to review the forecasting.

Table 2 During the forecasting of time series at SAA at CORSAN in Lajeado

Period 25 years	Variable (x, y)	Identifying the function in LAB Fit Curve Fitting and its perspective formula	red x^2	X^2	FD	r	r^2	Growth period	Yearly growth
Between 2008 and 2032	CRW x Time	(9) $y = \frac{A}{X^2} + B$, where A = -.7134E+15 and B = 0.1832E+09	1	6	6	0,88	0,77	51,59	2,15
Between 2008 and 2032	CTW x Time	(9) $y = \frac{A}{X^2} + B$, where A = -.7598E+15 and B = 0.1943E+09	1	6	6	0,93	0,87	56,88	2,37
Between 2008 and 2032	C x Time	(33) $y = A + \frac{B}{X} + \frac{C}{X^2}$, where A =0.2790E+11, B =-.1115E-15 and C =0.1114E+18	1	5	5	0,99	0,98	118,96	4,96
Between 2008 and 2032	Total population x Time	(140) $y = A.e^{\frac{(X-B)^2}{C}} + D$, where A =0.7907E+06, B =0.2075E+04 C =-.1042E+04 and D =0.5839E+05	1	4	4	0,86	0,75	105,08	4,38
Between 2008 and 2032	C x Total population x Time	(382) $y = A.X_1^3 + B.X_2^2 + C.X_1 + D.X_2$, where A =0.3200E+01, B =-.1278E+05, C =0.1277E+08 and D =0.3092E+01	1	4	4	0,99	0,98	118,69	4,94
Between 2008 and 2032	Supplied population x Total population x Time	(304) $y = A.X_2^{B.X_1}$, where A = 0.1656E+02 and B = 0,3645E-03	1	6	6	0,72	0,52	63,55	2,65

Table Continued

Period 25 years	Variable (x, y)	Identifying the function in LAB Fit Curve Fitting and its perspective formula	red x²	X²	FD	r	r²	Growth period	Yearly growth
Between 2008 and 2032	C per capita x Time	(181) $y = A.\sinh(B.X) + C.X^2$, where A =-.3773E+02, B =-.2022E-02 and C =-.2341E-03	1	5	5	0,56	0,31	19,36	0,81
Between 2008 and 2032	Per capita C x Supplied population x Time	(343) $y = A + (B.X_1) + \left(\frac{C}{(X_2)^2}\right)$, where A =-.1003E+05 B =-.5027E+01 C =0.2868E+12	1	5	5	0,98	0,97	50,12	2,09

Source: The chart was done by the author with Microsoft® Office Excel 2003 software.

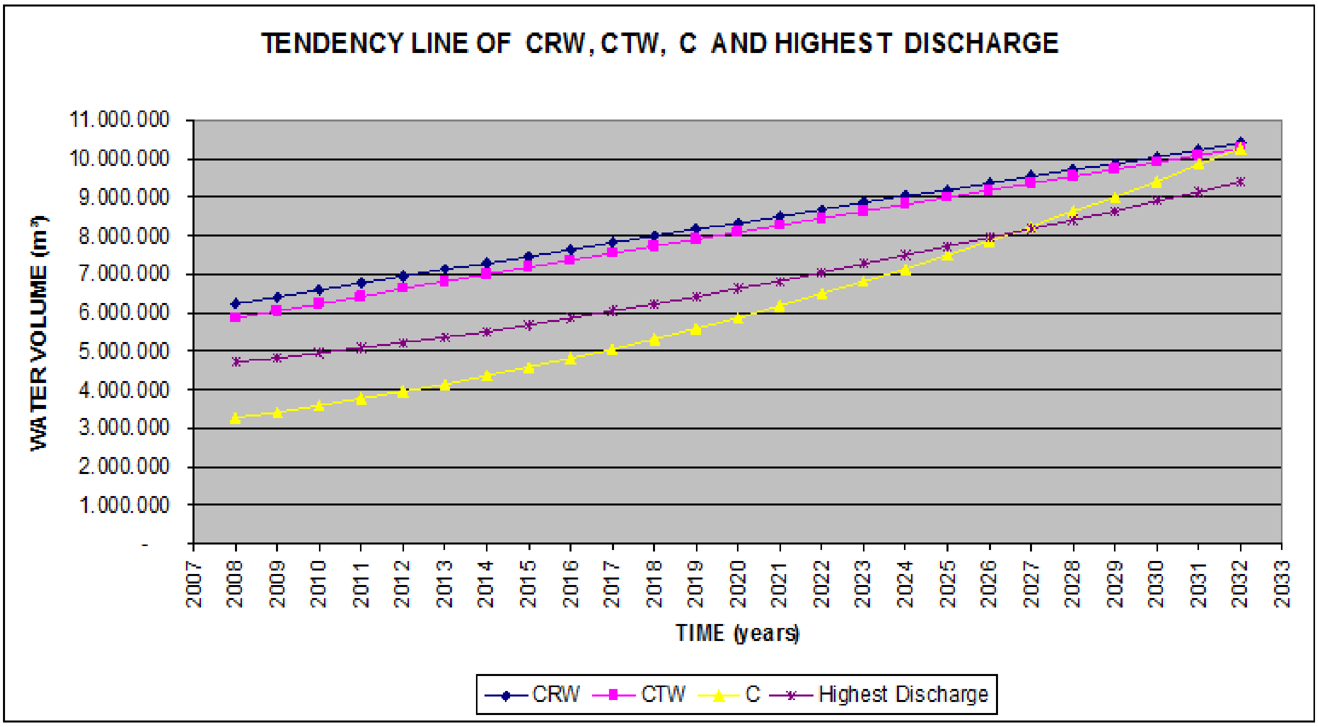

Figure 3 CRW volume tendency Lines x Time, function ($y = \frac{A}{X^2} + B$, where A = -.7134E+15 and B = 0.1832E+09 (m³), CTW volume x Time, function ($y = \frac{A}{X^2} + B$, where A= -.7598E+15 and B = 0.1943E+09), C volume x Total Population x Time, function ($y = A.X_1^3 + B.X_2^2 + C.X_1 + D.X_2$, where A =0.3200E+01, B = -.1278E+05, C = 0.1277E+08 and D = 0.3092E+01, and Highest Discharge according to CORSAN.[15]

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None.

Conflict of interest

The author declares there is no conflict of interest.

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