

Appendix

$$c1 = R * \delta$$

$$c2 = \delta^2$$

$$c3 = -M^2 - \frac{1}{k}$$

$$c4 = R * \delta^3$$

$$c5 = \delta^4$$

$$c6 = \frac{-\delta^2}{k}$$

$$c7 = R * Pr * \delta$$

$$c8 = \delta^2 * N_b$$

$$c9 = \delta^2 * N_t$$

$$c10 = N_b$$

$$c11 = N_t$$

$$c12 = R * Sc * \delta$$

$$c13 = \frac{\delta^2 * N_t}{N_b}$$

$$c14 = \frac{N_t}{N_b}$$

$$c15 = -8 * \pi^3 * \epsilon * (E_1 + E_2)$$

$$c16 = 4 * \pi^2 * \epsilon * E_3$$

$$A = \frac{\lambda(c1 - c4)}{1 - c5}$$

$$B = \frac{c3 - c6}{1 - c5}$$

$$m1 = \frac{-A + \sqrt{A^2 - 4B}}{2}$$

$$m2 = \frac{-A - \sqrt{A^2 - 4B}}{2}$$

$$F1(x, t) = \frac{-m1 \left(\frac{\text{Sinh}[m2 * \eta[x, t]] + \beta * m2 * \text{Cosh}[m2 * \eta[x, t]]}{\text{Sinh}[m1 * \eta[x, t]] + \beta * m1 * \text{Cosh}[m1 * \eta[x, t]]} \right) * \text{Exp}[(m2 - m1) * (x - \lambda * t)]}{m2}$$

$$F2(x, t) = \left(-F1[x, t] * \left(\beta + \frac{1}{m1} \right) * \text{Exp}[m1 * (x + \eta[x, t] - \lambda * t)] - \left(\beta + \frac{1}{m2} \right) * \text{Exp}[m2 * (x + \eta[x, t] - \lambda * t)] \right);$$

$$F3(x, t) = \left(\frac{-F1[x, t] * \text{Exp}[m1 * (x - \lambda * t)]}{(m1)^2} - \frac{1}{(m2)^2} * \text{Exp}[m2 * (x - \lambda * t)] - F2[x, t] * (x - \lambda * t) \right);$$

$$\begin{aligned}
F4(x, t) = & c1 * \left(\frac{\partial_x (F1[x, t])}{(m1)^2} * \text{Exp}[m1 * (x + \eta[x, t] - \lambda * t)] + \frac{(F1[x, t])}{m1} * \text{Exp}[m1 * (x + \eta[x, t] - \lambda * t)] \right) + \\
& \frac{1}{m2} * \text{Exp}[m2 * (x + \eta[x, t] - \lambda * t)] + \partial_x (F2[x, t]) * (x + \eta[x, t] - \lambda * t) + F2[x, t] + \partial_x (F3[x, t]) * \\
& (F1[x, t] * \text{Exp}[m1 * (x + \eta[x, t] - \lambda * t)] + \text{Exp}[m2 * (x + \eta[x, t] - \lambda * t)]) - \\
& \left(\frac{F1[x, t]}{m1} * \text{Exp}[m1 * (x + \eta[x, t] - \lambda * t)] + \frac{1}{m2} * \text{Exp}[m2 * (x + \eta[x, t] - \lambda * t)] + F2[x, t] \right) * \\
& \left(\frac{\partial_x (F1[x, t])}{m1} + F1[x, t] \right) * \text{Exp}[m1 * (x + \eta[x, t] - \lambda * t)] + \text{Exp}[m2 * (x + \eta[x, t] - \lambda * t)] + \partial_x (F2[x, t]);
\end{aligned}$$

$$\begin{aligned}
F5(x, t) = & c2 \left(F1[x, t] * m1 + 2 * \partial_x (F1[x, t]) + \frac{\partial_x \partial_x (F1[x, t])}{m1} \right) * \text{Exp}[m1 * (x + \eta[x, t] - \lambda * t)] \\
& + c2 * m2 * \text{Exp}[m2 * (x + \eta[x, t] - \lambda * t)] + c2 * \partial_x \partial_x (F2[x, t]) + m1 * F1[x, t] * \text{Exp}[m1 * (x + \eta[x, t] - \lambda * t)] \\
& + m2 * \text{Exp}[m2 * (x + \eta[x, t] - \lambda * t)] - c1 \left(\frac{\partial_t (F1[x, t])}{m1} - \lambda * t \right) * \text{Exp}[m1 * (x + \eta[x, t] - \lambda * t)] \\
& + c1 * \lambda * \text{Exp}[m2 * (x + \eta[x, t] - \lambda * t)] - c1 * \partial_t (F2[x, t]) + \frac{c3 * F1[x, t]}{m1} * \text{Exp}[m1 * (x + \eta[x, t] - \lambda * t)] \\
& + \frac{c3}{m2} * \text{Exp}[m2 * (x + \eta[x, t] - \lambda * t)] + c3 * F2[x, t];
\end{aligned}$$

$$F6(x, t) = -(c15 * \text{Cos}[2 * \pi(x - t)] + c16 * \text{Sin}[2 * \pi(x - t)]);$$

$$F7(x, t) = \left(\frac{-F5[x, t] + \sqrt{(F5[x, t])^2 - 4 * F4[x, t] * F6[x, t]}}{2 * F4[x, t]} \right);$$

$$F8(x, t) = \frac{(F7[x, t] * F1[x, t])}{(m1)^2} * \text{Exp}[m1 * (x - \lambda * t)];$$

$$F9(x, t) = \frac{(F7[x, t])}{(m2)^2} * \text{Exp}[m2 * (x - \lambda * t)];$$

$$F10(x, t) = F7[x, t] * F2[x, t];$$

$$F11(x, t) = F7[x, t] * F2[x, t] * (x - \lambda * t) + F7[x, t] * F3[x, t];$$

$$M1(x, t) = -\frac{1}{4} (\eta[x, t])^{-4} * (\partial_x (\eta[x, t]))^2 * (c8 + c9);$$

$$\begin{aligned}
M2(x, t) = & -\frac{1}{2} * c7 * (\eta[x, t])^{-2} * \partial_t (\eta[x, t]) + \frac{-1}{2} * c7 * (\eta[x, t])^{-2} * \\
& \partial_x (\eta[x, t]) * F10[x, t] - c7 * \partial_x (F10[x, t]) * \frac{1}{2 * \eta[x, t]} - \\
& c2 * \left(-\frac{1}{2} * (\eta[x, t])^{-2} * \partial_x \partial_x (\eta[x, t]) + (\partial_x (\eta[x, t]))^{-3} * \eta[x, t] \right);
\end{aligned}$$

$$M3(x, t) = c7 * m1 * F8[x, t] * -\frac{1}{2} * (\eta[x, t])^{-2} * \partial_x (\eta[x, t]);$$

$$M4(x, t) = c7 * m2 * F9[x, t] * -\frac{1}{2} * (\eta[x, t])^{-2} * \partial_x (\eta[x, t]);$$

$$M5(x, t) = -c7 * (\partial_x (F8[x, t])) * \frac{1}{2 * \eta[x, t]};$$

$$M6(x, t) = -c7 * (\partial_x (F9[x, t])) * \frac{1}{2 * \eta[x, t]};$$

$$M7(x, t) = -c7 * (\partial_x (F11[x, t])) * \frac{1}{2 * \eta[x, t]} - c10 * \frac{1}{4 * (\eta[x, t])^2} - c11 * \frac{1}{4 * (\eta[x, t])^2};$$

$$M8(x, t) = \frac{M1[x, t]}{12};$$

$$M9(x, t) = \frac{M2[x, t]}{6};$$

$$M10(x, t) = \frac{M7[x, t]}{2};$$

$$M11(x, t) = \frac{M3[x, t]}{(m1)^2};$$

$$M12(x, t) = \frac{M4[x, t]}{(m2)^2};$$

$$M13(x, t) = \frac{M5[x, t]}{(m1)^2} - \frac{2M3[x, t]}{(m1)^3};$$

$$M14(x, t) = \frac{M6[x, t]}{(m2)^2} - \frac{2M4[x, t]}{(m2)^3};$$

$$k1(x, t) = \frac{M1}{12} * \eta^4 - \frac{M2}{6} * \eta^3 + \frac{M3}{m1} * (-\eta * \text{Exp}[-\eta * m1]) - \frac{1}{(m1)^2} * \text{Exp}[-\eta * m1] -$$

$$\frac{M3}{(m1)^3} * \text{Exp}[-\eta * m1] + \frac{M4}{m2} * (-\eta * \text{Exp}[-\eta * m2]) - \frac{1}{(m2)^2} * \text{Exp}[-\eta * m2] - \frac{M4}{(m2)^3} * \text{Exp}[-\eta * m2]$$

$$+ \frac{M5}{(m1)^2} * \text{Exp}[-\eta * m1] + \frac{M6}{(m2)^2} * \text{Exp}[-\eta * m2] + \frac{M7}{2} * \eta^2;$$

$$k2(x, t) = \frac{M1}{12} * \eta^4 + \frac{M2}{6} * \eta^3 + \frac{M3}{m1} * \left(\frac{\eta}{m1} * \text{Exp}[\eta * m1] - \frac{1}{(m1)^2} * \text{Exp}[\eta * m1] \right)$$

$$- \frac{M3}{(m1)^3} * \text{Exp}[\eta * m1] + \frac{M4}{m2} * \left(\frac{\eta}{m2} * \text{Exp}[\eta * m2] - \frac{1}{(m2)^2} * \text{Exp}[\eta * m2] \right) -$$

$$\frac{M4}{(m2)^3} * \text{Exp}[\eta * m2] + \frac{M5}{(m1)^2} * \text{Exp}[\eta * m1] + \frac{M6}{(m2)^2} * \text{Exp}[\eta * m2] + \frac{M7}{2} * \eta^2;$$

$$F12(x, t) = \frac{k1[x, t] + T2[x, t]}{\eta[x, t]};$$

$$F13(x, t) = \frac{-(k1[x, t] + k2[x, t])}{2};$$

$$\begin{aligned} \text{M15}(x, t) = & \left(c12 * \left(-\frac{1}{2} * (\eta[x, t])^{-2} * \partial_t (\eta[x, t]) \right) + c12 * \text{F10}[x, t] * \frac{-1}{2} * (\eta[x, t])^{-2} * \partial_x (\eta[x, t]) \right) \\ & - c12 * \partial_x (\text{F10}[x, t]) * \frac{1}{2 * \eta[x, t]} - c2 * \left(-\frac{1}{2} * (\eta[x, t])^{-2} * \partial_x \partial_x (\eta[x, t]) + (\eta[x, t])^{-3} * \partial_x (\eta[x, t]) \right) \\ & - c13 * \left(-\frac{1}{2} * (\eta[x, t])^{-2} * \partial_x \partial_x (\eta[x, t]) + (\partial_x (\eta[x, t]))^{-3} * \eta[x, t] \right); \end{aligned}$$

$$\text{M16}(x, t) = c12 * m1 * \text{F8}[x, t] * -\frac{1}{2} * (\eta[x, t])^{-2} * \partial_x (\eta[x, t]);$$

$$\text{M17}(x, t) = c12 * m2 * \text{F9}[x, t] * -\frac{1}{2} * (\eta[x, t])^{-2} * \partial_x (\eta[x, t]);$$

$$\text{M18}(x, t) = -c12 * (\partial_x (\text{F8}[x, t])) * \frac{1}{2 * \eta[x, t]};$$

$$\text{M19}(x, t) = -c12 * (\partial_x (\text{F9}[x, t])) * \frac{1}{2 * \eta[x, t]};$$

$$\text{M20}(x, t) = -c12 * (\partial_x (\text{F11}[x, t])) * \frac{1}{2 * \eta[x, t]};$$

$$\text{M21}(x, t) = \frac{\text{M15}[x, t]}{6};$$

$$\text{M22}(x, t) = \frac{\text{M20}[x, t]}{2};$$

$$\text{M23}(x, t) = \frac{\text{M16}[x, t]}{(\text{m1})^2};$$

$$\text{M24}(x, t) = \frac{\text{M17}[x, t]}{(\text{m2})^2};$$

$$\text{M25}(x, t) = \left(\frac{\text{M18}[x, t]}{(\text{m1})^2} - \frac{2\text{M16}[x, t]}{(\text{m1})^3} \right);$$

$$\text{M26}(x, t) = \left(\frac{\text{M19}[x, t]}{(\text{m2})^2} - \frac{2\text{M17}[x, t]}{(\text{m2})^3} \right);$$

$$\begin{aligned} k3(x, t) = & \frac{\text{M15}}{6} * -\eta^3 + \frac{\text{M16}}{\text{m1}} \left(\frac{-\eta}{\text{m1}} * \text{Exp}[-\eta * \text{m1}] - \frac{1}{(\text{m1})^2} * \text{Exp}[-\eta * \text{m1}] \right) - \\ & \frac{\text{M16}}{(\text{m1})^3} * \text{Exp}[-\eta * \text{m1}] + \frac{\text{M17}}{\text{m2}} \left(\frac{-\eta}{\text{m2}} * \text{Exp}[-\eta * \text{m2}] - \frac{1}{(\text{m2})^2} * \text{Exp}[-\eta * \text{m2}] \right) - \\ & \frac{\text{M17}}{(\text{m2})^3} * \text{Exp}[-\eta * \text{m2}] + \frac{\text{M18}}{(\text{m1})^2} * \text{Exp}[-\eta * \text{m1}] + \frac{\text{M19}}{(\text{m2})^2} * \text{Exp}[-\eta * \text{m2}] + \frac{\text{M20}}{2} * \eta^2; \end{aligned}$$

$$k4(x,t) = \frac{M15}{6} * \eta^3 + \frac{M16}{m1} \left(\frac{\eta}{m1} * \text{Exp}[\eta * m1] - \frac{1}{(m1)^2} * \text{Exp}[\eta * m1] \right) -$$

$$\frac{M16}{(m1)^3} * \text{Exp}[\eta * m1] + \frac{M17}{m2} \left(\frac{\eta}{m2} * \text{Exp}[\eta * m2] - \frac{1}{(m2)^2} * \text{Exp}[\eta * m2] \right) -$$

$$\frac{M17}{(m2)^3} * \text{Exp}[\eta * m2] + \frac{M18}{(m1)^2} * \text{Exp}[\eta * m1] + \frac{M19}{(m2)^2} * \text{Exp}[\eta * m2] + \frac{M20}{2} * \eta^2;$$

$$F14(x,t) = \frac{k3[x,t] + T4[x,t]}{\eta[x,t]};$$

$$F15(x,t) = \frac{-(k3[x,t] + k4[x,t])}{2};$$

$$s1(x,t) = \left(\frac{1}{2} c8 + c9 \right) * \left(\eta[x,t]^{-2} * \partial_x(\eta[x,t]) \right) * \partial_x(M8[x,t]);$$

$$s2(x,t) = c7 * \partial_t(M8[x,t]) + c7 * F10[x,t] * \partial_x(M8[x,t]) + \left(\frac{1}{2} c8 + c9 \right) * \left(\eta[x,t]^{-2} * \partial_x(\eta[x,t]) \right) *$$

$$\partial_x(M9[x,t]) - 4 * c7 * \partial_x(F10[x,t]) * M8[x,t] - c2 * \partial_x \partial_x(M8[x,t]) + \left(\frac{1}{2} c8 \right) * \left(\eta[x,t]^{-2} * \partial_x(\eta[x,t]) \right) * \partial_x(M21[x,t]);$$

$$s3(x,t) = c7 * \partial_t(M9[x,t]) + c7 * F10[x,t] * \partial_x(M9[x,t]) + \left(\frac{1}{2} c8 + c9 \right) * \left(\eta[x,t]^{-2} * \partial_x(\eta[x,t]) \right) *$$

$$\partial_x(M10[x,t]) - 3 * c7 * \partial_x(F10[x,t]) * M9[x,t] - c2 * \partial_x \partial_x(M9[x,t]) + \left(\frac{1}{2} c8 \right) * \left(\eta[x,t]^{-2} * \partial_x(\eta[x,t]) \right)$$

$$* \partial_x(M22[x,t]) - 4 \left(c7 * \partial_x(F11[x,t]) + \frac{1}{2\eta[x,t]}(c10 + 2c11) \right) * M8[x,t];$$

$$s4(x,t) = c7 * \partial_t(M10[x,t]) + c7 * F10[x,t] * \partial_x(M10[x,t]) + \left(\frac{1}{2} c8 + c9 \right) * \left(\eta[x,t]^{-2} * \partial_x(\eta[x,t]) \right) *$$

$$\partial_x(F12[x,t]) - 2 * c7 * \partial_x(F10[x,t]) * M10[x,t] - 3 \left(c7 * \partial_x(F11[x,t]) + \frac{1}{2\eta[x,t]}(c10 + 2c11) \right) * M9[x,t] -$$

$$c2 * \partial_x \partial_x(M10[x,t]) + \left(\frac{1}{2} c8 \right) * \left(\eta[x,t]^{-2} * \partial_x(\eta[x,t]) \right) * \partial_x(M14[x,t]) - \frac{3}{2\eta[x,t]} * c10 * M21[x,t];$$

$$s5(x,t) = c7 * \partial_t(F12[x,t]) + c7 * F10[x,t] * \partial_x(F12[x,t]) + \left(\frac{1}{2} c8 + c9 \right) * \left(\eta[x,t]^{-2} * \partial_x(\eta[x,t]) \right)$$

$$* \partial_x(F13[x,t]) - c7 * \partial_x(F10[x,t]) * F12[x,t] - 2 \left(c7 * \partial_x(F11[x,t]) + \frac{1}{2\eta[x,t]}(c10 + 2c11) \right)$$

$$* M10[x,t] - c2 * \partial_x \partial_x(F12[x,t]) + \left(\frac{1}{2} c8 \right) * \left(\eta[x,t]^{-2} * \partial_x(\eta[x,t]) \right) * \partial_x(F15[x,t]) - \frac{1}{\eta[x,t]} * c10 * M22[x,t];$$

$$s6(x,t) = c7 * \partial_t(F13[x,t]) + c7 * F10[x,t] * \partial_x(F13[x,t]) - \left(c7 * \partial_x(F11[x,t]) + \frac{1}{2\eta[x,t]}(c10 + 2c11) \right)$$

$$* F12[x,t] - c2 * \partial_x \partial_x(F13[x,t]) - \frac{1}{\eta[x,t]} * c10 * F14[x,t];$$

$$s7(x,t) = c7 * m1 * F8[x,t] * \partial_x(M8[x,t]);$$

$$s8(x,t) = c7 * m2 * F9[x,t] * \partial_x(M8[x,t]);$$

$$\begin{aligned}
s9(x, t) &= c7*m1*F8[x, t]*\partial_x(M9[x, t]) - 4c7*\partial_x(F8[x, t])*M8[x, t]; \\
s10(x, t) &= c7*m2*F9[x, t]*\partial_x(M9[x, t]) - 4c7*\partial_x(F9[x, t])*M8[x, t]; \\
s11(x, t) &= c7*m1*\partial_x(M10[x, t])*F8[x, t] + \left(\frac{1}{2}c8 + c9\right)*(\eta[x, t])^{-2}* \\
&\partial_x(\eta[x, t])*\partial_x(M11[x, t]) - 3*c7*\partial_x(F8[x, t])*M9[x, t] - c7*m1* \\
&M11[x, t]*\partial_x(F10[x, t]) + \left(\frac{1}{2}c8\right)*(\eta[x, t])^{-2}*\partial_x(\eta[x, t])*\partial_x(M23[x, t]); \\
s12(x, t) &= c7*m2*\partial_x(M10[x, t])*F9[x, t] + \left(\frac{1}{2}c8 + c9\right)*(\eta[x, t])^{-2}*\partial_x(\eta[x, t])*\partial_x(M12[x, t]) - \\
&3*c7*\partial_x(F9[x, t])*M9[x, t] - c7*m2*M12[x, t]*\partial_x(F10[x, t]) + \left(\frac{1}{2}c8\right)*(\eta[x, t])^{-2}*\partial_x(\eta[x, t])*\partial_x(M24[x, t]); \\
s13(x, t) &= c7*\partial_t(M11[x, t]) + c7*m1*\partial_x(M12[x, t])*F8[x, t] + c7*F10[x, t]*\partial_x(M11[x, t]) + \\
&\left(\frac{1}{2}c8 + c9\right)*(\eta[x, t])^{-2}*\partial_x(\eta[x, t])*\partial_x(M13[x, t]) - 2*c7*\partial_x(F8[x, t])*M10[x, t] - c7*M11[x, t]* \\
&\partial_x(F10[x, t]) - c7*m1*\partial_x(F10[x, t])*M13[x, t] - \left(c7*\partial_x(F11[x, t]) + \frac{1}{2\eta[x, t]}(c10 + 2c11)\right)*m1*M11[x, t] - \\
&c2*\partial_x\partial_x(M11[x, t]) + \left(\frac{1}{2}c8\right)*(\eta[x, t])^{-2}*\partial_x(\eta[x, t])*\partial_x(M25[x, t]) - c10*\frac{1}{2\eta[x, t]}*m1*M23[x, t]; \\
s14(x, t) &= c7*\partial_t(M12[x, t]) + c7*m2*\partial_x(M12[x, t])*F9[x, t] + c7*F10[x, t]*\partial_x(M12[x, t]) + \left(\frac{1}{2}c8 + c9\right)* \\
&(\eta[x, t])^{-2}*\partial_x(\eta[x, t])*\partial_x(M14[x, t]) - 2*c7*\partial_x(F9[x, t])*M10[x, t] - c7*M12[x, t]*\partial_x(F10[x, t]) \\
&- c7*m2*\partial_x(F10[x, t])*M14[x, t] - \left(c7*\partial_x(F11[x, t]) + \frac{1}{2\eta[x, t]}(c10 + 2c11)\right)*m2*M12[x, t] \\
&- c2*\partial_x\partial_x(M12[x, t]) + \left(\frac{1}{2}c8\right)*(\eta[x, t])^{-2}*\partial_x(\eta[x, t])*\partial_x(M26[x, t]) - c10*\frac{1}{2\eta[x, t]}*m2*M24[x, t]; \\
s15(x, t) &= c7*m1*\partial_x(M11[x, t])*F8[x, t] - c7*m1*\partial_x(F8[x, t])*M11[x, t]; \\
s16(x, t) &= c7*m2*\partial_x(M12[x, t])*F9[x, t] - c7*m2*\partial_x(F9[x, t])*M12[x, t]; \\
s17(x, t) &= c7*m1*\partial_x(M12[x, t])*F8[x, t] + c7*m2*F9[x, t]*\partial_x(M11[x, t]) - c7*m2*M12[x, t]*\partial_x(F8[x, t]) - c7*m1*M11[x, t]*\partial_x(F9[x, t]); \\
s18(x, t) &= c7*\partial_t(M13[x, t]) + c7*m1*\partial_x(M13[x, t])*F8[x, t] + c7*F10[x, t]*\partial_x(M13[x, t]) - \\
&c7*\partial_x(F8[x, t])*F12[x, t] - \left(c7*\partial_x(F11[x, t]) + \frac{1}{2\eta[x, t]}(c10 + 2c11)\right)*M11[x, t] - (c7*\partial_x(F11[x, t]) + \\
&\frac{1}{2\eta[x, t]}(c10 + 2c11))*m1*M13[x, t] - c2*\partial_x\partial_x(M13[x, t]) - c10*\frac{1}{2\eta[x, t]}*M23[x, t] - c10*\frac{1}{2\eta[x, t]}*m1*M25[x, t]; \\
s19(x, t) &= c7*\partial_t(M14[x, t]) + c7*m2*\partial_x(M13[x, t])*F9[x, t] + c7*F10[x, t]*\partial_x(M14[x, t]) - \\
&c7*\partial_x(F9[x, t])*F12[x, t] - \left(c7*\partial_x(F11[x, t]) + \frac{1}{2\eta[x, t]}(c10 + 2c11)\right)*M12[x, t] - (c7*\partial_x(F11[x, t]) + \\
&\frac{1}{2\eta[x, t]}(c10 + 2c11))*m2*M14[x, t] - c2*\partial_x\partial_x(M14[x, t]) - c10*\frac{1}{2\eta[x, t]}*M24[x, t] - c10*\frac{1}{2\eta[x, t]}*m2*M26[x, t];
\end{aligned}$$

$$s20(x, t) = c7 * m1 * \partial_x (M13[x, t]) * F8[x, t] - c7 * \partial_x (F8[x, t]) * M11[x, t] - c7 * m1 * \partial_x (F8[x, t]) * M13[x, t];$$

$$s21(x, t) = c7 * m2 * \partial_x (M14[x, t]) * F9[x, t] - c7 * \partial_x (F9[x, t]) * M12[x, t] - c7 * m2 * \partial_x (F9[x, t]) * M14[x, t];$$

$$s22(x, t) = c7 * m1 * \partial_x (M14[x, t]) * F8[x, t] + c7 * m2 * \partial_x (M13[x, t]) * F9[x, t] - c7 * \partial_x (F8[x, t]) * M12[x, t] - c7 * m2 * \partial_x (F8[x, t]) * M14[x, t] - c7 * \partial_x (F9[x, t]) * M11[x, t] - c7 * m1 * \partial_x (F9[x, t]) * M13[x, t];$$

$$k5(x, t) = \frac{s1}{42}(-\eta^7) + \frac{s2}{30}(\eta^6) + \frac{s3}{20}(-\eta^5) + \frac{s4}{12}(\eta^4) + \frac{s5}{6}(-\eta^3) + \frac{s6}{2}(\eta^2) + \frac{s7}{(m1)^6} * \text{Exp}[-\eta * m1] * (120 - 96 * m1 * (-\eta) + 36 * (m1)^2 * \eta^2 - 81 * (m1)^3 * (-\eta^3) + (m1)^4 * \eta^4) + \frac{s8}{(m2)^6} * \text{Exp}[-\eta * m2] * (120 - 96 * m2 * (-\eta) + 36 * (m2)^2 * \eta^2 - 81 * (m2)^3 * (-\eta^3) + (m2)^4 * \eta^4) + \frac{s9}{(m1)^5} * \text{Exp}[-\eta * m1] * (-24 + 81 * m1 * (-\eta) - 6 * (m1)^2 * \eta^2 + (m1)^3 * (-\eta^3)) + \frac{s10}{(m2)^5} * \text{Exp}[-\eta * m2] * (-24 + 81 * m2 * (-\eta) - 6 * (m2)^2 * \eta^2 + (m2)^3 * (-\eta^3)) + \frac{s11}{(m1)^4} * \text{Exp}[-\eta * m1] * (6 - 4 * m1 * (-\eta) + (m1)^2 * \eta^2) + \frac{s12}{(m2)^4} * \text{Exp}[-\eta * m2] * (6 - 4 * m2 * (-\eta) + (m2)^2 * \eta^2) + \frac{s13}{(m1)^3} * \text{Exp}[-\eta * m1] * (-2 + m1 * (-\eta)) + \frac{s14}{(m2)^3} * \text{Exp}[-\eta * m2] * (-2 + m2 * (-\eta)) + \frac{s15}{4(m1)^3} * \text{Exp}[-2 * m1 * \eta] * (-1 + m1 * (-\eta)) + \frac{s16}{4(m2)^3} * \text{Exp}[-2 * m2 * \eta] * (-1 + m2 * (-\eta)) + \frac{s17}{(m1 + m2)^3} * \text{Exp}[-\eta * (m1 + m2)] * (-2 + m1 * (-\eta) + (m2) * (-\eta)) + \frac{s18}{(m1)^2} * \text{Exp}[-\eta * m1] + \frac{s19}{(m2)^2} * \text{Exp}[-\eta * m2] + \frac{s20}{4(m1)^2} * \text{Exp}[-2 * \eta * m1] + \frac{s21}{4(m2)^2} * \text{Exp}[-2 * \eta * m2] + \frac{s22}{(m1 + m2)^2} * \text{Exp}[-\eta * (m1 + m2)];$$

$$k6(x, t) = \frac{s1}{42}(\eta^7) + \frac{s2}{30}(\eta^6) + \frac{s3}{20}(\eta^5) + \frac{s4}{12}(\eta^4) + \frac{s5}{6}(\eta^3) + \frac{s6}{2}(\eta^2) + \frac{s7}{(m1)^6} * \text{Exp}[\eta * m1] * (120 - 96 * m1 * (\eta) + 36 * (m1)^2 * \eta^2 - 81 * (m1)^3 * (\eta^3) + (m1)^4 * \eta^4) + \frac{s8}{(m2)^6} * \text{Exp}[\eta * m2] * (120 - 96 * m2 * (\eta) + 36 * (m2)^2 * \eta^2 - 81 * (m2)^3 * (\eta^3) + (m2)^4 * \eta^4) + \frac{s9}{(m1)^5} * \text{Exp}[\eta * m1] * (-24 + 81 * m1 * (\eta) - 6 * (m1)^2 * \eta^2 + (m1)^3 * (\eta^3)) + \frac{s10}{(m2)^5} * \text{Exp}[\eta * m2] * (-24 + 81 * m2 * (\eta) - 6 * (m2)^2 * \eta^2 + (m2)^3 * (\eta^3)) + \frac{s11}{(m1)^4} * \text{Exp}[\eta * m1] * (6 - 4 * m1 * (\eta) + (m1)^2 * \eta^2) + \frac{s12}{(m2)^4} * \text{Exp}[\eta * m2] * (6 - 4 * m2 * (\eta) + (m2)^2 * \eta^2) + \frac{s13}{(m1)^3} * \text{Exp}[\eta * m1] * (-2 + m1 * (\eta)) + \frac{s14}{(m2)^3} * \text{Exp}[\eta * m2] * (-2 + m2 * (\eta)) + \frac{s15}{4(m1)^3} * \text{Exp}[2 * m1 * \eta] * (-1 + m1 * (\eta)) + \frac{s16}{4(m2)^3} * \text{Exp}[2 * m2 * \eta] * (-1 + m2 * (\eta)) + \frac{s17}{(m1 + m2)^3} * \text{Exp}[\eta * (m1 + m2)] * (-2 + m1 * (\eta) + (m2) * (\eta)) + \frac{s18}{(m1)^2} * \text{Exp}[\eta * m1] + \frac{s19}{(m2)^2} * \text{Exp}[\eta * m2] + \frac{s20}{4(m1)^2} * \text{Exp}[2 * \eta * m1] + \frac{s21}{4(m2)^2} * \text{Exp}[2 * \eta * m2] + \frac{s22}{(m1 + m2)^2} * \text{Exp}[\eta * (m1 + m2)];$$

$$T5(x, t) = \frac{k5[x, t] + T6[x, t]}{\eta[x, t]};$$

$$T6(x, t) = \frac{-(k5[x, t] + k6[x, t])}{2};$$

$$s23(x, t) = \frac{s1[x, t]}{42};$$

$$s24(x, t) = \frac{s2[x, t]}{30};$$

$$s25(x, t) = \frac{s3[x, t]}{20};$$

$$s26(x, t) = \frac{s4[x, t]}{12} + M8[x, t];$$

$$s27(x, t) = \frac{s5[x, t]}{6} + M9[x, t];$$

$$s28(x, t) = \frac{s6[x, t]}{2} + M10[x, t];$$

$$s29(x, t) = \frac{1}{2\eta[x, t]} + T5[x, t] + F12[x, t];$$

$$s30(x, t) = \frac{120*s7[x, t]}{(ml)^6} + \frac{-24*s9[x, t]}{(ml)^5} + \frac{6*s11[x, t]}{(ml)^4} + \frac{-2*s13[x, t]}{(ml)^3} + \frac{s18[x, t]}{(ml)^2} + M13[x, t];$$

$$s31(x, t) = \frac{120*s8[x, t]}{(m2)^6} + \frac{-24*s10[x, t]}{(m2)^5} + \frac{6*s12[x, t]}{(m2)^4} + \frac{-2*s14[x, t]}{(m2)^3} + \frac{s19[x, t]}{(m2)^2} + M14[x, t];$$

$$s32(x, t) = \frac{-96*ml*s7[x, t]}{(ml)^6} + \frac{81*ml*s9[x, t]}{(ml)^5} + \frac{-4*ml*s11[x, t]}{(ml)^4} + \frac{ml*s13[x, t]}{(ml)^3} + M11[x, t];$$

$$s33(x, t) = \frac{-96*m2*s8[x, t]}{(m2)^6} + \frac{81*m2*s10[x, t]}{(m2)^5} + \frac{-4*m2*s12[x, t]}{(m2)^4} + \frac{m2*s14[x, t]}{(m2)^3} + M12[x, t];$$

$$s34(x, t) = \frac{36*(ml)^2*s7[x, t]}{(ml)^6} + \frac{-6*(ml)^2*s9[x, t]}{(ml)^5} + \frac{(ml)^2*s11[x, t]}{(ml)^4};$$

$$s35(x, t) = \frac{36*(m2)^2*s8[x, t]}{(m2)^6} + \frac{-6*(m2)^2*s10[x, t]}{(m2)^5} + \frac{(m2)^2*s12[x, t]}{(m2)^4};$$

$$s36(x, t) = \frac{-81*(ml)^3*s7[x, t]}{(ml)^6} + \frac{(ml)^3*s9[x, t]}{(ml)^5};$$

$$s37(x, t) = \frac{-81*(m2)^3*s8[x, t]}{(m2)^6} + \frac{(m2)^3*s10[x, t]}{(m2)^5};$$

$$s38(x, t) = \frac{(ml)^4*s7[x, t]}{(ml)^6};$$

$$s39(x, t) = \frac{(m2)^4*s8[x, t]}{(m2)^6};$$

$$s40(x, t) = \frac{-s15[x, t]}{4(ml)^3} + \frac{s20[x, t]}{4(ml)^2};$$

$$s41(x, t) = \frac{-s16[x, t]}{4(m2)^3} + \frac{s21[x, t]}{4(m2)^2};$$

$$s42(x, t) = \frac{ml*s15[x, t]}{4(ml)^3};$$

$$s43(x, t) = \frac{m2*s16[x, t]}{4(m2)^3};$$

$$s44(x, t) = \frac{-2*s17[x, t]}{(m1 + m2)^3} + \frac{s22[x, t]}{(m1 + m2)^2};$$

$$s45(x, t) = \frac{(m1 + m2)*s17[x, t]}{(m1 + m2)^3};$$

$$s46(x, t) = F13[x, t] + T6[x, t] + \frac{1}{2}$$

$$R1(x, t) = -c13*\partial_x\partial_x(M8[x, t]);$$

$$R2(x, t) = c12*\partial_t(M21[x, t]) + c12*F10[x, t]*\partial_x(M21[x, t]) - c12*M21[x, t]*\partial_x(F10[x, t]) - c12*\partial_{x,x}(M21[x, t]) - c13*\partial_{x,x}(M9[x, t]);$$

$$R3(x, t) = c12*\partial_t(M22[x, t]) + c12*F10[x, t]*\partial_x(M22[x, t]) - 2*c12*M22[x, t]*\partial_x(F10[x, t]) - 3*c12*\partial_x(F11[x, t]) - c12*\partial_{x,x}(M22[x, t]) - c13*\partial_{x,x}(M10[x, t]) - 12*c14*M8[x, t];$$

$$R4(x, t) = c12*\partial_t(F14[x, t]) + c12*F10[x, t]*\partial_x(F14[x, t]) - c12*F14[x, t]*\partial_x(F10[x, t]) - 2*c12*M22[x, t]*\partial_x(F11[x, t]) - c12*\partial_{x,x}(F14[x, t]) - c13*\partial_{x,x}(F12[x, t]) - 6*c14*M9[x, t];$$

$$R5(x, t) = c12*\partial_t(F15[x, t]) + c12*F10[x, t]*\partial_x(F15[x, t]) - c12*F14[x, t]*\partial_x(F11[x, t]) - c12*\partial_{x,x}(F15[x, t]) - c13*\partial_{x,x}(F13[x, t]) - 2*c14*M10[x, t];$$

$$R6(x, t) = c12*m1*F8[x, t]*\partial_x(M21[x, t]);$$

$$R7(x, t) = c12*m2*F9[x, t]*\partial_x(M21[x, t]);$$

$$R8(x, t) = c12*m1*F8[x, t]*\partial_x(M22[x, t]) - 3c12*M21[x, t]*\partial_x(F8[x, t]) - c12*m1*M23[x, t]*\partial_x(F10[x, t]);$$

$$R9(x, t) = c12*m2*F9[x, t]*\partial_x(M22[x, t]) - 3c12*M21[x, t]*\partial_x(F9[x, t]) - c12*m2*M24[x, t]*\partial_x(F10[x, t]);$$

$$R10(x, t) = c12*\partial_t(M23[x, t]) + c12*m1*F8[x, t]*\partial_x(F14[x, t]) + c12*F10[x, t]*\partial_x(M23[x, t]) - 2c12*\partial_x(F8[x, t])*M22[x, t] - c12*\partial_x(F10[x, t])*(M23[x, t] + m1*M25[x, t]) - c12*m1*M23[x, t]*\partial_x(F11[x, t]) - c12*\partial_{x,x}(M23[x, t]) - c13*\partial_{x,x}(M11[x, t]) - c14*(m1)^2*M11[x, t];$$

$$R11(x, t) = c12*\partial_t(M24[x, t]) + c12*m2*F9[x, t]*\partial_x(F14[x, t]) + c12*F10[x, t]*\partial_x(M24[x, t]) - 2c12*\partial_x(F9[x, t])*M22[x, t] - c12*\partial_x(F10[x, t])*(M24[x, t] + m2*M26[x, t]) - c12*m2*M24[x, t]*\partial_x(F11[x, t]) - c12*\partial_{x,x}(M24[x, t]) - c13*\partial_{x,x}(M12[x, t]) - c14*(m2)^2*M12[x, t];$$

$$R12(x, t) = c12*m1*F8[x, t]\partial_x(M23[x, t]) - c12*M23[x, t]*m1*\partial_x(F8[x, t]);$$

$$R13(x, t) = c12*m2*F9[x, t]\partial_x(M24[x, t]) - c12*M24[x, t]*m2*\partial_x(F9[x, t]);$$

$$R14(x, t) = c12*m1*F8[x, t]\partial_x(M24[x, t]) + c12*F9[x, t]*m2*\partial_x(M23[x, t]) - c12*M24[x, t]*m2*\partial_x(F8[x, t]) - c12*M23[x, t]*m1*\partial_x(F9[x, t]);$$

$$\begin{aligned} R15(x, t) = & c12*\partial_t (M25[x, t]) + c12*m1*F8[x, t]*\partial_x (F15[x, t]) + c12*F10[x, t]*\partial_x (M25[x, t]) - \\ & c12*\partial_x (F8[x, t])*F14[x, t] - c12*\partial_x (F11[x, t])*(M23[x, t] + m1*M25[x, t]) \\ & - c12*\partial_{x,x} (M25[x, t]) - c13*\partial_{x,x} (M13[x, t]) - 2*c14*m1*M11[x, t] - c14*(m1)^2*M13[x, t]; \end{aligned}$$

$$\begin{aligned} R16(x, t) = & c12*\partial_t (M26[x, t]) + c12*m2*F9[x, t]*\partial_x (F15[x, t]) + c12*F10[x, t]*\partial_x (M26[x, t]) - \\ & c12*\partial_x (F9[x, t])*F14[x, t] - c12*\partial_x (F11[x, t])*(M24[x, t] + m2*M26[x, t]) \\ & - c12*\partial_{x,x} (M26[x, t]) - c13*\partial_{x,x} (M14[x, t]) - 2*c14*m2*M12[x, t] - c14*(m2)^2*M14[x, t]; \end{aligned}$$

$$R17(x, t) = c12*m1*F8[x, t]\partial_x (M25[x, t]) - c12*(M23[x, t] + m1*M25[x, t])*\partial_x (F8[x, t]);$$

$$R18(x, t) = c12*m2*F9[x, t]\partial_x (M26[x, t]) - c12*(M24[x, t] + m2*M26[x, t])*\partial_x (F9[x, t]);$$

$$\begin{aligned} R19(x, t) = & c12*m1*F8[x, t]\partial_x (M26[x, t]) + c12*F9[x, t]*m2*\partial_x (M25[x, t]) - c12*(M24[x, t] \\ & + m2*M26[x, t])*\partial_x (F8[x, t]) - c12*(M23[x, t] + m1*M25[x, t])*\partial_x (F9[x, t]); \end{aligned}$$

$$R20(x, t) = \frac{R1[x, t]}{30};$$

$$R21(x, t) = \frac{R2[x, t]}{20};$$

$$R22(x, t) = \frac{R3[x, t]}{12};$$

$$R23(x, t) = \frac{R4[x, t]}{6};$$

$$R24(x, t) = \frac{R5[x, t]}{2};$$

$$\begin{aligned} k7 = & R20*\eta^6 + R21*(-\eta)^5 + R22*\eta^4 + R23*(-\eta)^3 + R24*\eta^2 + R27*(-\eta)^3*\text{Exp}[-m1*\eta] + R28*(-\eta)^3*\text{Exp}[-m2*\eta] + \\ & R29*\eta^2*\text{Exp}[-m1*\eta] + R30*\eta^2*\text{Exp}[-m2*\eta] + R31*(-\eta)^1*\text{Exp}[-m1*\eta] + R32*(-\eta)^1*\text{Exp}[-m2*\eta] + \\ & R33*(-\eta)^1*\text{Exp}[-2*m1*\eta] + R34*(-\eta)^1*\text{Exp}[-2*m2*\eta] + R35*(-\eta)^1*\text{Exp}[-(m1 + m2)*\eta] + \\ & R36*\text{Exp}[-m1*\eta] + R37*\text{Exp}[-m2*\eta] + R38*\text{Exp}[-2*m1*\eta] + R39*\text{Exp}[-2*m2*\eta] + R40*\text{Exp}[-(m1 + m2)*\eta]; \end{aligned}$$

$$\begin{aligned} k8 = & R20*\eta^6 + R21*(\eta)^5 + R22*\eta^4 + R23*(\eta)^3 + R24*\eta^2 + R27*(\eta)^3*\text{Exp}[m1*\eta] + R28*(\eta)^3*\text{Exp}[m2*\eta] + \\ & R29*\eta^2*\text{Exp}[m1*\eta] + R30*\eta^2*\text{Exp}[m2*\eta] + R31*(\eta)^1*\text{Exp}[m1*\eta] + R32*(\eta)^1*\text{Exp}[m2*\eta] + \\ & R33*(\eta)^1*\text{Exp}[2*m1*\eta] + R34*(\eta)^1*\text{Exp}[2*m2*\eta] + R35*(\eta)^1*\text{Exp}[(m1 + m2)*\eta] + \\ & R36*\text{Exp}[m1*\eta] + R37*\text{Exp}[m2*\eta] + R38*\text{Exp}[2*m1*\eta] + R39*\text{Exp}[2*m2*\eta] + R40*\text{Exp}[(m1 + m2)*\eta]; \end{aligned}$$

$$R25 = \frac{k7 + T8}{\eta};$$

$$R26 = \frac{-(k7 + k8)}{2};$$

$$R27(x, t) = \frac{R6[x, t]}{(m1)^2};$$

$$R28(x,t) = \frac{R7[x,t]}{(m2)^2};$$

$$R29(x,t) = \frac{-6*R6[x,t]}{(m1)^3} + \frac{R8[x,t]}{(m1)^2};$$

$$R30(x,t) = \frac{-6*R7[x,t]}{(m2)^3} + \frac{R9[x,t]}{(m2)^2};$$

$$R31(x,t) = \frac{18*R6[x,t]}{(m1)^4} - \frac{4*R8[x,t]}{(m1)^3} + \frac{R10[x,t]}{(m1)^2};$$

$$R32(x,t) = \frac{18*R7[x,t]}{(m2)^4} - \frac{4*R9[x,t]}{(m2)^3} + \frac{R11[x,t]}{(m2)^2};$$

$$R33(x,t) = \frac{R12[x,t]}{4(m1)^2};$$

$$R34(x,t) = \frac{R13[x,t]}{4(m2)^2};$$

$$R35(x,t) = \frac{R14[x,t]}{(m1+m2)^2};$$

$$R36(x,t) = \frac{-24*R6[x,t]}{(m1)^5} + \frac{6*R8[x,t]}{(m1)^4} - \frac{2*R10[x,t]}{(m1)^3} + \frac{R15[x,t]}{(m1)^2};$$

$$R37(x,t) = \frac{-24*R7[x,t]}{(m2)^5} + \frac{6*R9[x,t]}{(m2)^4} - \frac{2*R11[x,t]}{(m2)^3} + \frac{R16[x,t]}{(m2)^2};$$

$$R38(x,t) = \frac{-1*R12[x,t]}{4*(m1)^3} + \frac{R17[x,t]}{4*(m1)^2};$$

$$R39(x,t) = \frac{-1*R13[x,t]}{4*(m2)^3} + \frac{R18[x,t]}{4*(m2)^2};$$

$$R40(x,t) = \frac{-2*R14[x,t]}{(m1+m2)^3} + \frac{R19[x,t]}{(m1+m2)^2};$$

$$R41(x,t) = R23[x,t] + M21[x,t];$$

$$R42(x,t) = R24[x,t] + M22[x,t];$$

$$R43(x,t) = R25[x,t] + \frac{1}{2*\eta[x,t]} + F14[x,t];$$

$$R44(x,t) = R26[x,t] + \frac{1}{2} + F15[x,t];$$

$$R45(x,t) = R31[x,t] + M23[x,t];$$

$$R46(x,t) = R32[x,t] + M24[x,t];$$

$$R47(x,t) = R36[x,t] + M25[x,t];$$

$$R48(x,t) = R37[x,t] + M26[x,t];$$