

Fractal geometry in medicine: an effective method for identifying tumors

Abstract

Fractals are geometric shapes characterized by self-similarity, where patterns repeat at different scales and are common in nature. The box-counting method is used to measure their dimension, but it has a key limitation: it only offers a global and homogeneous characterization, ignoring local variations in density or complexity within the structure. Multifractal analysis solves this problem. Instead of a single dimension, it provides a spectrum of dimensions that maps how regularity varies in different parts of the object, offering a more accurate description of complex and heterogeneous systems. These concepts are relevant to endocrinology and medicine through the study of biological systems with fractal geometry, such as the branching of blood vessels or neurons. Multifractal tools help researchers measure the complexity of these structures and dynamic processes (such as hormonal rhythms), potentially improving the diagnosis and understanding of various pathologies.

Keywords: geometry, tumors, hormonal, metabolic problems

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Introduction

The concept of a fractal, coined in 1975 by the mathematician Benoit Mandelbrot,¹⁻⁴ refers to a geometric figure whose essence, characterized by being irregular or fragmented, is reproduced at different levels of magnitude. This phenomenon is called self-similarity, and its analysis and theoretical formulation introduce us to the field of mathematics related to infinite processes.⁵

This characteristic can be identified in the shape of certain types of plants or trees, as in a cauliflower head, or in different varieties of ferns, where each leaf resembles the overall shape of the plant. Similarly, it is observed in the silhouette of some clouds, which from a distance appear as a single unit, but upon closer inspection reveal small fragments that are repeated at various scales.⁶

An example of what we now identify as a mathematical fractal was presented in 1915 by the Polish scientist Waclaw Franciszek Sierpiński, who lived from 1882 to 1969. His design, based on a triangle, generates a series of geometric figures that enclose an area approaching zero, while their perimeter approaches infinity. This characteristic suggests that, to effectively capture its level of irregularity and fragmentation, as well as its ability to occupy space, the notion arises of linking a non-integer dimension to a fractal object situated between line and surface. In the specific case of the Sierpiński triangle, the linked self-similarity dimension is approximately 1.585.^{7,8}

In reality, perfect and infinite self-similarity does not exist in biological objects. In reality, no completely faithful, reduced reproductions of the original object are found, and furthermore, only a limited number of levels of self-similarity can be observed. However, numerous components of nature exhibit complex forms,⁹⁻¹¹ and through fractal analysis, a fractal pattern can be recognized that, while not perfect, is an approximation. An example of this is its use in digital image processing, which is frequently employed in the field of medical diagnosis.

Considering that, although various cancer-related processes may be externally visible, their form can vary considerably depending on the scale of observation.⁵ In this context, a detailed analysis supported by these images helps to evaluate the different morphological

characteristics present.

Scientific studies¹² indicate that the initial stages of tumor formation can be identified using fractal and multifractal geometry techniques. These strategies facilitate the characterization of variations in the irregularity of cell, tissue, and vascular system contours as an abnormal mass develops. This allows for the assessment of the extent of damage to the original tissue, which would help reduce both diagnostic errors and ambiguity.

However, assigning a numerical value that accurately captures the irregular shape of these natural elements is challenging. One commonly used parameter is the box count dimension, which establishes a relationship between observations at different scales of the same examined fragment, facilitating the measurement of the rate at which irregularities manifest. This counting method usually yields good results when the examined contour conforms in some way to the mathematical property of self-similarity. However, the complexity of certain images can cause the value obtained for this parameter to not adequately reflect their underlying structure. This can be improved through multifractal.⁵

Multifractality deals with how the pixels defining the contour under analysis are distributed, and not just with simply counting the boxes or squares that cross that contour. This allows for a detailed exploration of the internal structure at a local level and of the variations in the studied morphology. Thus, instead of focusing on a single parameter or dimension, it is crucial to manage a diverse set of values, known as the multifractal spectrum,² facilitates the evaluation of different scale and density properties that might be present. In contrast, in the case of monofractality, such a spectrum would be limited to a set of overlapping values.

Multifractal imaging is increasingly used for detailed analysis of medical images, facilitated by recent technological advances.¹² In 2015, a team led by Igor Sokolov and Craig Woodworth presented findings demonstrating the existence of multifractal structures on the surfaces of cells in the pre-cancerous stage. These features are not found in healthy cells or in cells already affected by the disease within the same tissue. Therefore, by identifying these structures, it would be possible to diagnose this condition before a tumor develops.¹²

Discussion

Fractal geometry, albeit on a reduced scale, is already being used in endocrine systems to interpret the complexity and effectiveness of structures and hormone distribution, revealing patterns of self-similarity. This provides a way to describe the organization and texture of endocrine tissues and cells, offering a means of assessing complexity that goes beyond Euclidean measurements such as length, area, and volume. Therefore, this geometry offers a representation of the structures and dynamics of the human body that Euclidean geometry cannot capture, such as branching characteristics and self-similarity.

In the endocrine system,¹³ evidence shows that it is mainly used for modeling and evaluating structures and signals:

- 1) **Glandular structures:** the fractal approach is applied to define the irregular and complex morphology of glands and tissues, facilitating the identification of abnormal growths, such as tumors;
- 2) **Hormonal signals:** the fluctuation of hormones, which is dynamic and does not follow a linear course, can be examined using principles of chaos and fractals, allowing a better understanding of rhythms and dysfunctions within the system;
- 3) **Vascular networks:** endocrine glands have extensive irrigation through blood vessels, and this network follows efficient fractal patterns that optimize the exchange area, which can be modeled.

In its application to the pituitary gland (hypophysis), Grizzi et al.,¹⁴ argue that its study is crucial for understanding how fractals can measure the complexity of the blood vessels that nourish the pituitary gland, something essential for its endocrine function. The findings indicate that fractal dimension is a valid and objective measure of microvascular complexity in the pituitary gland, encompassing both physiological and pathological conditions, including the presence of tumors.

In the application of fractal dimension to the thyroid gland (image analysis), Carg et al.,¹⁵ focused on quantifying thyroid tumors using fractal dimension as an imaging biomarker. This study analyzed fractal dimension, calculated using the box-counting technique, to distinguish between normal and cancerous thyroid tissue. On the other hand, the research by Komatsu et al.,¹⁶ focused less on shape and more on the mathematical modeling of pulsatile hormonal signals (chaotic or fractal rhythms) of the HPA axis. They applied fractal analysis to objectively determine the presence of chronic inflammation in thyroid tissue from ultrasound images.

Bosse,¹⁷ and Kim,¹⁸ contribute to hormonal dynamics,^{17,18} addressing the Hypothalamus-Pituitary Axis, explaining how the concepts of fractal geometry and nonlinear chaos are valuable tools for measuring structure and metabolism, including the dynamics of pulsatile secretion of hormones such as TSH.

Promising research indicates that this emerging field of mathematics may play a vital role in the early and effective identification of irregularities and the detection of potentially cancerous problems in the cervix. These approaches facilitate the description of irregular variations that occur in the contours of cells, tissues, and vascular networks when an abnormal mass forms.

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Conflict of interest

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