Short Communication

An overview of the evolution of the devices running computer algebra systems and their educational implications

Abstract

Only back in late 60's and early 70's powerful computers were required to run the then just released computer algebra systems (brand new software mainly designed as a specialized aid for astronomy and high energy physics). In the late 80's they could be run in standard computers. And in 1995 the first calculator including a computer algebra system was released. Now some of these pieces of software are freely available for Smartphones. That this software has been ported to these electronic devices is a challenge to different aspects of mathematical and physics education, like: problem solving, assessment and standardized assessment.

Keywords: computer algebra systems, computers, calculators, smartphones, mathematical education

Volume 1 Issue 1 - 2017

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Received: May 05, 2017 | **Published:** June 20, 2017

Introduction

Computer algebra systems[1] have two main differences w.r.t. classic computer languages:

a) They perform numerical computations in exact arithmetic (by default), instead of in floating point arithmetic. Therefore, the user can directly compute calculations such as:

$$\frac{1}{3}+\frac{2}{5}=\frac{11}{15}$$

$$\sqrt{7}*\sqrt{3}-\sqrt{21}=0$$

Or:

100! =93326215443944152681699238856266700490715968264

381621468592963895217599993229915608941463976156518

286253697920827223758251185210916864000000000000000

00000000000

Note that no rounding or truncation takes place: the system performs the computations as usual in mathematics. This requires not using the built-in floating point arithmetic but to implement (in software) an exact arithmetic, what implies slower calculations. The speed of the computations can also be slowed by the growth of the expressions.

b) They can handle non-assigned variables, that is, variables in the mathematical sense, not in the usual computational sense. For instance, we can ask a CAS to perform the simplification:

$$(x+y)^2-(x-y)^2=4xy$$

(Where no values have been assigned to x and y). Therefore, CAS are at a higher-level than usual computer languages (in the sense of being farther away from the machine code and closer to humans' language). As a consequence of the possibility of handling symbols, symbolic manipulations such as symbolic differentiation and integration can be implemented. The incredible increase of microprocessors power makes it possible to have CAS available for Smartphones. This induces serious challenges in teaching mathematics as well and in the assessment of this subject.

The pioneers

Macsyma is one of the pioneer general purposes CAS. Its development began on 1968 by a set of researchers at the MIT and was written in a dialect of LISP. It was commercialised by different companies or organizations during the following years.[1,2,3] A 1982 version was released under GPL license in 1999 and is still available with the name Maxima[4] and it is maintained. Friendly user interfaces, such as wxMaxima[5] are also offered. Prof Anthony C Hearn began the development of REDUCE in 1963[6] and its first release dates back to 1968.[7] It is also a very powerful general purpose CAS[7,8,9] and it is written in a dialect of LISP too. The reason to write these CAS in LISP is that it is easy to handle lists in LISP, and this structure was used for implementing, for instance, exact arithmetic. Now REDUCE can be freely obtained too.[10]

In the mid 80's, the release of the general purpose CAS Mathematica[11] and Maple[12] probably marked the beginning of the decay of the pioneer CAS. These two CAS were written in C and have hundreds of thousands of users. There are many other general purpose CAS, like DERIVE, Axiom, MuPAD, Fermat, Risa/Asir, SageMath, etc. and specific purpose CAS, such as CoCoA, SINGULAR, PolyBoRi, etc.

Derive

In 1979 Prof. David Stoutemyer and Albert Rich, an applied logician, founded The Software House in Honolulu with the goal of creating a small yet powerful CAS. The first products of the company were: muLISP,[13] a tiny LISP dialect that could be run on small computers, and muMATH79,[14] a CAS written in muLISP. The last version was muMATH83. It was a standard (for the time) command-line interface CAS. It was surprisingly tiny and the hardware requirements were very modest. But the star of the company was DERIVE, which first version appeared in 1988. It was also designed for text terminals

[1]Most software and hardware names mentioned in this article are registered trademarks.

(MS-DOS), but it was menu-driven and very comfortable to use (Figure 1). A computer without a hard disk could directly run DERIVES from the 5 1/4" distribution diskette. DERIVE 4, included both text console and Windows versions. The latter version was much improved in the next DERIVE 5. The Software House was purchased Texas Instruments in 1999 and DERIVE was discontinued in 2007 in favour of TI-Nspire, a product developed from scratch by Texas Instruments and fully documented. The last version of DERIVE was Derive 6.1 (Figure 2). It was distributed in a CD although it occupied little more than 2 MB. A brief summary of its possibilities can be found in.[15]

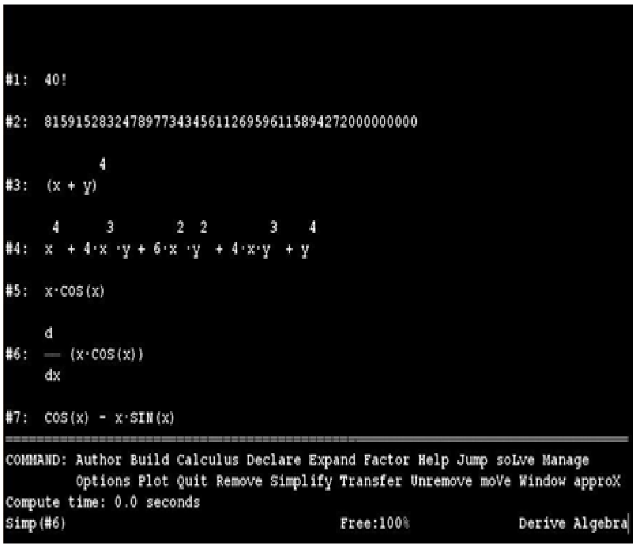

Figure 1 Screenshot of the text console version of DERIVE

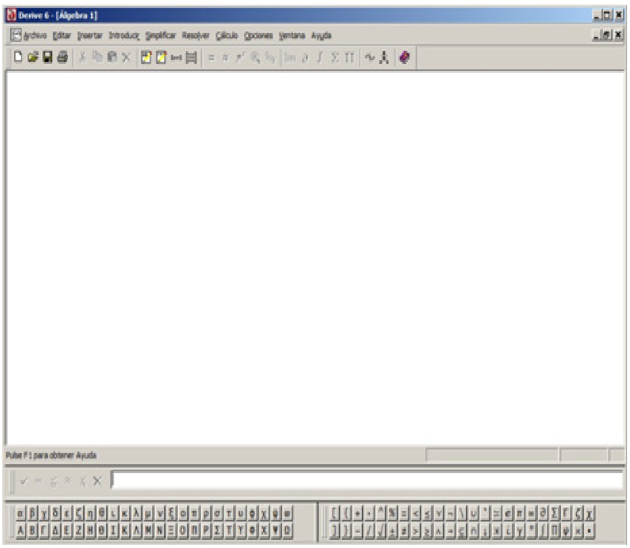

Figure 2 Screenshot of DERIVE 6.1.

Derive and education

In 1983 Klaus Aspetsberger and Gerhard Funk conducted in an Austrian high school what is probably the world's first educational experiment with a CAS (muMATH).[16] But it was the muMATH descendant DERIVES which was a real success, used at many high-schools and universities around the world. Most impressively, the Austrian experiments led to the creation by the Austrian government of an specialized research centre, the *Austrian Center for Didactics of Computer Algebra* (ACDCA)[17] and the acquisition of a country-wide license of DERIVE for Secondary Education. There are very many papers and books about DERIVE and education, so we shall not include a comprehensive literature about DERIVE. We will only give as example.[18,19] Regarding journals and bulletins, *The DERIVE User Group* (DUG) was founded by Josef Böhm in 1991 consists now of more than 500 members from all over the world and publishes *The Derive Newsletter* (DNL). The DUG was initially printed in paper and available to subscribers, but only the electronic version is nowadays produced and it is (freely) available from.[20] The DNL includes information and both technical and didactic articles about DERIVE and the TI symbolic calculators. Although the CAS DERIVE was long ago discontinued, the DNL is alive, demonstrating the impacts that DERIVE had on mathematics teaching around the world.

Meanwhile The International Derive Journal, founded at Plymouth University in 1994, followed a different evolution and has widened its aims and scope two times. It was firstly renamed in 1997 The International Journal of Computer Algebra in Mathematics Education (IJCAME) and was renamed again later as The International Journal of Technology in Mathematics Education (IJTME).[21] The influence of DERIVE in education was so strong that some of the early versions of wxMaxima resembled DERIVE's front end (Figure 3).

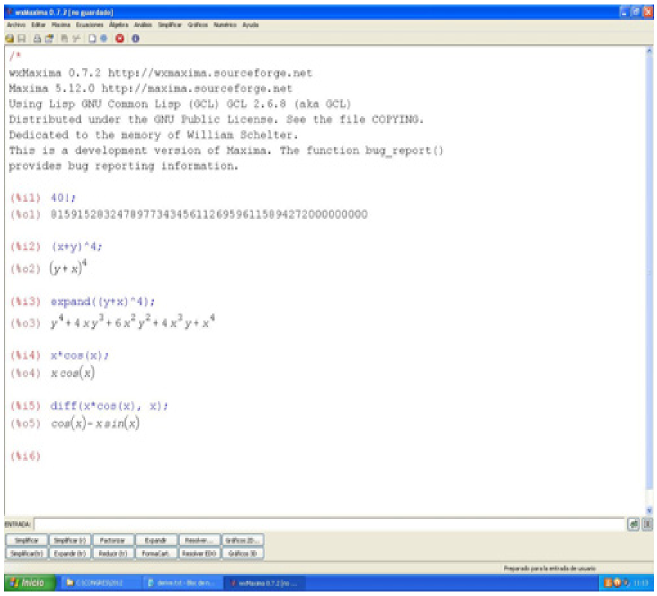

Figure 3 An early version of the front end of wxMaxima with an input line in the lower part and the output area above, that resembled DERIVE front end

Calculators and CAS

The developers of DERIVE desired to have a portable electronic device running a powerful CAS. A software derived from DERIVE was used in some Texas Instruments pocket calculators like the TI-92, launched in 1995 (Figure 4) and the TI-89 (Figure 5). Other models followed, and calculators running TI-nspire CAS are available (Figure 6).[22] Other companies such as Hewlett-Packard and Casio soon offered calculators running CAS too, like the HP49G, HP50g, etc. and Casio CFX 9970G, Classpad, etc., respectively.[23,24] These calculators have the possibilities of usual scientific calculators, as well as those of graphic calculators, and can perform the tasks of usual CAS. In my opinion, calculators including CAS are not widely spread, as scientific and graphic calculators are, probably due to their higher price.

Citation: Roanes-Lozano E. An overview of the evolution of the devices running computer algebra systems and their educational implications. *Electric Electron Tech Open Acc J.* 2017;1(1):7–11. DOI: 10.15406/eetoaj.2017.01.00002

Figure 4 A TI-92 (1995). The first calculator including a CAS (symbolic calculator).

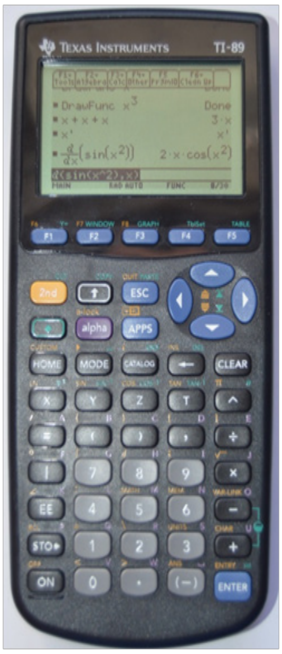

Figure 5 The TI 89 symbolic calculator.

Figure 6 The first TI-nspire calculator.

CAS for smartphones

Now Maxima and REDUCE have been ported to Android OS and are freely available for tablets and Smartphones (Figures 7 & 8). Android Reduce was started in 2011 by the then student Ahmad Akra at Cambridge University under the supervision of Prof. Arthur C. Norman. Two user interfaces exist, one that makes the screen look like a calculator and another one with text input very similar to the computer version of REDUCE. Maxima were ported to Android by Yasuaki Honda, a Maxima developer, in 2012. Modern smartphones are faster and have more memory available than the desktop computers available not many years ago. Therefore, the algebraic tasks that can be faced are not neglectable, and many students carry their smartphones with them everywhere.

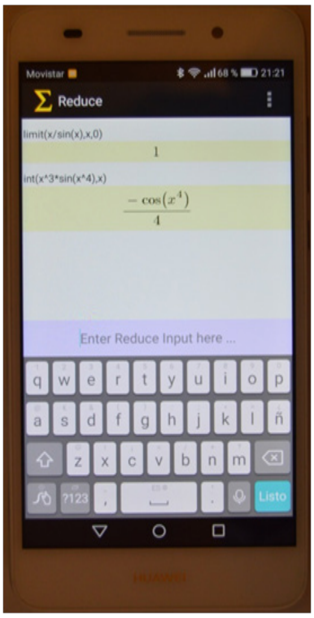

Figure 7 A limit that requires applying L'Hôpital's rule and a simple indefinite integration computed using REDUCE on a smartphone

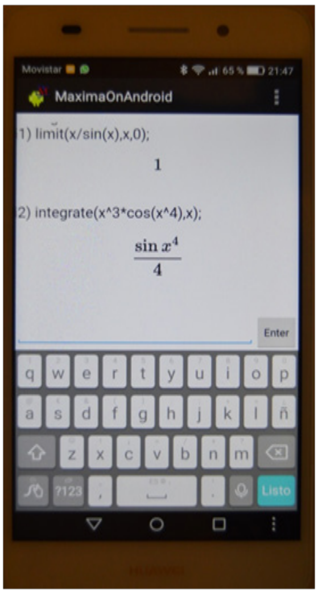

Figure 8 The same limit and the same indefinite integration computed using Maxima on a smartphone.

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CAS at the classroom and assessment

CAS run on computers induce a revolution in the classroom:

a) They change the characteristics of many problems. Therefore the curriculum has to be revisited.

b) They trivialize some problems that were typical of Maths Olympiads (the *eureka effect* therefore disappears).

Example 1: How many final zeroes have 100!? (Section 1).

Example 2: A farmer has a fence made of pieces that can be assembled in rectangular shapes. As the number of pieces is limited, the perimeter is fixed. Is there a way to maximize the area inside the fence? It is easy to investigate with a CAS which seems to be the solution and to prove it (Figure 9). The first guess can be easily made thanks to an accurate plot, but plotting is not so easy and takes time if done by hand.

c) They transform long cumbersome problems, where errata can easily arise, into trivial ones.

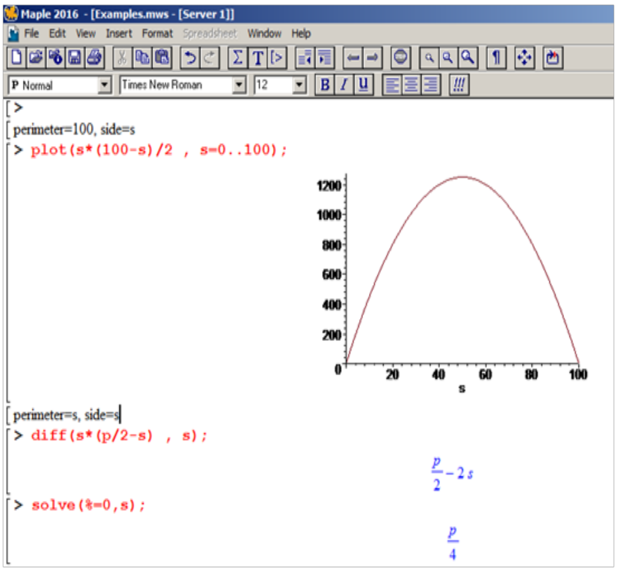

Figure 9 Investigating and proving with the CAS Maple.

Example 3: An exercise where a system of linear Diophantine equations about animals in a zoo and the total number of animals, tails, legs, wings, eyes, etc. (with several equations).

Example 4: A tedious integration by partial fractions. For instance, the CAS Maple can compute and justify the result of $\int \frac{x^3+x+1}{x^2-3x}dx$ (Figure 10).

Example 5: An astronomical problem involving long trigonometric expressions.

d) If CAS is not used by all students, the class is divided into two groups when solving problems (as solving timings can be very different). All students should be given the same access to technology and to technological instruction (this is sometimes a problem, as training the students so that they can take advantage of the possibilities of technology takes some time, and many times teachers consider it does not worthwhile).

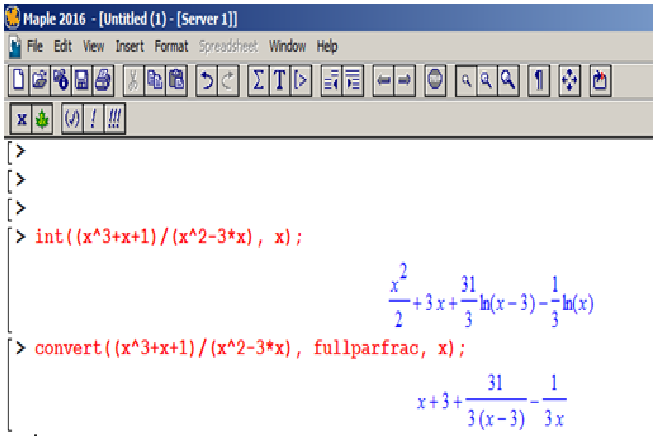

Figure 10 A computation performed and justified using Maple.

Example 6: Let us consider the following exercise: find the points of the ellipsoid $\frac{x^2}{7}+\frac{y^2}{5}+\frac{z^2}{25}-1$ lying on plane x+y=0 such that they maximize the distance to point (0,0,0). For instance, a Maple expert can realize that it provides a command that directly solves the problem, returning the extrema (Figure 11). Although not difficult, this problem takes some time to solve with pen and paper.

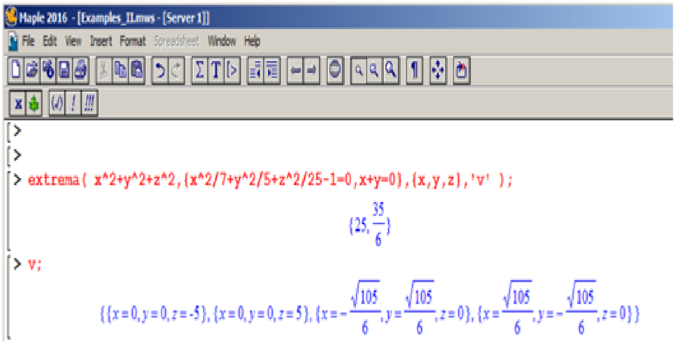

Figure 11 A problem solved with a single line of Maple code.

CAS clearly influences assessment

a) CAS make the computer skills of the student influence the marks obtained in some subjects like mathematics (consider, for instance, Example 6).

b) Training the students with the aid of technology can be dangerous if the rules of the standardized assessment are not clear. For instance, college entrance examinations in Spain traditionally included a tedious differentiation or integration and function plotting (justified by analysing its minima, maxima, inflection points and asymptotes), that are trivial or, at least, much simplified, with the help of a calculator including a CAS.

c) Taking into consideration i) above, exams have to be adapted to the technology allowed in the examinations.

Teacher's attitude towards CAS

Many mathematics teachers are aware of the possibilities of CAS: both from the point of view of their advantages and of what students can do with them, but many others are still unaware. In the first subset, I have witnessed different extreme attitudes:

1) The exultant: tries to use the CAS in most classroom activities and home exercises.
2) The "ostrich": is afraid of his/her students knowing of the existence of CAS, as "students will not want to learn limits, differentiation, integration, etc., because the machine does everything quickly and effortlessly".

In my opinion, CAS should be used wisely, as should scientific calculators. Understanding arithmetic operations and when to use them is a need in the present society. Mental calculus is necessary for simple calculations. Large calculations should be performed with a calculator. For instance, accountants do not need to be very clever in mental calculus nowadays, as they used to be in the past, as they now use spreadsheets. Similarly, a college level science student has to understand the concepts and ideas underlying, for instance, limits, differentiation and integration, should also be able to calculate mentally or with pen and paper not very long examples, but should be allowed to compute them with a CAS when they are long and tedious, as calculating them by hand is useless in such case. The fact that CAS is now (freely) available for Smartphones makes it necessary for the educational authorities to address these issues, especially assessment. The Spanish case can be specially biased, as almost all secondary students own Smartphones.

Conclusion

The electronic devices required to run CAS have evolved in the last 50 years from big computers to calculators and Smartphones. Although it is far more comfortable to use them on a computer, with a big screen and a real keyboard, the access to CAS has completely changed. As students have at their fingertips these powerful tools, different educational issues such as curricula, assessment and standardized assessment should be revisited.

Acknowledgements

This work was partially supported by the research projects TIN-2015-66471-P (Government of Spain) and CASI-CAM S2013/ICE-2845 (Comunidad Autónoma de Madrid).

Conflict of interest

The author declares no conflict of interest.

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