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On statistical properties, estimation of parameter and applications of new X-Lindley distribution

Abstract

In this paper some of the important statistical properties and applications of new X-Lindley distribution which were not studied earlier has been studied. The statistical properties based on moments including coefficient of variation, coefficient of Skewness, coefficient of kurtosis and index of dispersion have been derived and their behaviors examined graphically. Moment generating function, characteristics function, and conditional mean and conditional variance has been discussed derived. The reliability properties including survival function, hazard function, reverse hazard function, mean residual life function, stochastic ordering and stress-strength reliability have also been discussed along with deviation from mean and median, Bonferroni and Lorenz curve and their indices. The sequential probability ratio test and approximation for the average sample number have been discussed. The parameters of the distribution are estimated using method of moments, maximum likelihood estimation, maximum product spacing estimation, least square estimation, weighted least square estimation, Crammer-Von Mises estimation and Anderson-Darling estimation. A simulation study is carried out using acceptance-rejection method of simulation to know the consistency of the estimate values of the parameter. A goodness of fit measures has been shown on two real life datasets from the engineering field and compared with almost all one parameter lifetime distributions introduced in statistics literature.

Keywords: hazard function, mean residual life function, estimation, simulation study, applications

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Hosenur Rahman Prodhani, Rama Shanker
Department of Statistics, Assam University, India

Correspondence: Rama Shanker, Department of Statistics, Assam University, Silchar, Assam, India-788011

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Introduction

In statistical distribution theory, the exponential distribution is the earliest and most widely applied one-parameter lifetime distribution. Its simplicity and tractability have made it an essential tool for modelling lifetime data across diverse fields such as engineering, medicine, and reliability studies. However, its inherent assumption of a constant hazard rate often fails to capture the complexities observed in real-life datasets. This limitation has motivated researchers to explore alternative lifetime models with more flexible structures. To address this need, Lindley[1] introduced the Lindley distribution as a convex combination of exponential and gamma distribution. Later, Ghitany et al.[2] provided a comprehensive study of its properties and applications. Shanker et al.[3] have comparative study between exponential and Lindley distributions and concluded that on some datasets exponential distribution provides better fit as compared to the Lindley distribution and on some dataset Lindley distribution provides better fit as compared to the exponential distribution.

During recent decade several one parameter lifetime distributions have been proposed in statistics literature by different researchers and it has been observed that each has some advantages and disadvantages over the others in terms of statistical and reliability properties, applications and goodness of fit. For example, Shanker[4,5] introduced Akash distribution and Shanker distribution and discussed their statistical and reliability properties along with applications. Shanker and Shukla[6] proposed two-parameter Akash distribution and discussed the properties, estimation of parameters and applications. Shanker[7-12] have introduced Sujatha distribution, Aradhana distribution, Amarendra distribution, Garima distribution, Devya distribution and Shambhu distribution and discussed their statistical properties, estimation of the parameter using maximum likelihood estimation along with their applications. Shanker[13,14] have introduced Suja distribution and Rama distribution. Chouia and Zeghdoudi[15] introduced X-Lindley distribution (XLD), Shanker[16] introduced Uma distribution, Shanker[17,18] have introduced Komal distribution and Pratibha distribution and discussed their properties and applications Shanker et al.[19] introduced a new one parameter lifetime distribution known as Ravi distribution with applications in survival analysis of cancer patients. While these distributions have expanded the field of lifetime modelling, many of them exhibit certain limitations when evaluated from both mathematical and applied perspectives. In particular, some models lack tractable forms for estimation, while others fail to provide consistently good fits across diverse datasets. Moreover, simulation-based assessments of estimator performance, which are essential for verifying the consistency and reliability of estimation methods, have received limited attention in much of the existing literature.

To address these challenges, Nawel et al.[20] introduced the one-parameter lifetime distribution, known as the new X-Lindley distribution (NXLD). In this study, the fundamental statistical properties and parameter of NXLD are estimated using different estimation methods, including the method of moments, maximum likelihood estimation, maximum product spacing estimation, least squares estimation, weighted least squares estimation, Cramér-Von Mises estimation, and Anderson-Darling estimation. Importantly, a comprehensive simulation study has been conducted to evaluate the consistency and efficiency of the proposed estimators, thereby filling a gap left by earlier works. The flexibility of the NXLD and its strong inferential framework highlight its potential as a valuable alternative for lifetime data modelling in reliability and survival analysis.

The main purpose of this paper is to study those properties of NXLD which have not been studied earlier and examine the goodness of fit of NXLD in comparison to all one parameter lifetime

distributions introduced so far in statistics literature. The pdf of NXLD is unimodal and decreasing. The coefficient of variation, coefficient of Skewness, coefficient of kurtosis, index of dispersion, mean deviation about mean and median, and Bonferroni and Lorenz curves of NXLD have been discussed. The reliability properties have also been investigated, including the survival function, hazard function, reverse hazard function, mean residual life function, stochastic ordering, and stress-strength reliability. It is shown that the NXLD has an increasing hazard function, whereas both the reverse hazard function and the mean residual life function are always decreasing in nature. The sequential probability ratio test (SPRT) has been developed, and the average sample number (ASN) has been derived. A simulation study has been conducted to compare the ASN values under different hypotheses and for different levels of Type I and Type II errors. The parameters of the NXLD are estimated using different classical methods. A simulation study has been carried out to examine the consistency of these estimators. To assess the goodness of fit, the NXLD has been compared with a number of existing one parameter lifetime distributions.

New X-Lindley distribution

The NXLD is defined by its probability density function (pdf) and cumulative distribution function (CDF) as follows:

$$f(x,\theta)=\frac{\theta}{2}(1+\theta x)e^{-\theta x}\ ;x>0,\theta>0 \quad (1)$$

$$F(x,\theta)=1-\left(1+\frac{\theta x}{2}\right)e^{-\theta x}\ ;x>0,\theta>0 \quad (2)$$

Theorem 1: If $X\sim NXLD(\theta)$, then

The mode of the distribution is at $x=0$, and (ii) the pdf is decreasing function.

Proof: The mode of a distribution is the value of at which the pdf attains its maximum value. To determine the mode, we consider the function to be maximized. Taking the first derivative of $f(x;\theta)$ defined in (1) and equating to 0, we have

$$f'(x;\theta)=-\frac{\theta^3}{2}xe^{-\theta x}=0\Rightarrow x=0$$

Now, taking the second derivative of $f(x;\theta)$, we have

$$f''(x;\theta)=-\frac{\theta^3}{2}(1-\theta x)e^{-\theta x}$$

Evaluating at $x=0$, we obtained $f''(0;\theta)=-\frac{\theta^3}{2}<0$,

which confirms that $x=0$ is local maximum. Hence, the mode of NXLD is obtained at $x=0$. This indicates that the distribution is unimodal with the mode located at the origin. Such behavior is not unusual among lifetime distributions; for instance, the exponential distribution also exhibits a mode at zero. A mode at zero often reflects a relatively high likelihood of very early failures, which is the characteristic of "infant mortality" phenomena in reliability and survival analysis.

Moreover, since $f'(x;\theta)<0$ for all $x>0$. The function is strictly decreasing over its range. This completes the proof.

The behavior of the pdf and the CDF of NXLD for different values of the parameter are presented in the Figures 1 & 2 respectively.

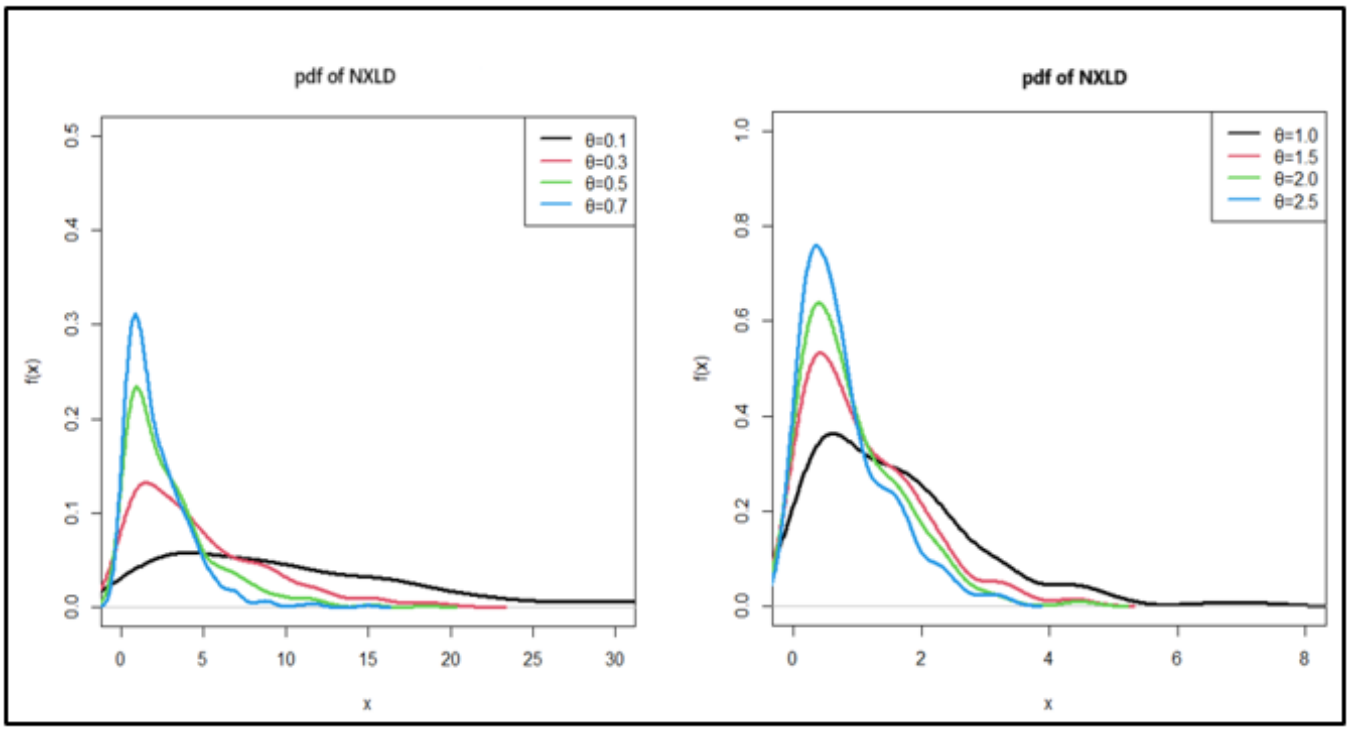

Figure 1 PDF of NXLD.

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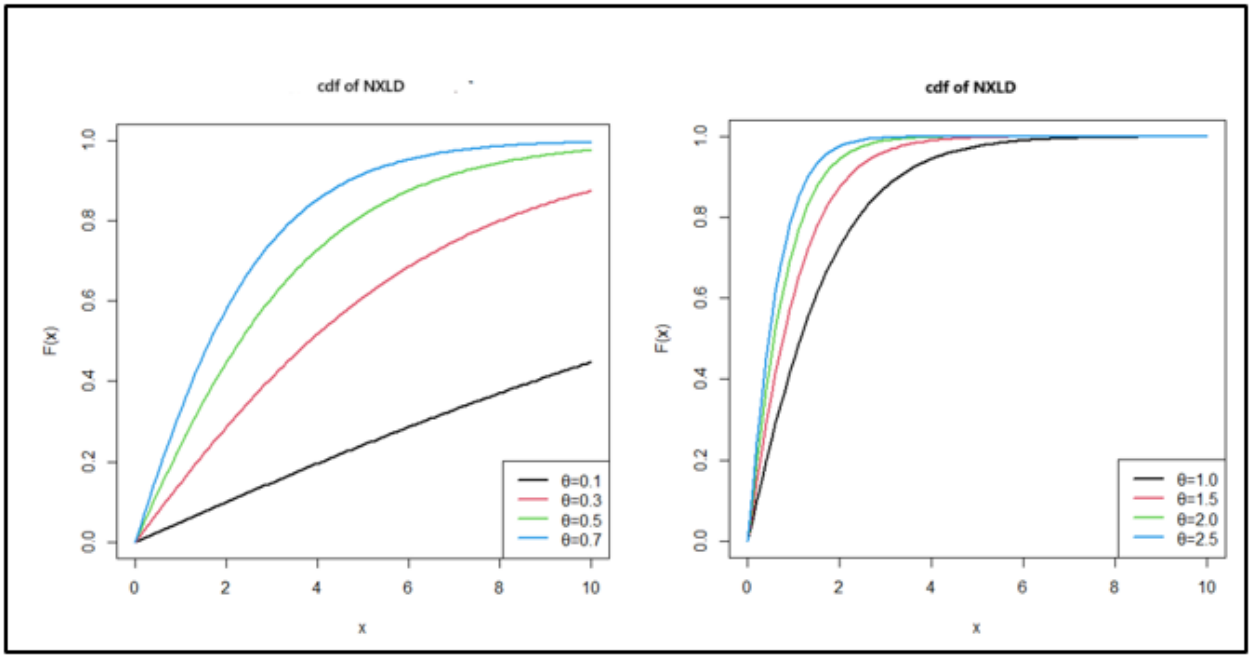

Figure 2 CDF of NXLD.

Descriptive measures based on moments

The th moments about origin of NXLD are given by

$$\mu_r' = \int_0^\infty x^r f(x,\theta)\,dx = \frac{r!(r+2)}{2\theta^r}; r = 1,2,3... \tag{3}$$

The first four moments about origin and the central moments of NXLD are given by

$$\mu_1' = \frac{3}{2\theta}\ \mu_2' = \frac{4}{\theta^2}\ \mu_3' = \frac{15}{\theta^3}\ \mu_4' = \frac{72}{\theta^4}\ \mu_2 = \frac{7}{4\theta^2}$$

$$\mu_3 = \frac{15}{4\theta^3}\ \mu_4 = \frac{333}{16\theta^4}$$

The descriptive moment based measures including coefficient of variation (CV), coefficient of Skewness (CS), coefficient of kurtosis (CK) and index of dispersion (ID) of NXLD are given by

$$CV = \frac{\sqrt{\mu_2}}{\mu_1'} = \frac{\sqrt{7}}{3} = 0.8819\ CS = \frac{\mu_3^{\,2}}{\mu_2^{\,3}} = \frac{30}{7^{\frac{3}{2}}} = 1.6198$$

$$CK = \frac{\mu_4}{\mu_2^{\,2}} = \frac{333}{49} = 6.7959\ ID = \frac{\mu_2}{\mu_1'} = \frac{7}{6\theta}$$

The CV of NXLD is 0.8819, indicating that the standard deviation is 88.19% of the mean i.e. a sign of moderate relative variability. The CS is 1.6198, suggesting that NXLD is positively skewed with a longer right tail similar to distributions like the exponential and gamma. The CK is 6.7959, which exceeds 3 and indicates that NXLD is leptokurtic i.e. characterized by a sharper peak with heavier tails than the normal distribution. NXLD is equi-dispersed for $\theta = 1.1666$, over-dispersed for and under-dispersed for $\theta > 1.1666$. The over-dispersion, equi-dispersion and under-dispersion of several one parameter lifetime distributions for values of parameter are presented in Table 1.

Table 1 Over-dispersion, equi-dispersion and under-dispersion of distributions

Distributions	Over-dispersion $(\mu < \sigma^2)$	Equi-dispersion $(\mu = \sigma^2)$	Under-dispersion $(\mu > \sigma^2)$
NXLD	$\theta < 1.1666$	$\theta = 1.1666$	$\theta > 1.1666$
XLD	$\theta < 1.3075$	$\theta = 1.3075$	$\theta > 1.3075$
Pratibha	$\theta < 1.4035$	$\theta = 1.4035$	$\theta > 1.4035$
Komal	$\theta < 1.1587$	$\theta = 1.1587$	$\theta > 1.1587$
Garima	$\theta < 1.1642$	$\theta = 1.1642$	$\theta > 1.1642$
Shanker	$\theta < 1.1715$	$\theta = 1.1715$	$\theta > 1.1715$
Sujatha	$\theta < 1.3642$	$\theta = 1.3642$	$\theta > 1.3642$
Ravi	$\theta < 1.0938$	$\theta = 1.0938$	$\theta > 1.0938$
Uma	$\theta < 1.7384$	$\theta = 1.7384$	$\theta > 1.7384$
Akash	$\theta < 1.5154$	$\theta = 1.5154$	$\theta > 1.5154$
Lindley	$\theta < 1.1700$	$\theta = 1.1700$	$\theta > 1.1700$
Rama	$\theta < 1.9501$	$\theta = 1.9501$	$\theta > 1.9501$
Suja	$\theta < 2.4931$	$\theta = 2.4931$	$\theta > 2.4931$
Amarendra	$\theta < 1.5257$	$\theta = 1.5257$	$\theta > 1.5257$
Devya	$\theta < 1.4516$	$\theta = 1.4516$	$\theta > 1.4516$
Shambhu	$\theta < 1.1490$	$\theta = 1.1490$	$\theta > 1.1490$
Aradhana	$\theta < 1.5257$	$\theta = 1.5257$	$\theta > 1.5257$
Exponential	$\theta < 1$	$\theta = 1$	$\theta > 1$

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Theorem 2: If $X \sim \mathrm{NXLD}(\theta)$, the moment generating function of NXLD is defined as

$$M_X(t)=\sum_{r=0}^{\infty}\frac{(r+2)}{2}\left(\frac{t}{\theta}\right)^r;r=0,1,2,...$$

Proof: We have

$$M_X(t)=\sum_{r=0}^{\infty}\frac{t^r}{r!}\mu_r';r=0,1,2,... \qquad (4)$$

Now, substituting the expression of μ_r' given in (3) into (4), we get

$$M_X(t)=\sum_{r=0}^{\infty}\frac{(r+2)}{2}\left(\frac{t}{\theta}\right)^r;r=0,1,2,...$$

Theorem 3. If $X \sim \mathrm{NXLD}(\theta)$, the characteristics function of NXLD is defined as

$$\Phi_X(t)=\sum_{r=0}^{\infty}\frac{(r+2)}{2}\left(\frac{it}{\theta}\right)^r;r=0,1,2,...$$

Proof: We have

$$\Phi_X(t)=\sum_{r=0}^{\infty}\frac{(it)^r}{r!}\mu_r';r=0,1,2,... \qquad (5)$$

Now, substituting the expression of μ_r' given in (3) into (5), we get

$$\Phi_X(t)=\sum_{r=0}^{\infty}\frac{(r+2)}{2}\left(\frac{it}{\theta}\right)^r;r=0,1,2,...$$

Conditional moments of NXLD

The conditional mean and variance of NXLD can be obtained as

$$E(X>X=x)=\frac{1}{S(x;\theta)}\int_x^{\infty}t\,f(t;\theta)\,dt-x=\frac{\theta^2x^2+3\theta x+3}{\theta(2+\theta x)}$$

$$V(X>X=x)=E(X^2>X=x)-\{E(X>X=x)\}^2=\frac{\theta^2x^2+6\theta x+7}{\theta^2(2+\theta x)^2}$$

Where,

$$E(X^2>X=x)=\frac{1}{S(x;\theta)}\int_x^{\infty}t^2\,f(t;\theta)\,dt-x=\frac{\theta^3x^3+4\theta^2x^2+8\theta x+8}{\theta^2(2+\theta x)}$$

Reliability properties

Survival function

The Survival function of NXLD is given by Figure 3

$$S(x;\theta)=1-F(x;\theta)=\left(1+\frac{\theta x}{2}\right)e^{-\theta x}$$

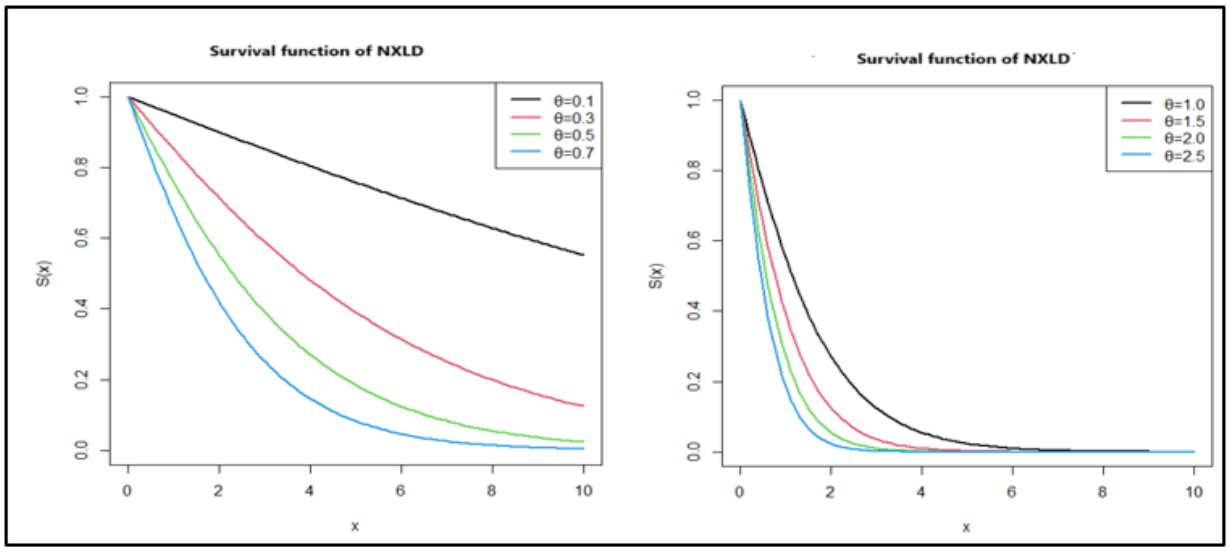

Figure 3 Survival function of NXLD.

Hazard function

The hazard function of NXLD is given by

$$h(x;\theta)=\frac{f(x;\theta)}{S(x;\theta)}=\frac{\theta(1+\theta x)}{2+\theta x}$$

Theorem 4: The hazard function of NXLD is always strictly increasing

Proof: The first derivative of the hazard function is obtained as

$$h'(x;\theta)=\frac{\theta^2}{(2+\theta x)^2}$$

Since $\theta>0$, then $(2+\theta x)^2>0$ for all $x>0$. Thus

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$h'(x;\theta) > 0$ for all $x > 0$ .Hence, $h(x;\theta)$ is always strictly increasing. The hazard function of NXLD is present in the Figure 4.

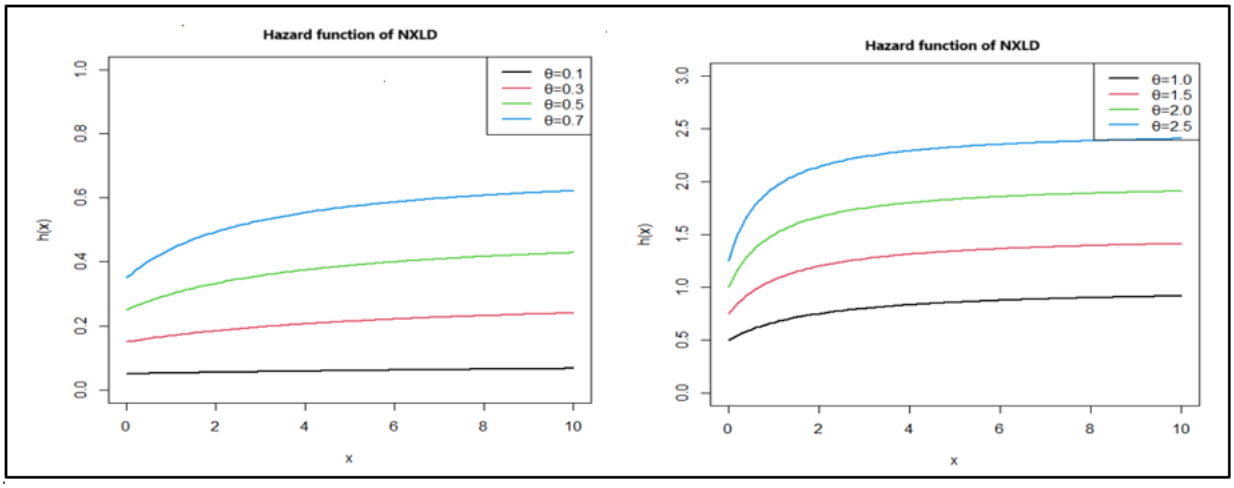

Figure 4 Hazard function of NXLD.

Reverse hazard function

The reverse hazard function of NXLD is given by

$$r(x;\theta) = \frac{f(x;\theta)}{F(x;\theta)} = \frac{\theta(1+\theta x)e^{-\theta x}}{2-(2+\theta x)e^{-\theta x}}$$

Theorem 5: The reverse hazard function of NXLD is always strictly decreasing

Proof: The first derivative of the reverse hazard function is obtained as

$$r'(x;\theta) = -\frac{\theta^2\left(2\theta x + e^{-\theta x}\right)e^{-\theta x}}{\left[(2+\theta x)e^{-\theta x} - 2\right]^2} < 0 \text{ For all values of } \theta > 0 \text{ and } x > 0$$

Thus, $r(x;\theta)$ is always strictly decreasing. The reverse hazard function of NXLD is presented in the Figure 5.

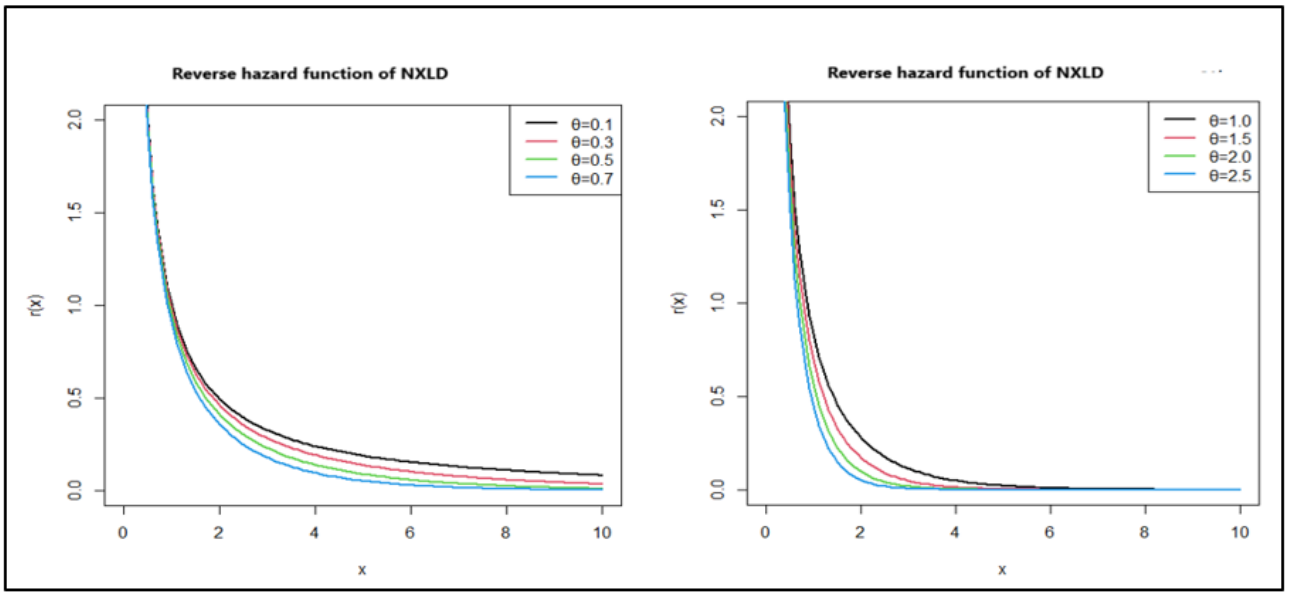

Figure 5 Reverse hazard function of NXLD.

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Mean residual life function

The mean residual life function of NXLD is given by

$$m(x;\theta)=\frac{1}{S(x;\theta)}\int_{x}^{\infty} t\,f(t;\theta)\,dt-x=\frac{(3+\theta x)}{\theta(2+\theta x)}$$

The mean residual life function at their two-extreme points are obtained as

$m(0)=\frac{3}{2\theta}$ And $m(\infty)=0$.

Theorem 6. Mean residual life function of NXLD is always decreasing.

Proof: the first derivative of the mean residual life function of NXLD is obtained as

$$m'(x;\theta)=-\frac{1}{(2+\theta x)^2}<0 \quad \text{For all values of } \theta>0 \text{ and } x>0$$

Thus, $m(x;\theta)$ is always decreasing. The graphical representation of the mean residual life function of NXLD present in the Figure 6.

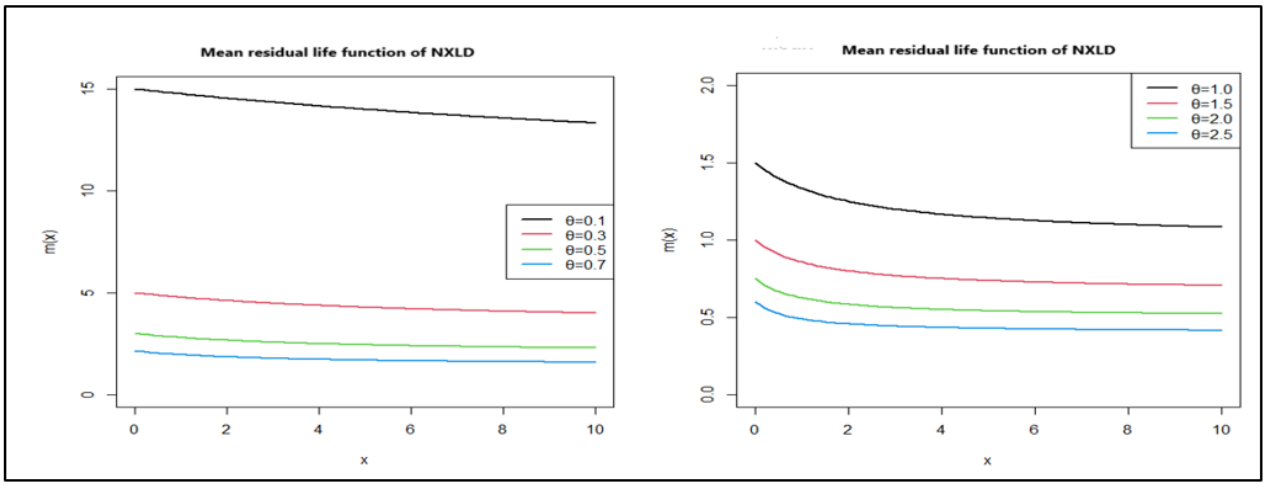

Figure 6 Mean residual life function of NXLD.

Stress-strength reliability

The stress-strength reliability of a component illustrates the life of the component which has random strength that is subjected to a random stress Y. When the stress of the component Y applied to it exceeds the strength of the component X, the component fails instantly, and the component will function satisfactorily until $X>Y$. Therefore, $R=P(Y<X)$ is a measure of the component reliability and is known as stress-strength reliability in statistical literature. It has extensive applications in almost all areas of knowledge especially in engineering such as structures, deterioration of rocket motors, static fatigue of ceramic components, aging of concrete pressure vessels, etc.

$$R=P(Y<X)=\int_0^\infty P(Y<X\mid X=x)\,f(x)\,dx=\int_0^\infty f(x)F(x)\,dx$$

The stress- strength reliability of NXLD can be obtained as

$$R=1-\frac{\theta_1\left(4\theta_1^2+5\theta_2^2+13\theta_1\theta_2\right)}{4(\theta_1+\theta_2)^3}$$

Mean deviation about mean and median

Mean deviation about the mean and the mean deviation about median of a random variable X having PDF $f(x)$ and CDF $F(x)$ are defined by

$$\delta_1(x)=\int_0^\infty |x-\mu|\,f(x)dx=2\mu F(\mu)-2\int_0^\mu x\,f(x)dx$$

And

$$\delta_2(x)=\int_0^\infty |x-M|\,f(x)dx=-\mu+2\int_M^\infty x\,f(x)dx$$

respectively, where $\mu=E(X)$ and $M=Median(X)$ using the pdf and expressions for the mean of NXLD, we get

$$\int_0^\mu x\,f(x;\theta)\,dx=\mu-\frac{\left(\theta^2\mu^2+3\theta\mu+3\right)e^{-\theta\mu}}{2\theta} \text{ And}$$

$$\int_0^M x\,f(x;\theta)=\mu-\frac{\left(\theta^2M^2+3\theta M+3\right)e^{-\theta M}}{2\theta}$$

Using above expressions and some algebraic simplifications, the mean deviation about the mean $\delta_1(x)$, and the mean deviation about the median $\delta_2(x)$ of NXLD are obtained as

$$\delta_1(x)=\frac{(\theta\mu+3)e^{-\theta\mu}}{\theta} \text{ and}$$

$$\delta_2(x)=\frac{\left(\theta^2M^2+3\theta M+3\right)e^{-\theta M}}{2\theta}-\mu$$

Bonferroni and lorenz curves and their indices

The Bonferroni and Lorenz curves[21] and Bonferroni and Gini indices have wide applications in economics to study income and

poverty, but it also used in other fields like reliability, demography, insurance and medicine. The Bonferroni and Lorenz curves are defined as

$$\mathrm{B(p)}=\frac{1}{p\mu}\int_0^q xf(x)dx=\frac{1}{p\mu}\left[\int_0^\infty xf(x)dx-\int_q^\infty xf(x)dx\right]=\frac{1}{p\mu}\left[\mu-\int_q^\infty xf(x)dx\right]$$

And

$$\mathrm{L(p)}=\frac{1}{\mu}\int_0^q xf(x)dx=\frac{1}{\mu}\left[\int_0^\infty xf(x)dx-\int_q^\infty xf(x)dx\right]=\frac{1}{\mu}\left[\mu-\int_q^\infty xf(x)dx\right]$$

Respectively or equivalently

The Bonferroni and Gini indices are obtained as

$B=1-\int_0^1 B(p)dp$ and $G=1-2\int_0^1 \mathrm{L}(p)dp$, respectively.

The Bonferroni and Lorenz curves of NXLD can be obtained as

$$\mathrm{B(p)}=\frac{1}{p}\left[1-\frac{1}{3}\left(\theta^2q^2+3\theta q+3\right)e^{-\theta q}\right] \text{ and}$$

$$L(\mathrm{p})=1-\frac{1}{3}\left(\theta^2q^2+3\theta q+3\right)e^{-\theta q}$$

The Bonferroni and Gini indices of NXLD can be obtained as

$$\mathrm{B}=1-\frac{1}{3}\left(\theta^2q^2+3\theta q+3\right)e^{-\theta q} \text{ and}$$

$$G=\frac{2}{3}\left(\theta^2q^2+3\theta q+3\right)e^{-\theta q}-1$$

Statistical inference

In this section, we have discussed sequential probability ratio test (SPRT) of NXLD along with its operating characteristic (OC) function and average sample number (ASN) function. We have also discussed the simulation results of ASN function for different values of parameter.

Sequential probability ratio test

Let $X_i\ (i=1,2,...,n)$ be random variables following NXLD (θ). The joint pdf of $X_i\ (i=1,2,...,n)$ can be expressed as

$$\prod_{i=1}^n f(x_i;\theta)=\left(\frac{\theta}{2}\right)^n\prod_{i=1}^n(1+\theta x_i)e^{-\theta\sum_{i=1}^n x_i}$$

In this test we have consider the null hypothesis $H_0:\theta=\theta_0$ against the alternative hypothesis $H_1:\theta=\theta_1$. In SPRT, we collect observations one at a time. When observation $X_i=x_i$ has been made, we have to decide between the following options:

i Accept the null hypothesis and stop observation.

ii Reject the null hypothesis and stop observation.

Defer decision until we have collected another piece of information X_{i+1}.

We have

$$L_m=\frac{\prod_{i=1}^n f(x_i;\theta_1)}{\prod_{i=1}^n f(x_i;\theta_0)}=\left(\frac{\theta_1}{\theta_0}\right)^n\frac{\prod_{i=1}^n(1+\theta_1x)}{\prod_{i=1}^n(1+\theta_0x)}e^{(\theta_0-\theta_1)\sum_{i=1}^n x_i}$$

We have to reject H_0 if

$$L_m=\left(\frac{\theta_1}{\theta_0}\right)^n\frac{\prod_{i=1}^n(1+\theta_1x)}{\prod_{i=1}^n(1+\theta_0x)}e^{(\theta_0-\theta_1)\sum_{i=1}^n x_i}\geq A$$

That is, s

We have to accept H_0 if

$$L_m=\left(\frac{\theta_1}{\theta_0}\right)^n\frac{\prod_{i=1}^n(1+\theta_1x)}{\prod_{i=1}^n(1+\theta_0x)}e^{(\theta_0-\theta_1)\sum_{i=1}^n x_i}\leq B$$

That is,

$$\log L_m=n\left[\log\theta_1-\log\theta_0\right]+\sum_{i=1}^n\left[\log(1+\theta_1x_i)-\log(1+\theta_0x_i)-(\theta_1-\theta_0)x_i\right]\leq\log B$$

Here the constants A and B are determined based on probability of type I error (α) and probability of type II error (β) and their approximate values are given by $A=\frac{1-\beta}{\alpha}$ and $B=\frac{\beta}{1-\alpha}$.

Operating characteristic function

Definition 1: The operating characteristic (OC) function gives the probability of accepting the null hypothesis H_0 when the true parameter value is θ and can be computed using the following formula

$$L(\theta)=\lambda=P_\theta(\text{accept } H_0)=\frac{A^h-1}{A^h-B^h}$$

Where is the non-zero solution of the equation?

$$E\left(e^{Z_i}\right)=\int_0^\infty\left[\frac{f(x_i,\theta_1)}{f(x_i,\theta_0)}\right]^h f(x,\theta)dx=1$$

$$Z_i=\log\frac{f(x,\theta_1)}{f(x,\theta_0)}$$

Approximation for the average sample number

Since, the sequential probability ratio test is sequential in nature, it is of interest to determine the average number of observations we might need to collect in order to reach a decision.

Definition 2: The average sample number (ASN) is the average

number of observations that we would have to collect in order to make a decision regarding the statistical hypothesis put forth and can be computed with the formula

$$E(N)=\frac{\log B\,L(\theta)+\log A\left[1-L(\theta)\right]}{E(Z)}$$

$$E(Z)=\log\theta_1-\log\theta_0+E\left[\log\left(\frac{1+\theta_1 x}{1+\theta_0 x}\right)\right]-(\theta_1-\theta_0)E(x),$$

The ASN under the null hypothesis H_0 and alternative hypothesis H_1 are approximately obtained as

$$E(N\mid H_0)\approx\frac{|\log B|}{|E_{\theta_0}(Z)|}\quad\text{and}\quad E(N\mid H_1)\approx\frac{\log A}{|E_{\theta_1}(Z)|}$$

Simulation Results for ASN

A simulation study has been conducted to compare the ASN for different hypothesis and various level of type I error α and type II error β. The ASN obtained using the acceptance-rejection method of simulation, based on 5000 independent replicates in each scenario. Here $E(N)$ denotes the theoretical approximate ASN required to make a decision and $\overline{n}$ represents the average sample number observed in the simulation along with their respective standard error (SE). The values of $E(N)$ and $\overline{n}$ under the null hypothesis H_0 and alternative hypothesis H_1 and for different combination α and β are presented in the Table 2&3 respectively.

Table 2 ASN (theoretical and simulated) under $H_0:\theta=0.02$ vs $H_1:\theta=0.05$ and $H_0:\theta=0.1$ vs $H_1:\theta=0.5$ respectively

α or β	Under H_0			Under H_1			Under H_0			Under H_1		
	$E(N)$	$\overline{n}$	SE	$E(N)$	$\overline{n}$	SE	$E(N)$	$\overline{n}$	SE	$E(N)$	$\overline{n}$	SE
0.01	7.47	9.10	0.23	10.59	11.46	0.14	1.52	3.04	0.07	4.44	5.28	0.05
0.05	4.79	6.31	0.16	6.79	7.50	0.10	0.97	2.35	0.05	2.85	3.52	0.04
0.10	3.57	4.82	0.12	5.06	5.56	0.09	0.73	2.02	0.04	2.12	2.72	0.03
0.20	2.25	3.07	0.07	3.20	3.59	0.06	0.46	1.56	0.03	1.34	1.82	0.03

Table 3 ASN (theoretical and simulated) under $H_0:\theta=1$ vs $H_1:\theta=2$ and $H_0:\theta=3$ vs $H_1:\theta=1$ respectively

α or β	Under H_0			Under H_1			Under H_0			Under H_1		
	$E(N)$	$\overline{n}$	SE	$E(N)$	$\overline{n}$	SE	$E(N)$	$\overline{n}$	SE	$E(N)$	$\overline{n}$	SE
0.01	13.55	17.06	0.39	16.82	17.67	0.23	7.86	8.76	0.1	4.47	6.1	0.14
0.05	8.68	11.28	0.29	10.78	11.11	0.16	5.04	5.77	0.08	2.86	4.62	0.11
0.10	6.48	8.01	0.20	8.04	8.52	0.15	3.76	4.61	0.07	2.14	3.41	0.08
0.20	4.09	4.73	0.12	5.07	5.17	0.09	2.37	2.91	0.04	1.35	2.44	0.05

From table 2 and 3, we have the following conclusions:

i The results confirms that the simulated ASN, closely match with the theoretical counterparts with very small SEs confirming stability.

ASN decreases with increasing α and β

ASN is smaller when the difference between null and alternative hypotheses is larger, highlighting the efficiency of the test procedure.

Therefore, the proposed SPRT of NXLD is efficient, reliable and consistent across different parameter settings, making it suitable for practical applications.

Estimation of the parameter

Let $(x_1,x_2,\ldots,x_n)$ be a random sample from NXLD(θ) and $x_{(1)}<x_{(2)}<\ldots<x_{(n)}$ be their corresponding order statistics.

Method of moments

The parameter is estimated by the method of moments, first equating the theoretical moments to the sample moments and solving the equation provides the required estimate of the parameter.

$$\mu_1'=\overline{x}\Rightarrow\frac{3}{2\theta}=\overline{x}\Rightarrow\hat{\theta}=\frac{3}{2\overline{x}}$$

Maximum likelihood estimation

The log-likelihood function of NXLD can be obtained as

$$\log L=n\log\theta-n\log 2+\sum_{i=1}^{n}\log(1+\theta x_i)-n\theta\overline{x}.$$

Citation: Prodhani HR, Shanker R. On statistical properties, estimation of parameter and applications of new X-Lindley distribution. *Biom Biostat Int J.* 2025;14(2):96–110. DOI: 10.15406/bbij.2025.14.00439

Now, $\frac{d}{d\theta}\log L = \frac{n}{\theta} - \sum_{i=1}^{n}\frac{x_i}{(1+\theta x_i)} - n\bar{x}$

The maximum likelihood estimates (MLE), $\hat{\theta}$ of θ is the solution of the equation $\frac{\partial \log L}{\partial \theta} = 0$ and is given by

$$\frac{\partial \log L}{\partial \theta} = \frac{n}{\theta} + \sum_{i=1}^{n}\frac{x_i}{(1+\theta x_i)} - n\bar{x} = 0$$

This equation is not in the closed form, so the parameter is estimated using the stats 4 package available in R software.

$$\frac{\partial^2 \log L}{\partial \theta^2} = -\frac{n}{\theta^2} - \sum_{i=1}^{n}\frac{x_i^2}{(1+\theta x_i)^2}$$

The fisher information matrix can be obtained as follows:

$$I(\theta) = -E\left(\frac{d^2}{d\theta^2}\log L\right) = \frac{n}{\theta^2} + \sum_{i=1}^{n} E\left[\frac{x_i^2}{(1+\theta x_i)^2}\right]$$

Theorem 8: MLE estimate of NXLD is consistent i.e. as $n \to \infty$, $P\{|\hat{\theta} - \theta| > \varepsilon\} \to 0$.

Proof: By Chebyshev's inequality, we have

$$P\{|\hat{\theta} - \theta| > \varepsilon\} \le \frac{V(\hat{\theta})}{\varepsilon^2}$$

The asymptotic variance of the MLEs is given by:

$V(\hat{\theta}) = \frac{1}{n} I^{-1}(\hat{\theta})$, as $n \to \infty$, $V(\hat{\theta}) \to 0$

Thus, $P\{|\hat{\theta} - \theta| > \varepsilon\} \to 0$ as $n \to \infty$. Hence, $\hat{\theta} \xrightarrow{P} \theta$

Maximum product spacing estimation

The maximum product spacing estimate (MPSE) $\hat{\theta}$ of θ can be obtained numerically by maximizing the following function with respect to θ.

$$MPS = \frac{1}{n+1}\sum_{i=1}^{n+1}\log\left[F(x_{(i)},\theta) - F(x_{(i-1)},\theta)\right]$$

Least squares estimation

The least squares estimate (LSE) $\hat{\theta}$ of θ can be obtained numerically by minimizing the following function with respect to θ.

$$LSE = \sum_{i}^{n}\left(F(x_{(i)},\theta) - \frac{i}{n+1}\right)^2$$

Weighted least squares estimate

The weighted least squares estimate (WLSE) $\hat{\theta}$ of θ can be obtained numerically by minimizing the following function with respect to θ.

$$WLSE = \sum_{i}^{n} w_i\left(F(x_{(i)},\theta) - \frac{i}{n+1}\right)^2, \text{ where}$$

$$w_i = \frac{(n+1)^2(n+2)}{i(n-i+1)}$$

Cramer-von mises estimation

The Cramer-Von Mises estimate (CVME) $\hat{\theta}$ of θ can be obtained numerically by minimizing the following function with respect to θ.

$$CVM = \frac{1}{12n} + \sum_{i=1}^{n}\left(F(x_{(i)},\theta) - \frac{2i-1}{2n}\right)^2$$

Anderson-darling estimation

The Anderson-Darling estimates (ADE) of $\hat{\theta}$ of θ can be obtained numerically by minimizing the following function with respect to θ.

$$AD = -n - \sum_{i=1}^{n}\frac{2i-1}{n}\left[\log F(x_{(i)}) + \log\left\{1 - F(x_{(n+1-i)})\right\}\right]$$

Simulation study

To assess the consistency of the estimates of the parameter obtained by different methods of estimation for NXLD, a simulation study has been conducted. The investigation involved examining biases, mean square errors (MSEs) of different estimates including MLE, MPSE, LSE, WLSE and CVME for NXLD discussed in table 4 using the formulae

$$Bias = \frac{1}{n}\sum_{i=1}^{n}(\hat{\eta} - \eta),\ \text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(\hat{\eta} - \eta)^2$$

The acceptance-rejection method of simulation has been used to generate random sample from the NXLD. The biases and MSEs of the MLE, MPSE, LSE, WLSE, CVME and ADE of the parameter of NXLD are decreasing for increasing sample size as shown in table 4. This supports the first order asymptotic theory of MLE. From the table 4, it is observed that MPSE and ADE consistently exhibit the least bias and MSE across all sample sizes and all values of θ. This indicate that the estimate provided by MPSE and ADE are precise and accurate, but in case of very small sample size ($n = 20, 40, 60$) MPSE provides a better estimate as compared to the ADE; otherwise both are performing equally well.

MLE performs equally well, especially when sample sizes are large (e.g. $n = 200, 300$). The bias and MSE for MLE decreases as the sample size increases, which is expected because MLE is asymptotically efficient, that is, it becomes better with large data. LSE, WLSE and CVME have comparatively higher bias and MSE in small sample size. This indicates that these methods are less effective especially for the limited sample size. As the sample size increases, all the methods tend to perform better but the improvement is most significant for MPSE, ADE and MLE. Finally, we conclude that MPSE provides best estimate in general due to consistently least bias and MSE across the scenarios. ADE comes in second and MLE comes in

third, particularly when the sample sizes are increasing. MPSE, ADE and MLE are the most popular techniques for estimating parameter in NXLD because other techniques are relatively less efficient.

Table 4 Descriptive statistics of NXLD

Parameter	Sample size	MLE		MPS		LSE		WLSE		CME		ADE	
		Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
	20	0.054	0.01	0.045	0.01	0.062	0.0115	0.058	0.011	0.06	0.011	0.05	0.0095
	40	0.032	0.006	0.028	0	0.038	0.0063	0.035	0.006	0.04	0.006	0.03	0.0052
	60	0.022	0.004	0.019	0	0.026	0.0042	0.024	0.004	0.03	0.004	0.02	0.0035
$\theta = 0.5$	80	0.017	0.003	0.015	0	0.02	0.0032	0.018	0.003	0.02	0.003	0.02	0.0027
	100	0.014	0.002	0.012	0	0.016	0.0025	0.015	0.002	0.02	0.002	0.01	0.0021
	200	0.007	0.001	0.006	0	0.008	0.0013	0.007	0.001	0.01	0.001	0.01	0.0011
	300	0.0030	0.0008	0.0030	0.0007	0.0040	0.0009	0.0040	0.0008	0.0040	0.0008	0.0030	0.0007
	20	0.075	0.02	0.063	0.02	0.086	0.0221	0.081	0.021	0.08	0.022	0.07	0.0184
	40	0.045	0.011	0.039	0.01	0.053	0.0122	0.049	0.012	0.05	0.012	0.04	0.0101
	60	0.031	0.007	0.027	0.01	0.036	0.0083	0.033	0.008	0.04	0.008	0.03	0.0069
$\theta = 0.7$	80	0.024	0.006	0.021	0.01	0.028	0.0063	0.026	0.006	0.03	0.006	0.02	0.0052
	100	0.019	0.005	0.017	0	0.022	0.005	0.02	0.005	0.02	0.005	0.02	0.0042
	200	0.009	0.002	0.008	0	0.011	0.0026	0.01	0.002	0.01	0.003	0.01	0.0021
	300	0.006	0.002	0.005	0	0.007	0.0017	0.006	0.002	0.01	0.002	0.01	0.0014
	20	0.108	0.041	0.09	0.04	0.123	0.0453	0.115	0.043	0.12	0.044	0.1	0.0378
	40	0.064	0.022	0.056	0.02	0.075	0.025	0.07	0.024	0.07	0.024	0.06	0.0207
$\theta = 1.0$	60	0.044	0.015	0.038	0.01	0.051	0.0169	0.047	0.016	0.05	0.016	0.04	0.0141
	80	0.034	0.011	0.03	0.01	0.039	0.0128	0.036	0.012	0.04	0.012	0.03	0.0106
	100	0.027	0.009	0.024	0.01	0.031	0.0102	0.029	0.01	0.03	0.01	0.03	0.0084
	200	0.013	0.005	0.012	0	0.015	0.0051	0.014	0.005	0.02	0.005	0.01	0.0043
	300	0.009	0.003	0.008	0	0.01	0.0034	0.009	0.003	0.01	0.003	0.01	0.0028
	20	0.162	0.091	0.135	0.08	0.185	0.1019	0.173	0.096	0.18	0.099	0.14	0.085
	40	0.096	0.05	0.084	0.04	0.112	0.0563	0.105	0.053	0.11	0.055	0.09	0.0468
	60	0.066	0.034	0.057	0.03	0.076	0.0381	0.071	0.036	0.07	0.037	0.06	0.0317
$\theta = 1.5$	80	0.051	0.026	0.044	0.02	0.059	0.0288	0.055	0.027	0.06	0.028	0.05	0.024
	100	0.041	0.021	0.036	0.02	0.047	0.0229	0.043	0.022	0.05	0.022	0.04	0.0191
	200	0.02	0.01	0.018	0.01	0.023	0.0115	0.022	0.011	0.02	0.011	0.02	0.0096
	300	0.013	0.007	0.012	0.01	0.015	0.0076	0.014	0.007	0.02	0.007	0.01	0.0064
	20	0.216	0.162	0.18	0.14	0.247	0.1812	0.23	0.171	0.24	0.176	0.19	0.1512
	40	0.128	0.089	0.112	0.08	0.149	0.1	0.139	0.095	0.15	0.097	0.12	0.0834
	60	0.088	0.061	0.076	0.05	0.101	0.0678	0.095	0.064	0.1	0.066	0.08	0.0564
$\theta = 2.0$	80	0.068	0.046	0.059	0.04	0.078	0.0512	0.073	0.048	0.08	0.05	0.06	0.0426
	100	0.054	0.037	0.048	0.03	0.062	0.0409	0.058	0.039	0.06	0.04	0.05	0.034
	200	0.027	0.018	0.024	0.02	0.031	0.0205	0.029	0.019	0.03	0.02	0.03	0.017
	300	0.018	0.012	0.016	0.01	0.021	0.0136	0.019	0.013	0.02	0.013	0.02	0.0113

Applications

The applications and goodness of fit of NXLD have been discussed with two real lifetime datasets relating to failure time data from engineering. The total time on test (TTT) plot, box plot and density plot of dataset 1 and 2 are presented in the figure 7. The descriptive summary of dataset 1 and 2 are presented in the table 5. The dataset 1 and 2 are as follows

Table 5 Summary of the datasets 1 and 2

Datasets	Minimum	1st Quartile	Median	Mean	Variance	3rd Quartile	Maximum
1	0.1	13.5	48.5	45.67	1078.161	81.25	86
2	0.39	4.688	8.315	10.498	78.101	12.47	36.63

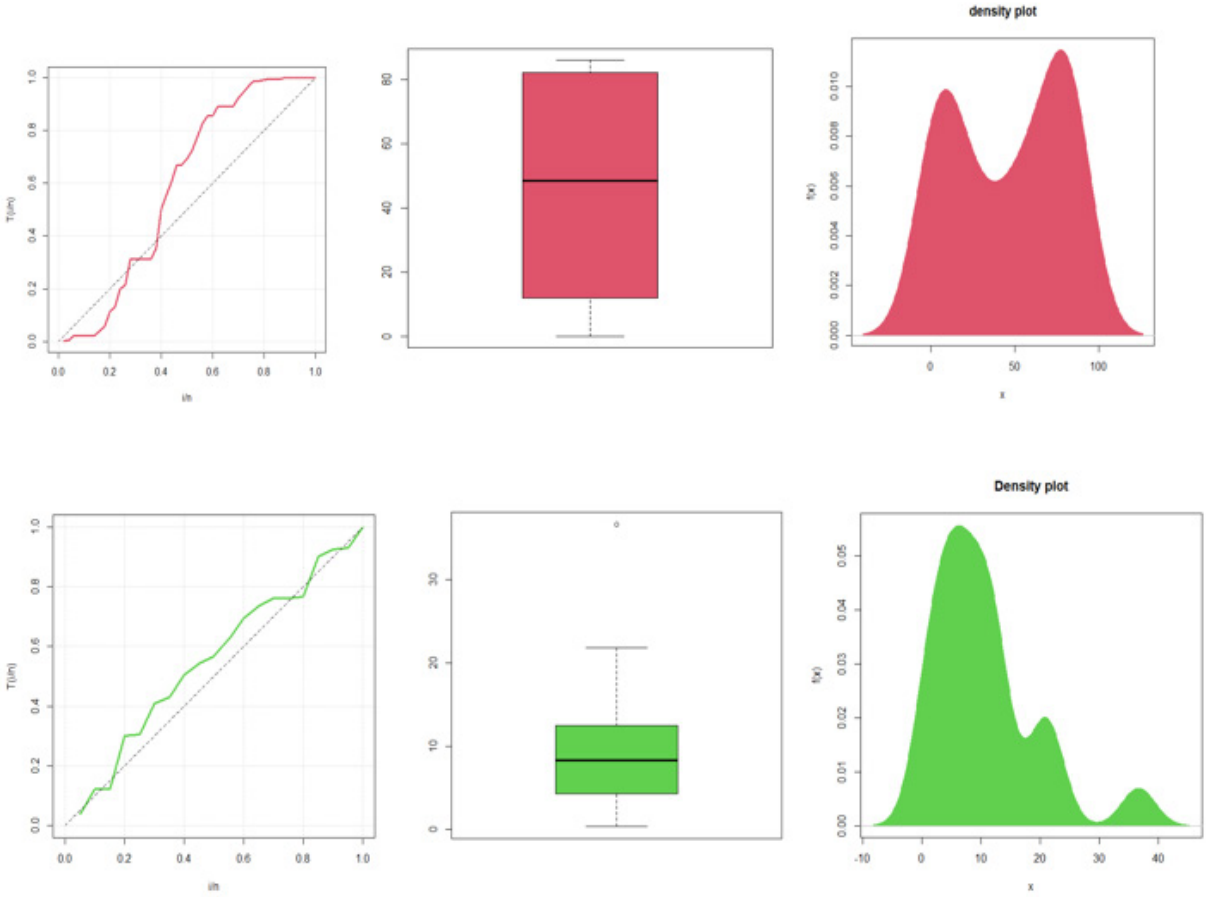

Figure 7 TTT-plot, box-plot and density plot of the dataset 1 and 2.

Dataset 1: The following bathtub shaped data presents a bat-tub shaped failure rate, discussed by Aarset,[22] consists of times to first failure of fifty devices and the values are:

0.1, 0.2, 1.0, 1.0, 1.0, 1.0, 1.0, 2.0, 3.0, 6.0, 7.0, 11.0, 12.0, 18.0, 18.0, 18.0, 18.0, 18.0, 21.0,32.0, 36.0, 40.0, 45.0, 45.0, 47.0, 50.0, 55.0, 60.0, 63.0, 63.0, 67.0, 67.0, 67.0, 67.0, 72.0,75.0, 79.0, 82.0, 82.0, 83.0, 84.0, 84.0, 84.0, 85.0, 85.0, 85.0, 85.0, 85.0, 86.0, 86.0.

Dataset 2: The following skewed to right, a complete set of data, discussed by Murthy et al.,[23] reports the failure times of 20 electric bulbs and the observations are:

1.32, 12.37, 6.56, 5.05, 11.58, 10.56, 21.82, 3.60, 1.33, 12.62, 5.36, 7.71, 3.53, 19.61, 36.63, 0.39, 21.35, 7.22, 12.42, 8.92.

From the Table 5, it is observed that variance is greater than mean for both the datasets that is both datasets are over-dispersed. For dataset 1, the median is greater than mean and for dataset 2, median is less than mean that is dataset 1 is negatively skewed and dataset 2 is positively skewed. From the figure 7, the TTT plot indicate that dataset one has bathtub shape hazard rate and for dataset 2, hazard rate initially constant then increases and again becomes constant over the time.

In order to compare the lifetime distributions, values of $-2\log L$, Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Consistent Akaike Information Criterion (CAIC), Hannan-Quinn Information Criterion (HQIC), Kolmogorov-Smirnov (K-S) Statistics and the corresponding probability value (p-value) for the above two datasets have been computed using the formulae

$$AIC = -2\log L + 2p \quad BIC = -2\log L + p\log(n)$$
$$CAIC = -2\log L + \frac{2pn}{n-p-1}$$

Citation: Prodhani HR, Shanker R. On statistical properties, estimation of parameter and applications of new X-Lindley distribution. *Biom Biostat Int J.* 2025;14(2):96–110. DOI: 10.15406/bbij.2025.14.00439

$$HQIC = -2\log L + 2p\log\left[\log(n)\right]$$

$$K\text{-}S = \underset{x}{Sup}\,|F_m(x) - F_o(x)|$$

Where, $\mathbf{p}$ = number of parameters, n = sample size, $F_m(x)$ = empirical CDF of considered distribution and $F_o(x)$ = CDF of considered distribution

The estimation of the parameter along with their standard error (SE) and goodness of fit measures of the considered distributions of the dataset 1 and 2 are presented in the table 6 & 8. The estimates values of the parameter, K-S values along with their p-values of NXLD using the different estimation methods are presented in the table 7 & 9. The confidence interval of the estimate values of the parameter for the dataset 1 and 2 are presented in table 10. The fitted plot, quantile-quantile (Q-Q) plot, probability-probability (P-P) plot, empirical cumulative density function (ECDF) plot, profile plot of the parameter are presented in figure 8- 10 respectively (Table 6-10, Figure 7-10).

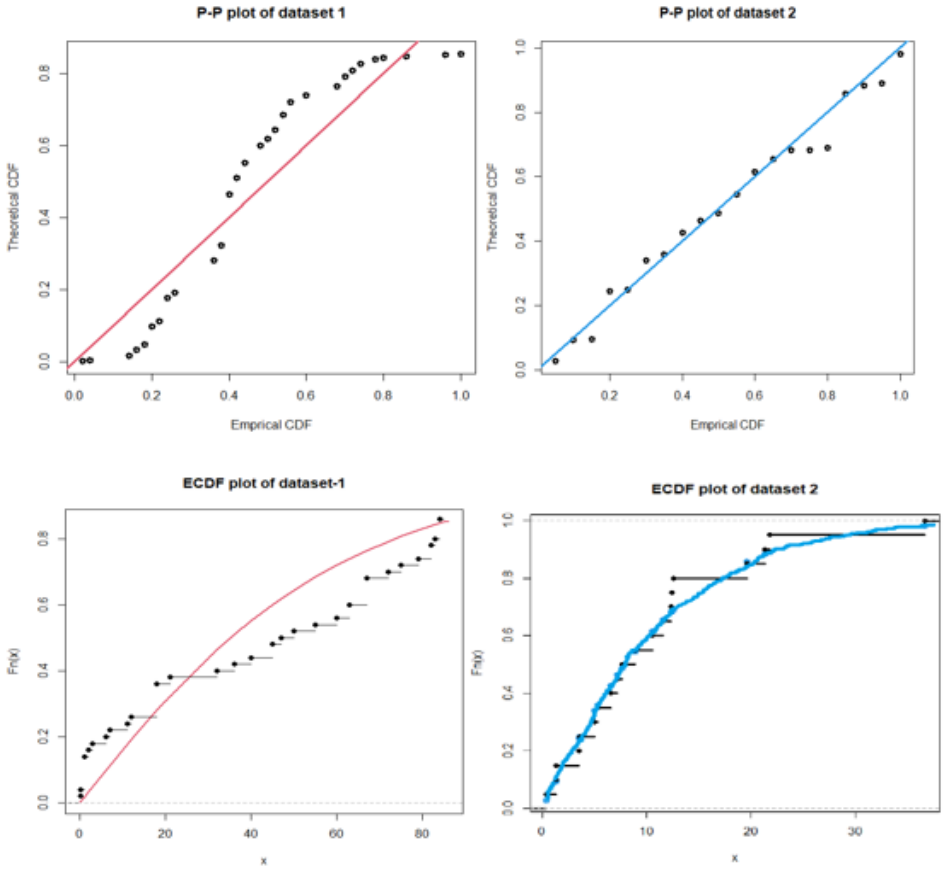

Figure 8 Q-Q plot, P-P plot and ECDF plot of dataset 1 and 2.

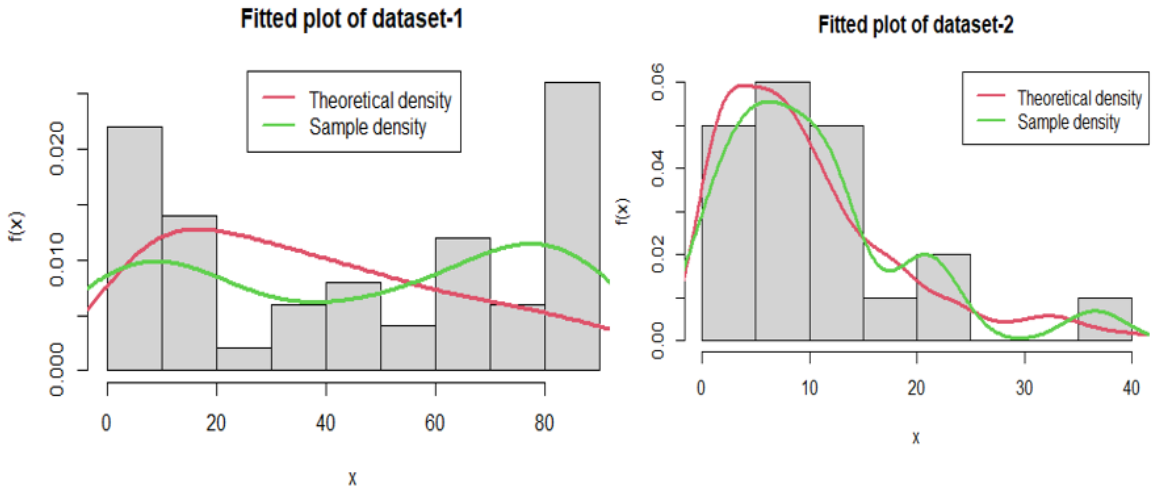

Figure 9 Fitted plot of dataset 1 and 2

Citation: Prodhani HR, Shanker R. On statistical properties, estimation of parameter and applications of new X-Lindley distribution. *Biom Biostat Int J.* 2025;14(2):96–110. DOI: 10.15406/bbij.2025.14.00439

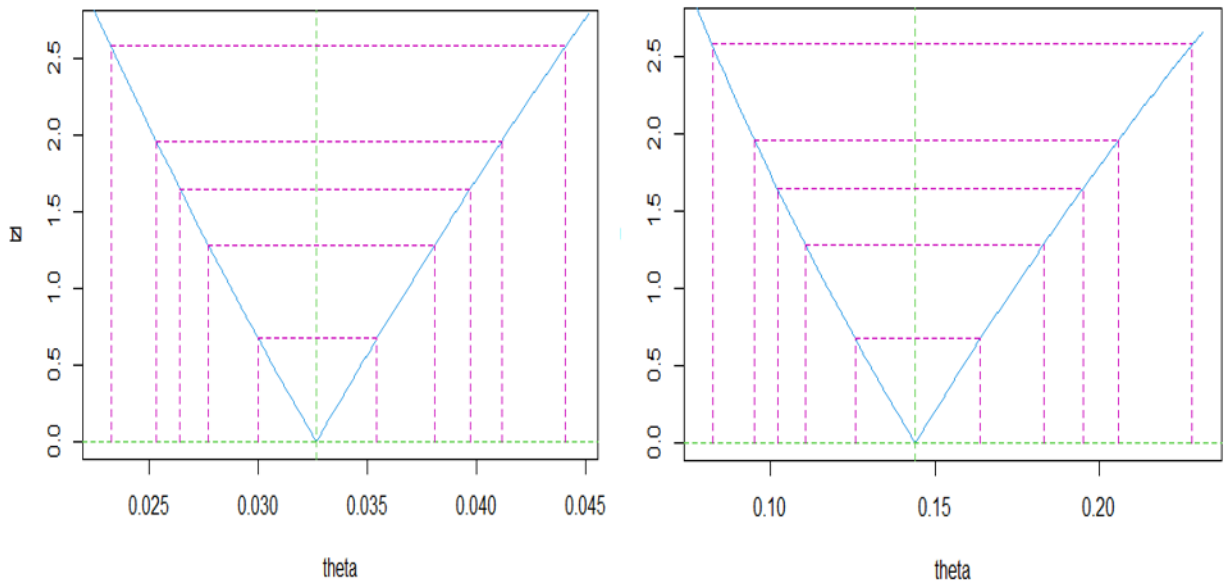

Figure 10 Profile plot of the dataset 1 and 2.

Table 6 Goodness of fit measures of dataset 1

Distributions	$\hat{\theta}$ (SE)	$-2\log L$	AIC	CAIC	BIC	HQIC	K-S	p-value
NXLD	0.0326 (0.0040)	480.37	482.37	482.45	484.28	483.09	0.1623	0.1823
XLD	0.0421 (0.0042)	496.67	498.67	500.58	498.75	499.39	0.2466	0.0871
Lindley	0.0429 (0.0043)	502.81	504.81	504.89	506.72	505.53	0.1944	0.06401
Pratibha	0.0653 (0.0053)	552.67	554.67	554.75	556.58	555.39	0.2284	0.0171
Komal	0.0429 (0.0042)	502.48	504.48	504.56	506.39	505.20	0.2129	0.0320
Garima	0.0325 (0.0040)	480.34	482.34	482.42	484.25	483.06	0.1977	0.0577
Sujatha	0.0649 (0.0052)	542.04	544.04	544.12	x545.95	544.76	0.2612	0.0040
Ravi	0.1532 (0.0081)	751.49	753.49	753.49	755.40	754.21	0.3246	0.0001
Uma	0.0875 (0.0061)	593.63	595.63	595.71	597.54	596.35	0.2820	0.0014
Akash	0.0656 (0.0053)	547.13	549.13	549.21	551.04	549.85	0.2446	0.0086
Shanker	0.0440 (0.0044)	516.23	518.23	518.31	520.14	518.95	0.1939	0.0648
Exponential	0.0218 (0.0031)	482.13	484.13	484.21	486.04	484.85	0.1870	0.07156
Rama	0.0875 (0.0061)	598.89	600.89	600.97	602.80	601.61	0.2574	0.2574
Suja	0.1094 (0.0069)	653.96	655.96	656.04	657.87	656.68	0.2933	0.0008
Amarendra	0.0869 (0.0061)	589.91	591.91	591.99	593.82	592.63	0.2755	0.0019
Devya	0.1088 (0.0068)	642.27	644.27	644.35	646.18	644.99	0.2929	0.0008
Shambhu	0.1307 (0.0075)	1014.89	1016.89	1016.97	1018.80	1017.61	0.3223	0.0001
Aradhana	0.0642 (0.0052)	538.49	540.49	540.57	542.40	541.21	0.2316	0.0153

Table 7 K-S values with respect to different estimation methods for NXLD of dataset 1

	MLE	MPSE	LSE	WLSE	CVME	ADE
$\hat{\theta}$	0.0326 (0.0040)	0.0321 (0.004)	0.0306 (0.0015)	0.033	0.0282 (0.0087)	0.0298 (0.0040)
K-S	0.1623	0.1537	0.1673	0.1578	0.1958	0.171
p-value	0.1823	0.2337	0.1563	0.209	0.0607	0.1404

Citation: Prodhani HR, Shanker R. On statistical properties, estimation of parameter and applications of new X-Lindley distribution. *Biom Biostat Int J.* 2025;14(2):96–110. DOI: 10.15406/bbij.2025.14.00439

Table 8 Goodness of fit measures of dataset 2

Distributions	$\hat{\theta}$ (SE)	$-2\log L$	AIC	CAIC	BIC	HQIC	K-S	P-value
NXLD	0.1440 (0.0281)	133.11	135.11	135.33	136.10	135.30	0.0921	0.9968
XLD	0.1654 (0.0264)	133.01	135.01	136.00	135.23	135.20	0.2653	0.1287
Lindley	0.1762 (0.0280)	133.44	135.44	135.66	136.43	135.63	0.1122	0.9684
Pratibha	0.2745 (0.0352)	139.09	141.09	141.31	142.08	141.28	0.1477	0.7396
Komal	0.1745 (0.0275)	133.33	135.33	135.55	136.32	135.52	0.0992	0.9914
Garima	0.1408 (0.0273)	133.18	135.18	135.40	136.17	135.37	0.1255	0.9218
Sujatha	0.2689 (0.0345)	137.54	139.54	139.76	140.53	139.73	0.1294	0.9037
Ravi	0.6597 (0.0553)	174.29	176.29	176.51	177.28	176.48	0.2734	0.0934
Uma	0.3743 (0.0413)	145.47	147.47	147.69	148.46	147.66	0.1973	0.3926
Akash	0.2786 (0.0355)	138.47	140.47	140.69	141.46	140.66	0.1607	0.6434
Shanker	0.1885 (0.0292)	134.65	136.65	136.87	137.64	136.84	0.1172	0.9539
Exponential	0.0952 (0.0212)	134.04	136.04	136.26	137.03	136.23	0.1255	0.9218
Rama	0.3784 (0.0420)	147.18	149.18	149.40	150.17	149.37	0.2126	0.3049
Suja	0.4754 (0.0474)	157.63	159.63	159.85	160.62	159.82	0.2210	0.2650
Amarendra	0.3653 (0.0405)	144.80	146.80	147.02	147.79	146.99	0.1943	0.4098
Devya	0.4620 (0.0459)	154.05	156.05	156.27	157.04	156.24	0.2100	0.3176
Shambhu	0.5583 (0.0507)	244.03	246.03	246.25	247.02	246.22	0.1914	0.4295
Aradhana	0.2615 (0.0338)	137.00	139.00	139.22	139.99	139.19	0.1288	0.9067

Table 9 K-S values with respect to different estimation methods for NXLD of dataset 2

	MLE	MPSE	LSE	WLSE	CVME	ADE
	0.1440 (0.0281)	0.1338 (0.0264)	0.1375 (0.0165)	0.1407 (0.0061)	0.1093 (0.0643)	0.1120 (0.0234)
K-S	0.0921	0.1093	0.1005	0.0892	0.1025	0.1514
p-value	0.9968	0.956	0.979	0.998	0.9741	0.7131

Table 10 Confidence interval of the estimate parameter of NXLD for datasets 1 and 2

Dataset	90% CI		95% CI		99% CI	
	Lower	Upper	Lower	Upper	Lower	Upper
1	0.0264	0.0397	0.0253	0.0412	0.0233	0.0441
2	0.1022	0.195	0.0951	0.2058	0.0823	0.2281

From the table 6 and 8, it has been observed that NXLD has the least AIC, BIC, CAIC, HQIC and K-S values as compared to X-LD, Lindley distribution Pratibha distribution, Komal distribution, Garima distribution, Sujatha distribution, Ravi distribution, Uma distribution, Akash distribution, Shanker distribution, exponential distribution, Rama distribution, Suja distribution, Amarendra distribution, Devya distribution, Shambhu distribution and Aradhana distribution except in case of dataset 2, the XLD provides negligible lower AIC, BIC, CAIC, HQIC values but has greater K-S values as compared to NXLD. So, NXLD provides a better fit as compared to Pratibha distribution, Komal distribution, Garima distribution, Sujatha distribution, Ravi distribution, Uma distribution, Akash distribution, Shanker distribution, Lindley distribution, exponential distribution, Rama distribution, Suja distribution, Amarendra distribution, Devya distribution, Shambhu distribution and Aradhana distribution. These results are validated graphically in figure 8 and 9. From table 7, for the dataset 1, the MPSE provides the least K-S values as compared to all the estimation methods. From table 9, the WLSE provides the least K-S values as compared to all the estimation methods. Thus, MPSE is better for dataset 1 and WLSE is better for dataset 2 in terms of fit.

Conclusion

Some interesting and useful statistical and reliability properties of NXLD have been studied. , The pdf exhibits is unimodal and decreasing. The coefficient of variation, coefficient of Skewness, coefficient of kurtosis, index of dispersion, mean deviation about mean and median, and Bonferroni and Lorenz curves of NXLD have been discussed. The reliability properties have also been investigated, including the survival function, hazard function, reverse hazard function, mean residual life function, stochastic ordering, and stress-strength reliability. It is shown that the NXLD has an increasing hazard function, while both the reverse hazard function and the mean residual life function are always decreasing in nature.

Furthermore, a sequential probability ratio test (SPRT) has been developed, and the average sample number (ASN) has been derived. A simulation study has been conducted to compare the ASN values under different hypotheses and for different levels of Type I and Type II errors.

The parameters of the NXLD are estimated using different classical methods, including maximum likelihood estimation (MLE), maximum

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product of spacing estimation (MPSE), least squares estimation (LSE), weighted least squares estimation (WLSE), Cramér–von Mises estimation (CME), and Anderson–Darling estimation (ADE). A simulation study has been carried out to examine the consistency of these estimators. As a result, we found that MPSE, ADE and MLE are the most popular techniques for estimating parameter in NXLD because other techniques are relatively less efficient. To assess the goodness-of-fit, the NXLD has been compared with a number of existing lifetime distributions, namely XLD, Lindley, Pratibha, Komal, Garima, Sujatha, Ravi, Uma, Akash, Shanker, Exponential, Rama, Suja, Amarendra, Devya, Shambhu, and Aradhana distributions. The results indicate that the NXLD consistently provides the lowest AIC, BIC, CAIC, HQIC, and K-S statistics, demonstrating a superior fit relative to these models. These findings are further supported by graphical validations using fitted density plots, Q-Q plots, P-P plots, and empirical cumulative distribution function (ECDF) plots. Additionally, 95% confidence intervals for the parameters are obtained and illustrated through profile plots. Thus, the NXLD can be regarded as a flexible distribution capable of effectively handling engineering datasets characterized by positive or negative Skewness, over-dispersion, as well as bathtub shaped and increasing hazard rate behaviors.

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Conflicts of interest

None.

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